

P wave bottomonium spectral functions in the QGP from lattice NRQCD

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Outline

- quarkonia and heavy ion collisions
- bottomonium at finite temperature on the lattice
- bottomonium spectral functions in the QGP
S waves vs P waves
- conclusion

Quarkonia and the QGP

quarkonia as a thermometer for the quark-gluon plasma

Matsui & Satz 86

- tightly bound states of charm quarks ($J/\psi, \dots$) or bottom quarks (Υ, \dots) survive to higher temperatures
- broader states melt at lower temperatures

melting pattern informs about temperature of the QGP

- relevant for heavy-ion collisions
- quantitative predictions required

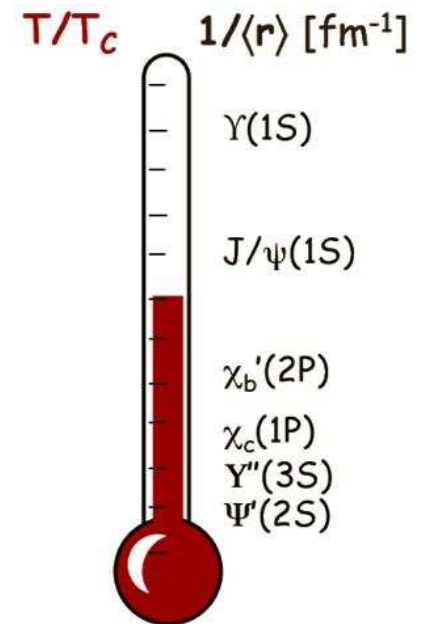


figure by A. Mocsy

Quarkonia and the QGP

how to find the response of quarkonia to the QGP?

- potential models
- lattice QCD

at $T > 0$:

- plethora of potential models: (seemingly) conflicting results
- interpretation of lattice correlators hindered by thermal (periodic) boundary conditions

re-addressed using first-principle approach:

- effective field theories (EFTs) and separation of scales

Laine et al 07-..., Brambilla et al 08-...

Non-Relativistic QCD

this talk:

- use NRQCD, one of the EFTs, nonperturbatively
- no potential model / no weak coupling

lattice QCD:

- heavy quarks with NRQCD

requirement $M \gg T$

bottomonium: $M_b \sim 4.5 \text{ GeV}$ $T \sim 150 - 400 \text{ MeV}$

use of NRQCD very well motivated

FASTSUM collaboration



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+ Alessandro Amato, Wynne Evans, Pietro Giudice, Tim Harris, Aoife Kelly

PRL (2011), JHEP (2011, 2013), in preparation

Two-flavour QCD

- many time slices: highly anisotropic lattices ($a_s/a_\tau = 6$)
- lattice spacing: $a_\tau^{-1} \simeq 7.35$ GeV, $a_s \simeq 0.162$ fm
- lattice size: $12^3 \times N_\tau$

N_τ	80	32	28	24	20	18	16
T/T_c	0.42	1.05	1.20	1.40	1.68	1.86	2.09
N_{cfg}	250	1000	1000	500	1000	1000	1000

- bottom quark: NRQCD

mean-field improved action with tree-level coefficients, including up to $\mathcal{O}(v^4)$ terms

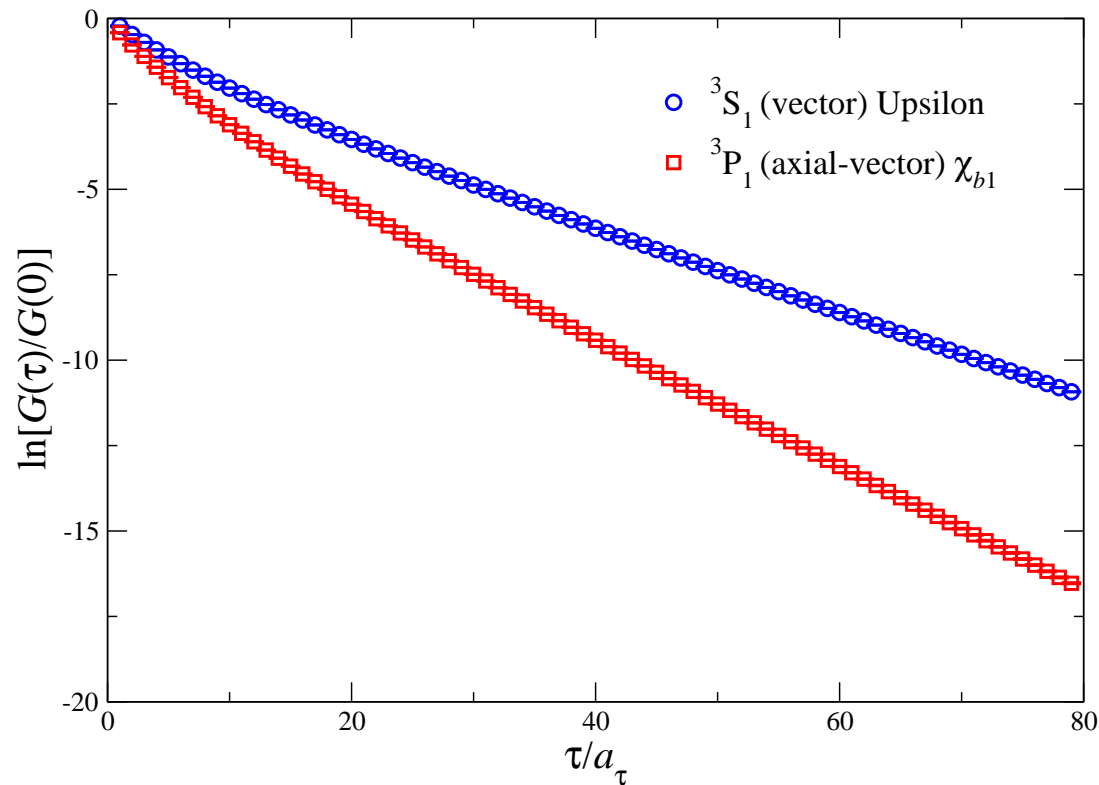
Davies et al 94

- in progress: extension to $N_f = 2 + 1$

see talks by Tim Harris, Alessandro Amato, Chris Allton

Spectrum at zero temperature

- euclidean correlators not periodic:
NRQCD is initial-value problem



Υ (S-wave) and χ_{b1} (P-wave)

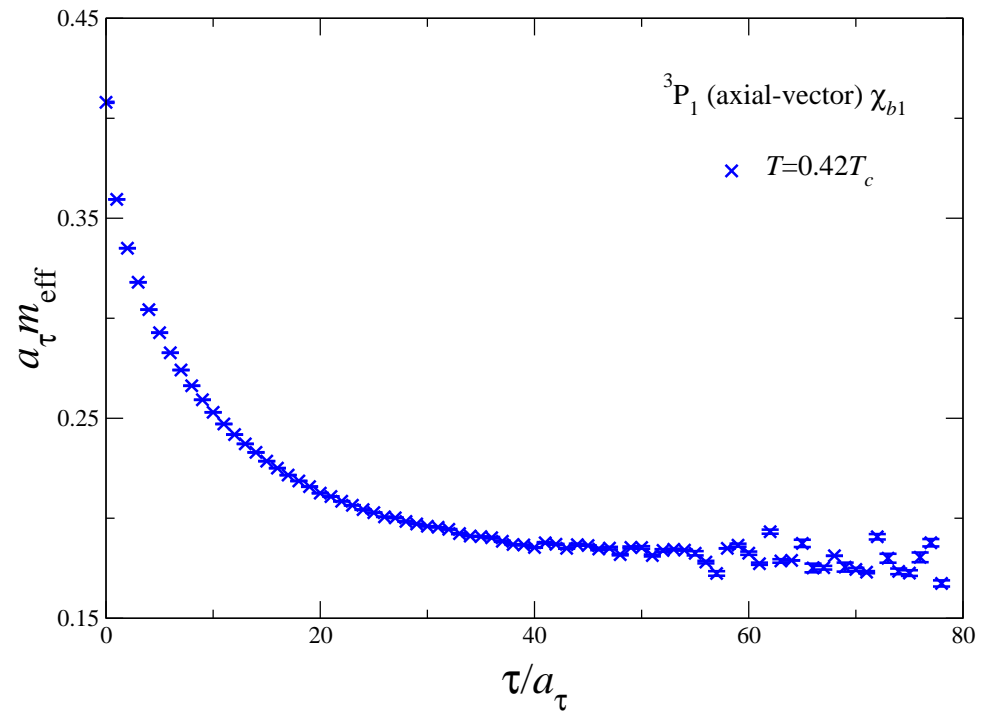
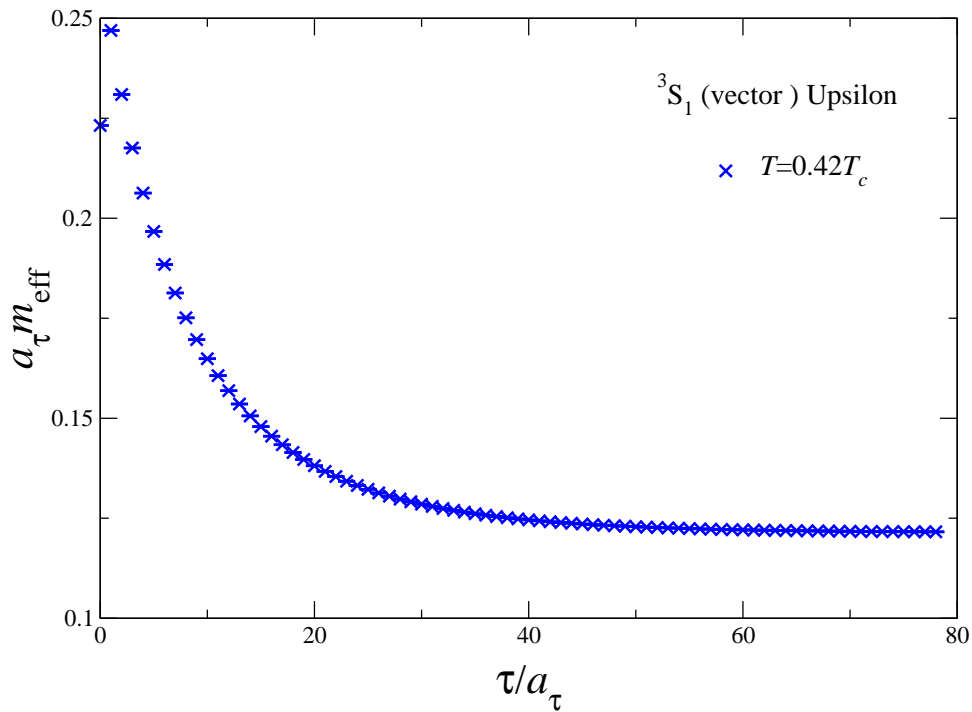
Increasing the temperature

effective mass plot

$$m_{\text{eff}} = -\log [G(\tau)/G(\tau - a_\tau)]$$

Υ S wave

χ_{b1} P wave



$$T/T_c = 0.42$$

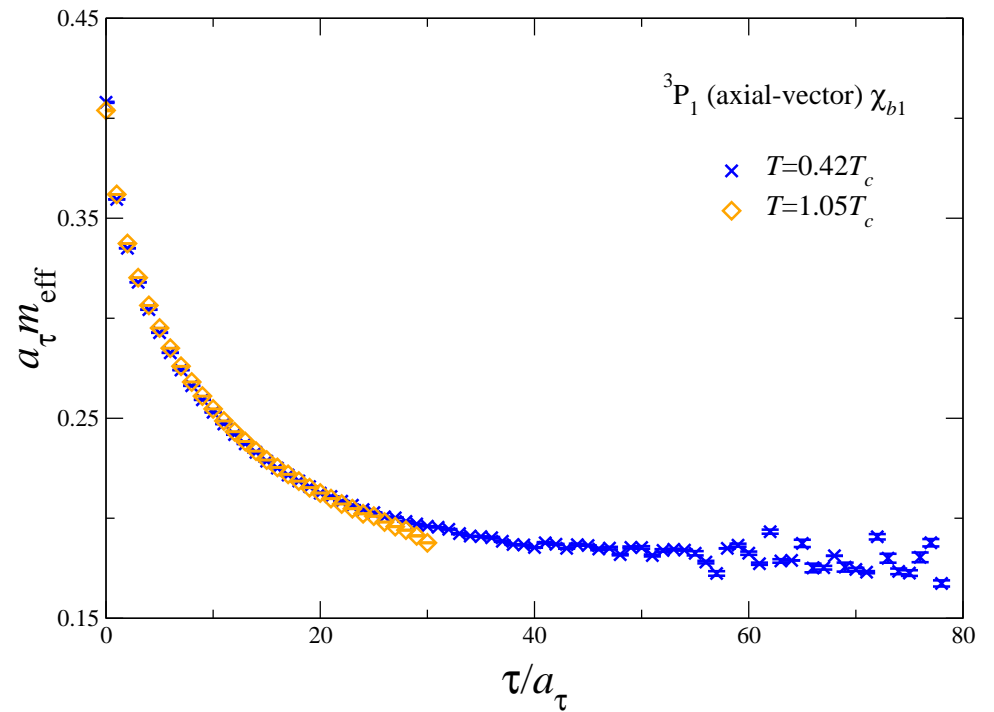
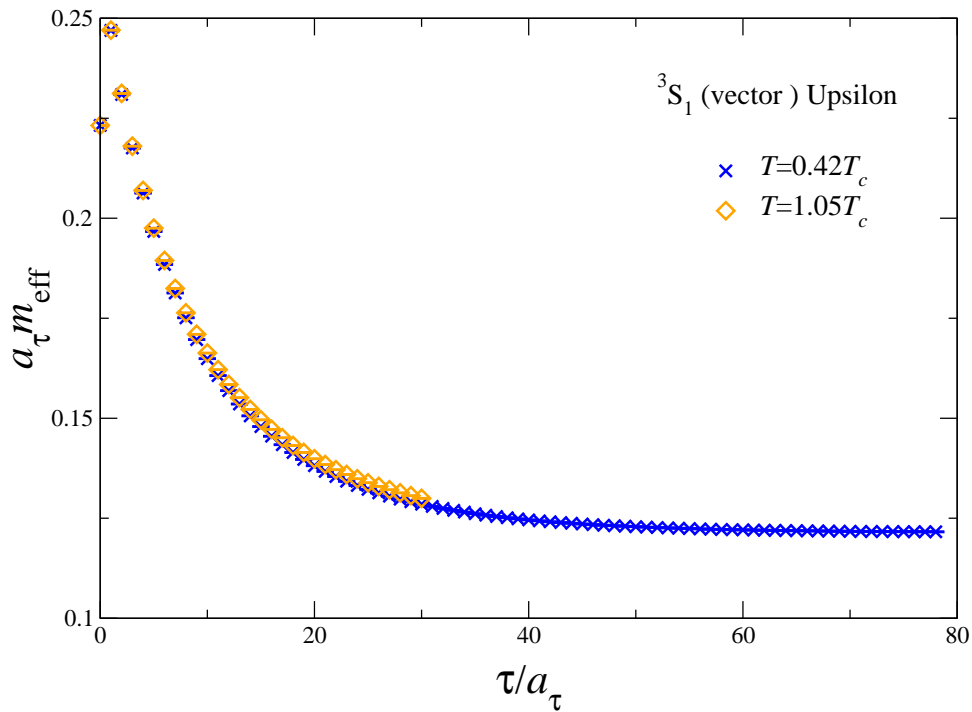
Increasing the temperature

effective mass plot

$$m_{\text{eff}} = -\log [G(\tau)/G(\tau - a_\tau)]$$

Υ S wave

χ_{b1} P wave



$$T/T_c = 1.05$$

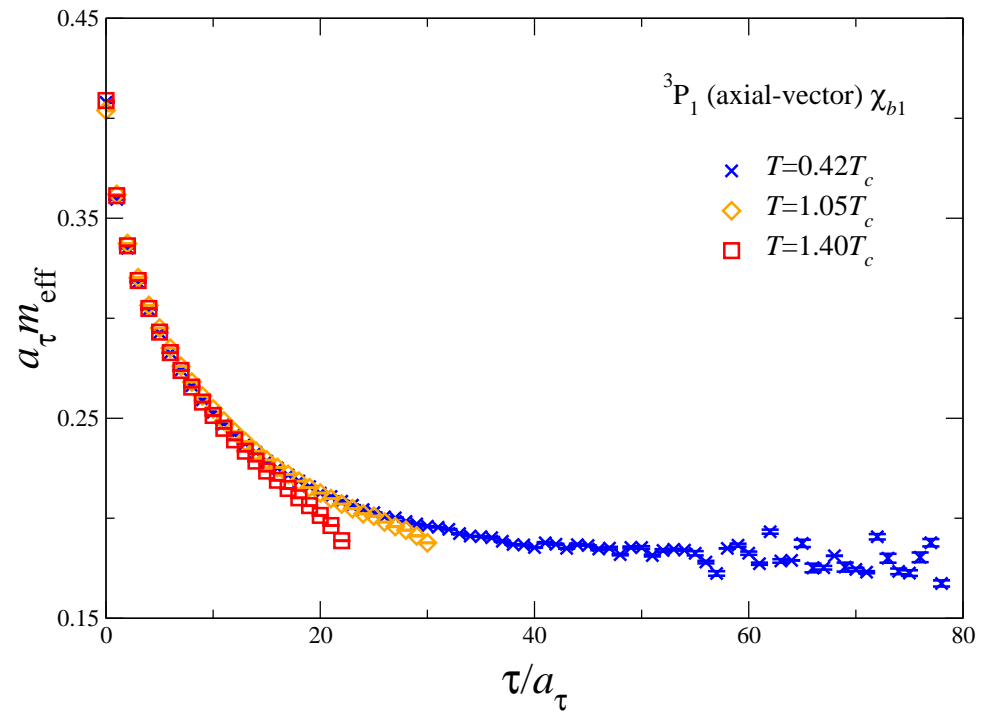
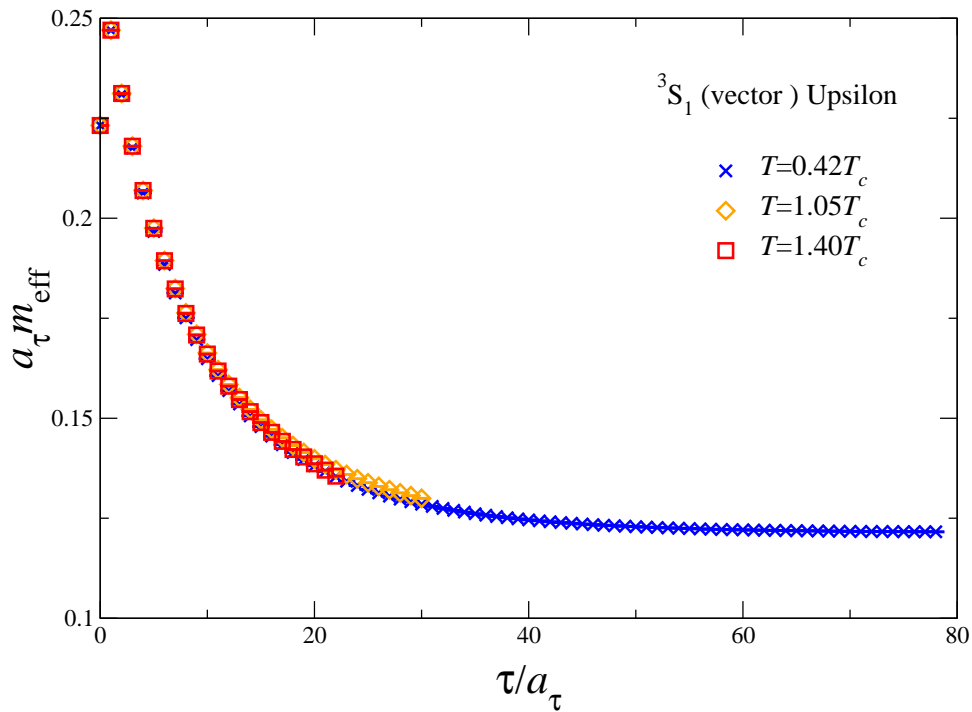
Increasing the temperature

effective mass plot

$$m_{\text{eff}} = -\log [G(\tau)/G(\tau - a_\tau)]$$

Υ S wave

χ_{b1} P wave



$$T/T_c = 1.40$$

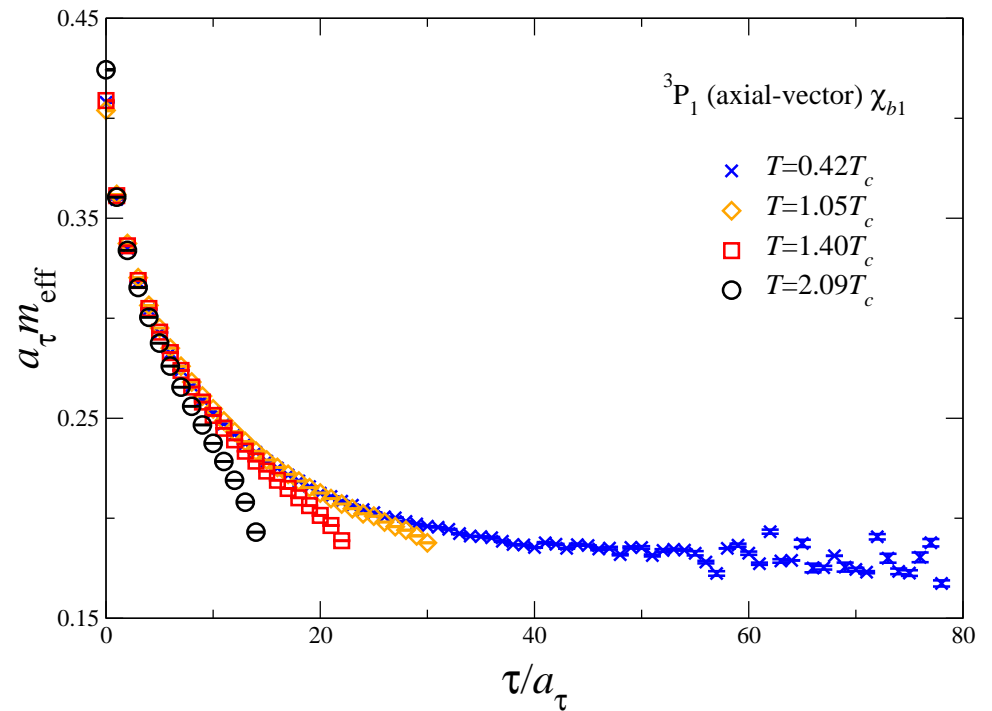
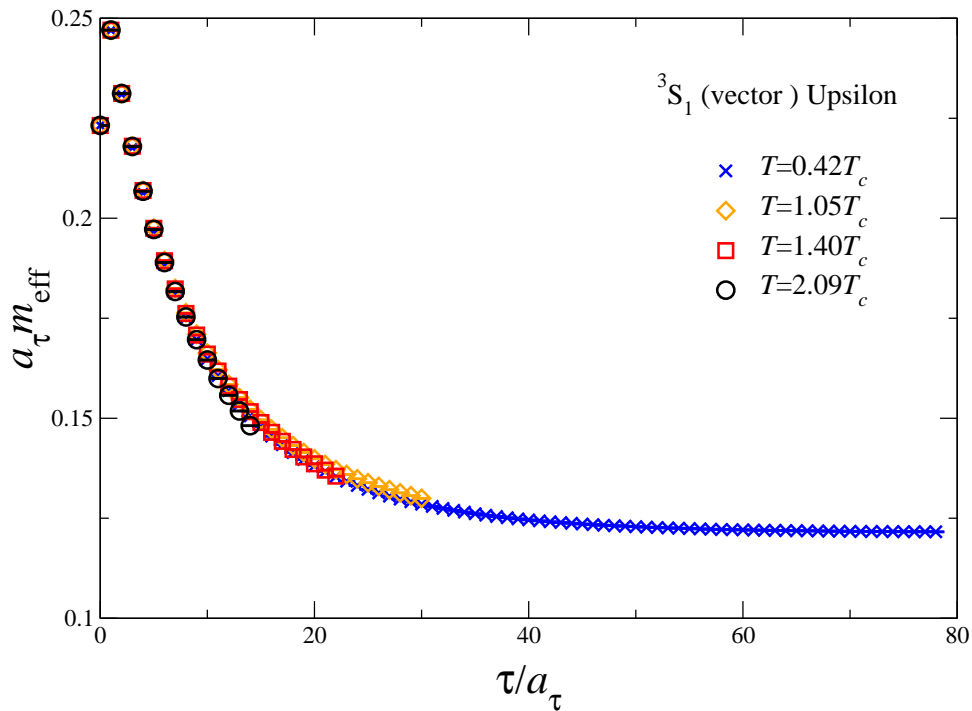
Increasing the temperature

effective mass plot

$$m_{\text{eff}} = -\log [G(\tau)/G(\tau - a_\tau)]$$

Υ S wave

χ_{b1} P wave



$$T/T_c = 2.09$$

little T dependence

substantial T dependence
no exponential decay
melting

Quarkonia at finite temperature

from euclidean correlators to spectral functions

$$G(\tau, \mathbf{p}) = \int d\omega K(\tau, \omega) \rho(\omega, \mathbf{p}) \quad K(\tau, \omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$

- use Maximal Entropy Method (MEM)
- first discussed quite some time ago ...

Asakawa & Hatsuda 1999, 2001

Karsch, Petreczky et al 2002

...

- ... but full of pitfalls and obstacles

Quarkonia at finite temperature

relativistic formulation:

- melting of quarkonia obscured by constant contribution

Umeda 07, Petreczky et al 07-09

NRQCD:

- constant contribution absent
- no thermal boundary condition
- simple spectral relation $G(\tau) = \int d\omega e^{-\omega\tau} \rho(\omega)$

why?

- factor out heavy quark mass scale: $\omega = 2M + \omega'$
- $M \gg T$: thermal effects exponentially suppressed

Quarkonia at finite temperature

- no thermal boundary conditions
- simple spectral relation $G(\tau) = \int d\omega e^{-\omega\tau} \rho(\omega)$

example:

correlators for free quarks with kinetic energy $E_{\mathbf{p}} = \frac{\mathbf{p}^2}{2M}$

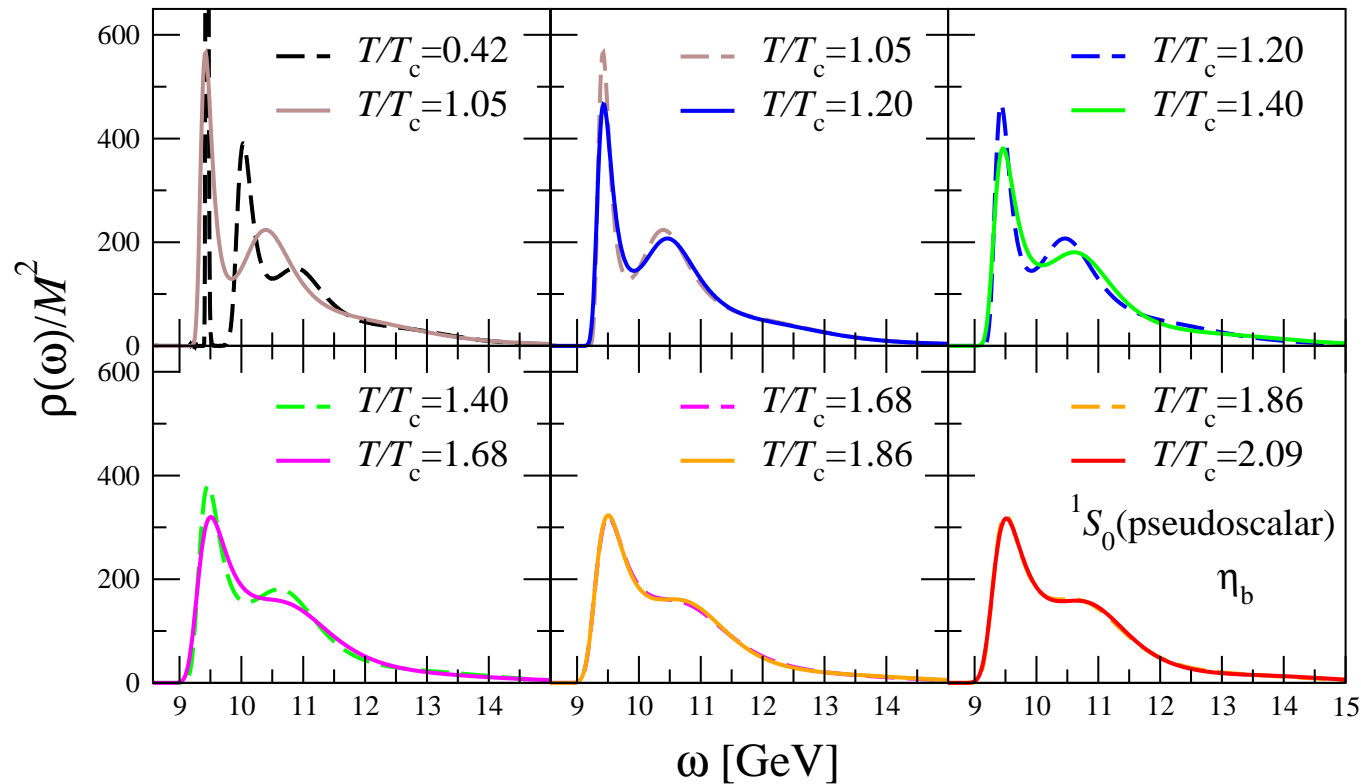
$$\begin{aligned} G_S(\tau) &\sim \int d^3p \exp(-2E_{\mathbf{p}}\tau) & \rho_S(\omega) &\sim \int d^3p \delta(\omega - 2E_{\mathbf{p}}) \\ G_P(\tau) &\sim \int d^3p \mathbf{p}^2 \exp(-2E_{\mathbf{p}}\tau) & \rho_P(\omega) &\sim \int d^3p \mathbf{p}^2 \delta(\omega - 2E_{\mathbf{p}}) \end{aligned}$$

Burnier, Laine & Vepsäläinen 08

- temperature dependence only enters via medium !

S wave at finite temperature

- temperature dependence in η_b channel



- S wave ground state survives – excited states suppressed

P waves at finite temperature

NRQCD:

no exponential decay – what to expect?

- consider free quarks with kinetic energy $E_{\mathbf{p}} = \frac{p^2}{2M}$

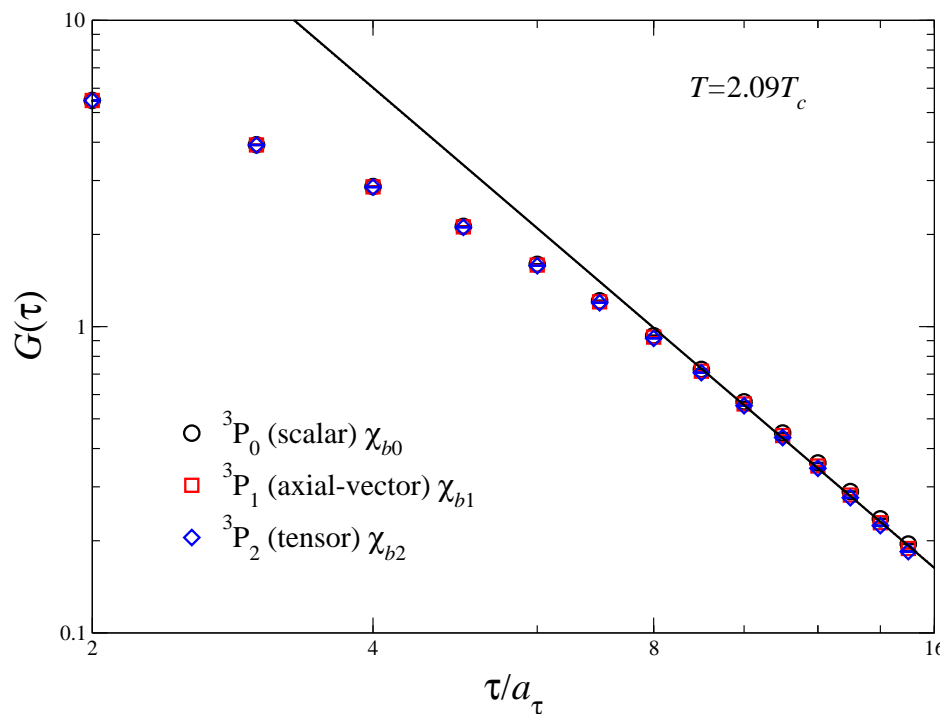
$$G_S(\tau) \sim \int d^3p e^{-2E_{\mathbf{p}}\tau} \sim \frac{1}{\tau^{3/2}}$$

$$G_P(\tau) \sim \int d^3p \mathbf{p}^2 e^{-2E_{\mathbf{p}}\tau} \sim \frac{1}{\tau^{5/2}}$$

- power decay at large euclidean times
(provided that the threshold is at $\omega = 2M$, see Tim's talk)

P waves at finite temperature

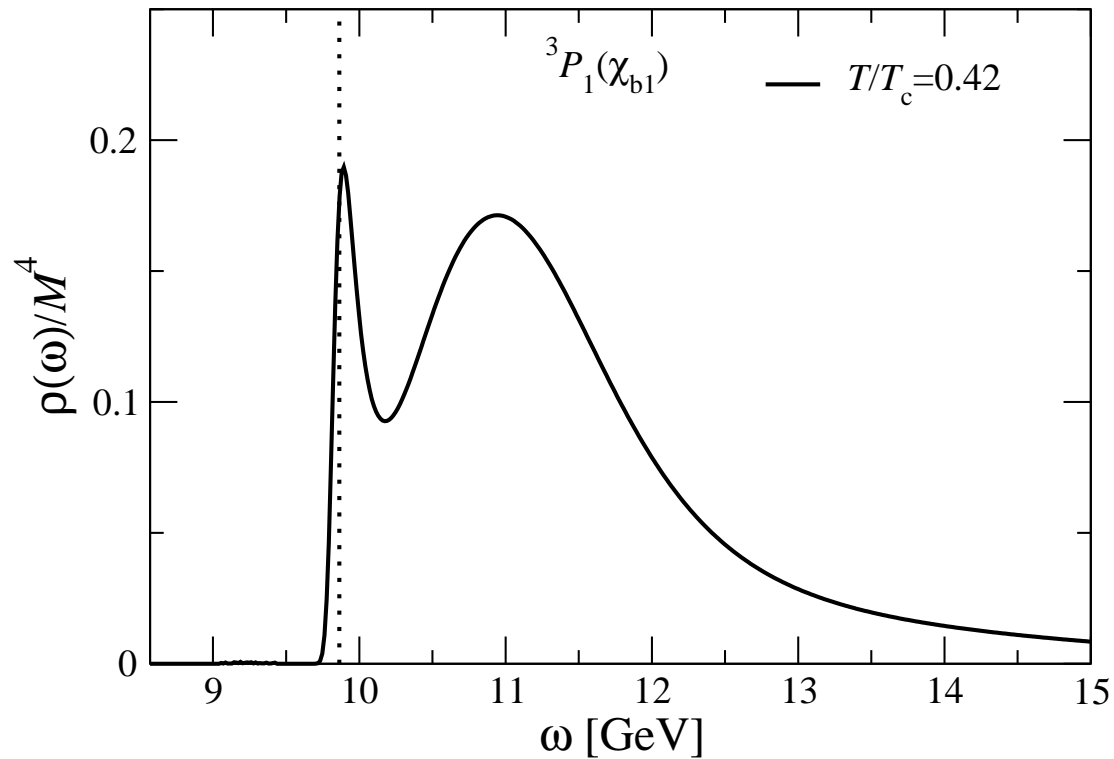
- correlator on log-log scale:
power decay at large euclidean times



- fit: $G(\tau) \sim 1/\tau^\gamma$, $\gamma = 2.605(1)$
without interactions: $\gamma = 5/2$
- spectral analysis?

P waves at finite temperature

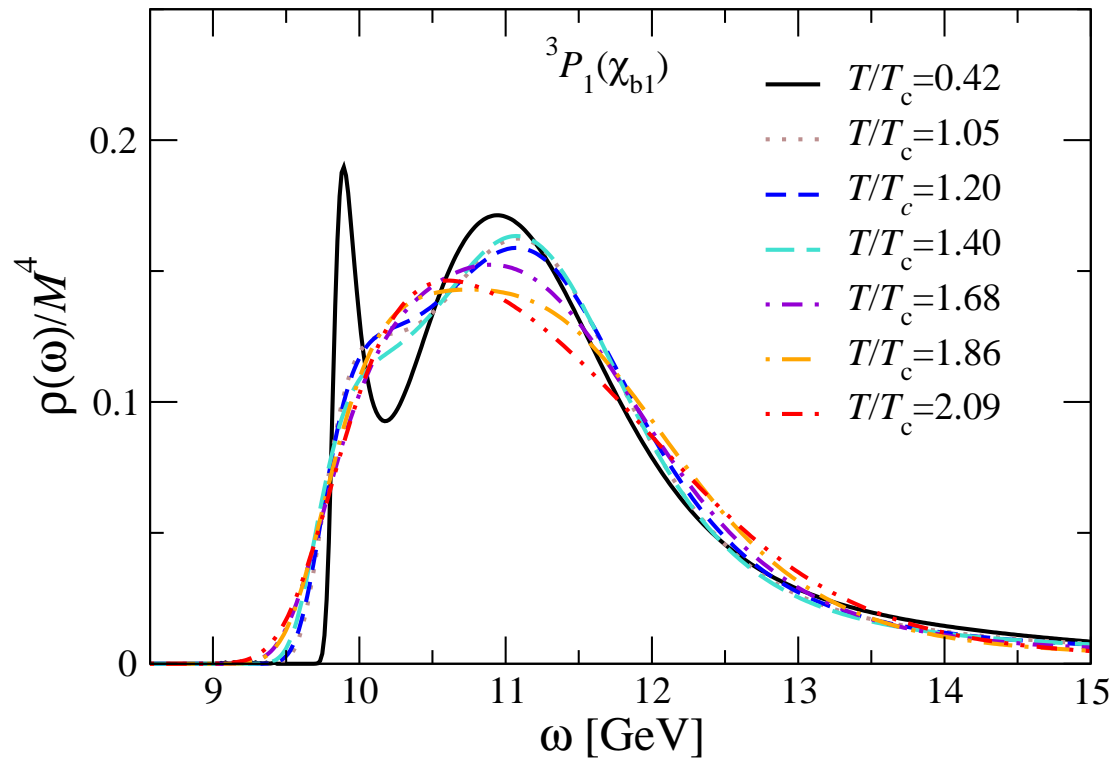
- χ_{b1} axial-vector channel



- groundstate below T_c , agreement with exp. fit

P waves at finite temperature

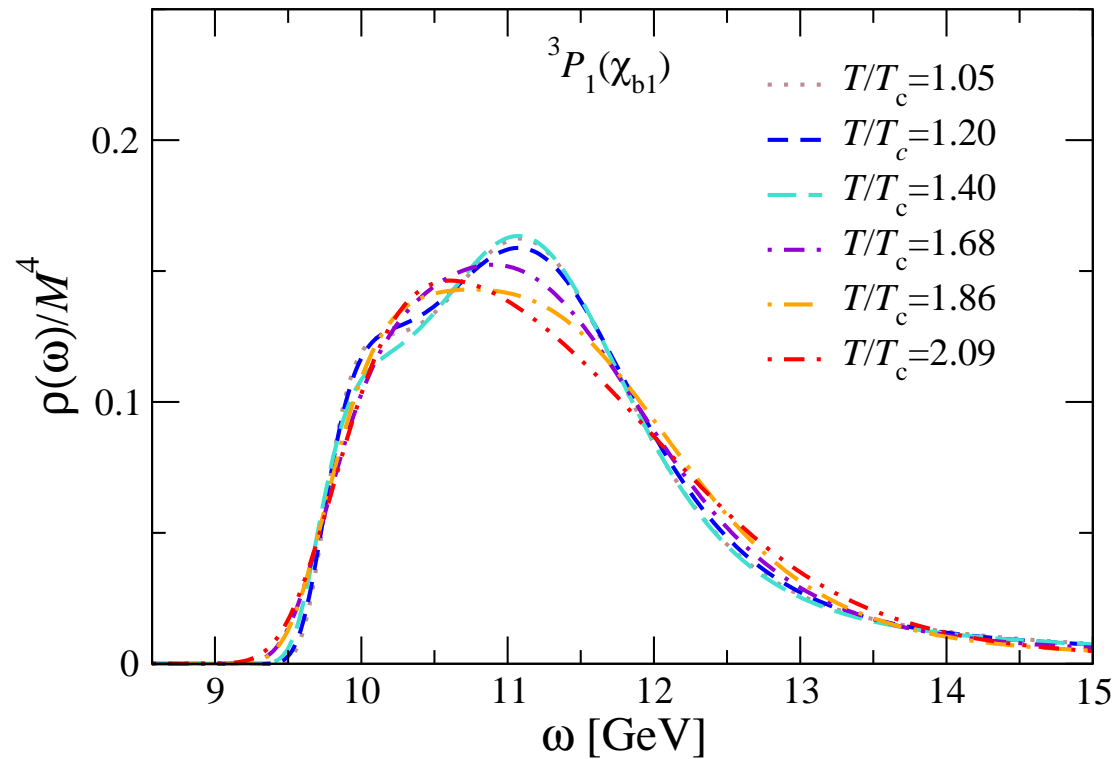
- χ_{b1} axial-vector channel



- melting immediately above T_c
- consistent with correlator decay

P waves at finite temperature

- χ_{b1} axial-vector channel



- melting immediately above T_c
- consistent with correlator decay

Systematics

systematic checks for MEM:

- default model dependence

- ω range: $\omega_{\min} < \omega < \omega_{\max}$

sensitivity to ω_{\min} , additive constant in NRQCD !

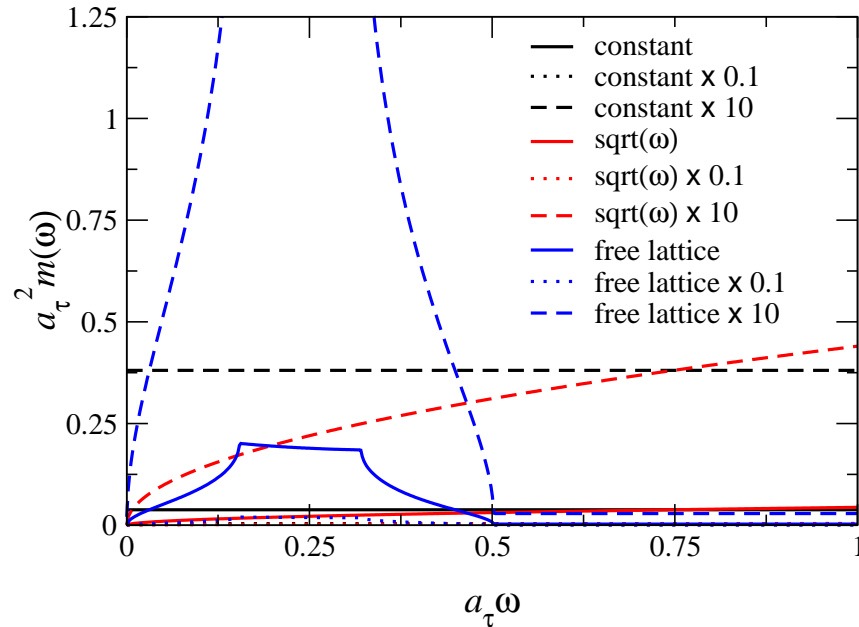
- number of configurations

high-precision data essential, rel. error $\sim 10^{-4}$

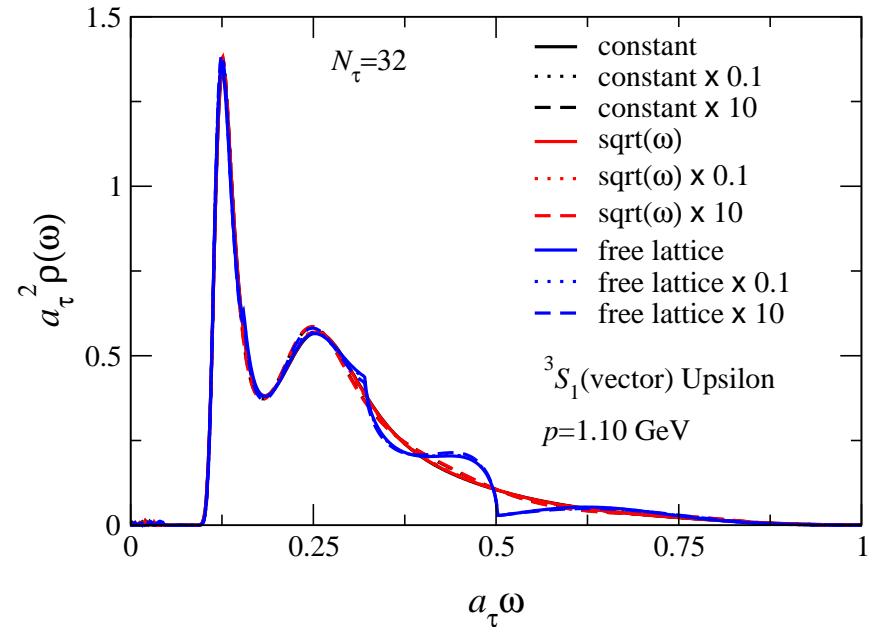
- τ range: $\tau_{\min} \leq \tau \leq \tau_{\max} \leq N_{\tau} - 1$

Systematics

default model dependence (S wave, nonzero momentum):



MEM input



MEM output

- insensitivity to default model
- smooth default model (without cusps) preferred

Systematics

systematic checks for MEM:

- default model dependence

- ω range: $\omega_{\min} < \omega < \omega_{\max}$

sensitivity to ω_{\min} , additive constant in NRQCD !

- number of configurations

high-precision data important, rel. error $\sim 10^{-4}$

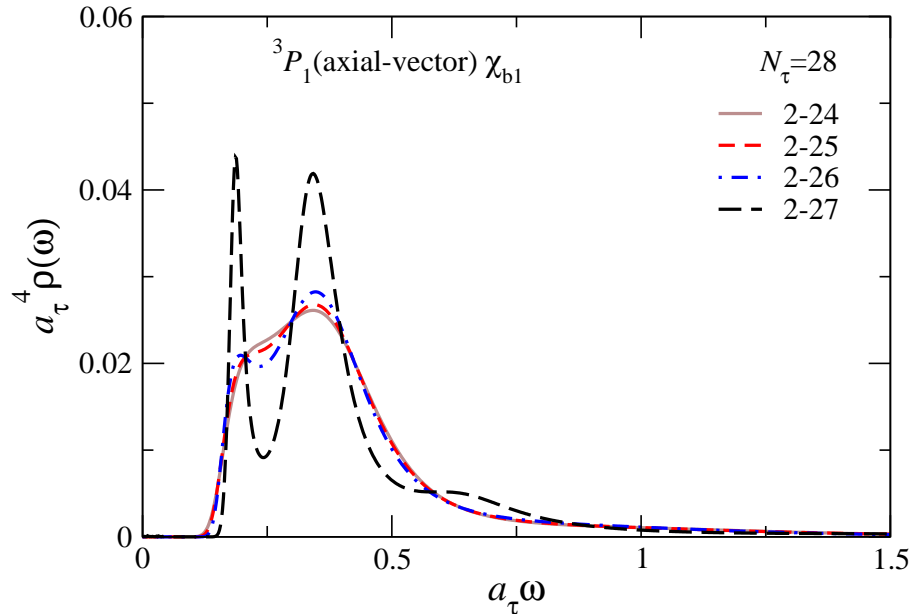
- τ range: $\tau_{\min} \leq \tau \leq \tau_{\max} \leq N_{\tau} - 1$

main source of uncertainty for P waves !

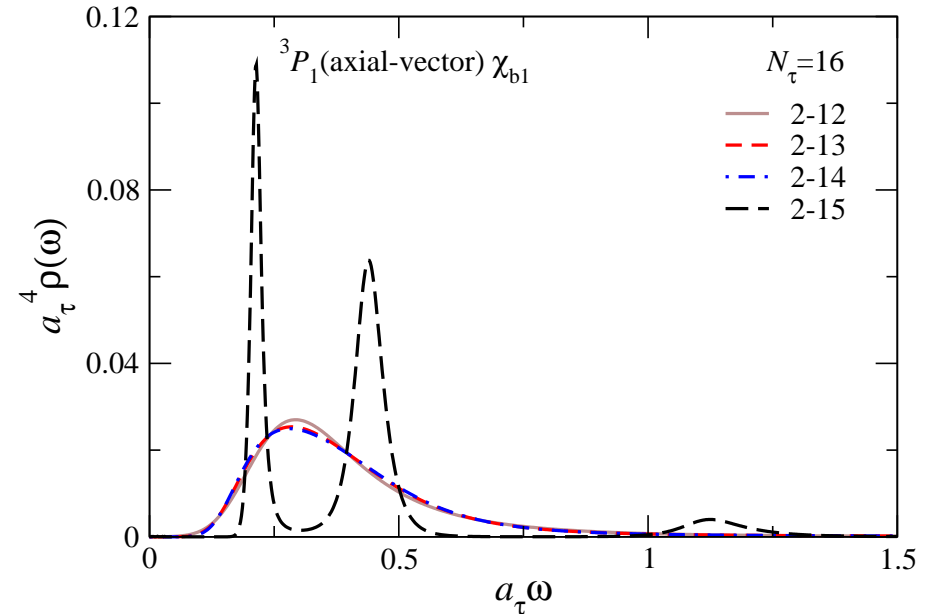
periodicity of gluons vs initial-value problem

Systematics

τ range: $\tau_{\min} = 2 \leq \tau \leq \tau_{\max} = N_{\tau} - 1, -2, -3, -4$



$N_{\tau} = 28$



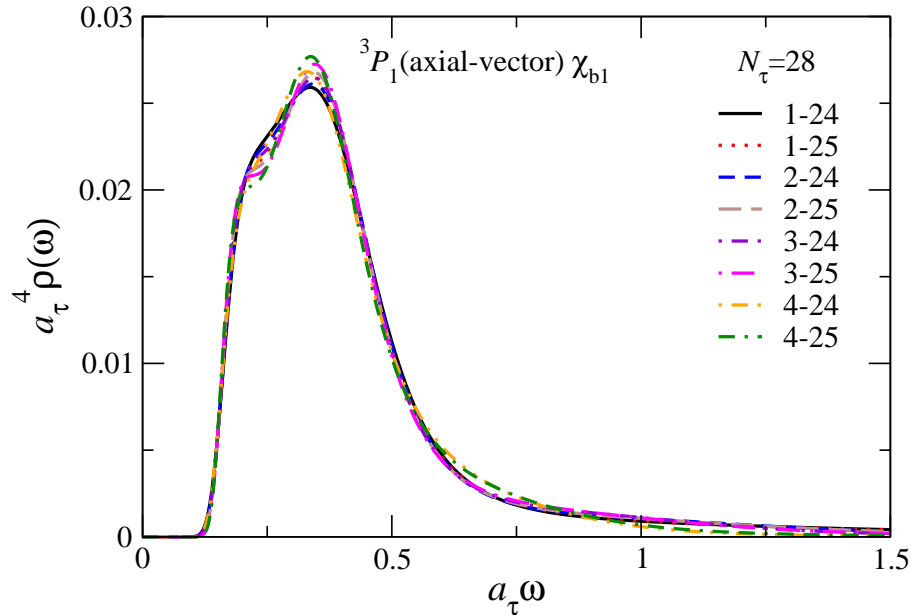
$N_{\tau} = 16$

- strong dependence on inclusion of final time slice(s)!
- related (?) to similar observations of τ dependence of Wilson loops and static meson correlators

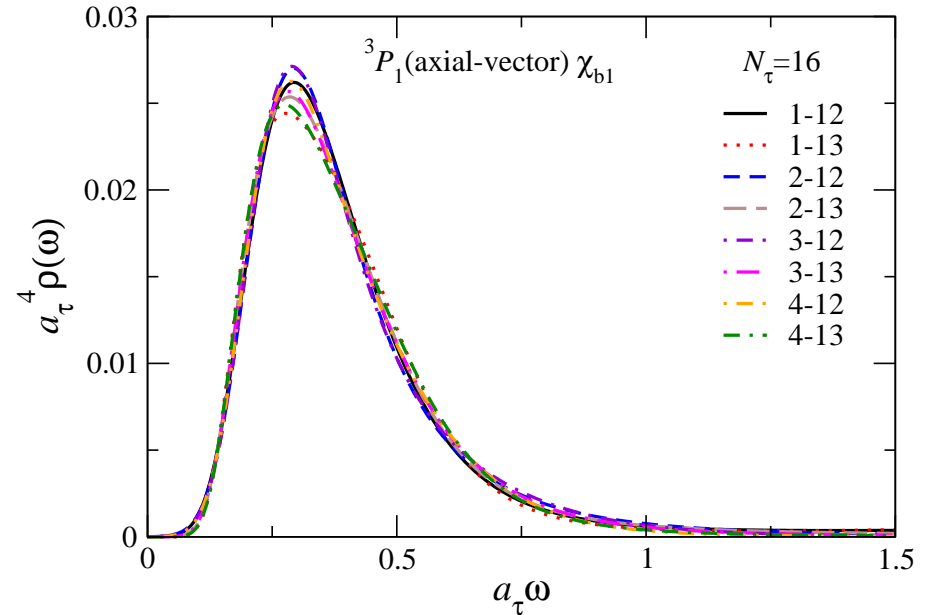
Rothkopf et al 11, Bazavov & Petreczky 13

Systematics

τ range: $\tau_{\min} = 1, 2, 3, 4 \leq \tau \leq \tau_{\max} = N_{\tau} - 3, -4$



$N_{\tau} = 28$

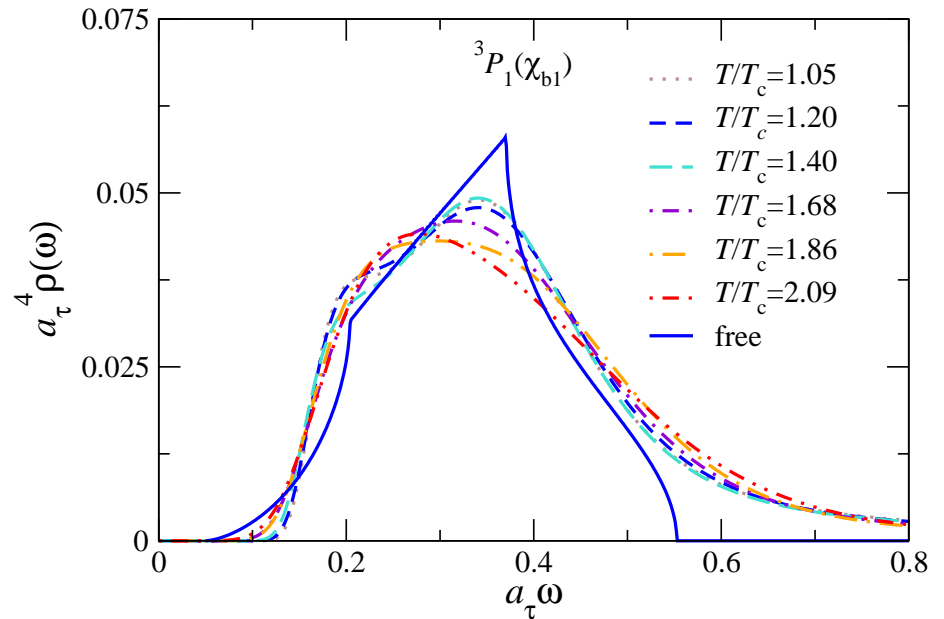
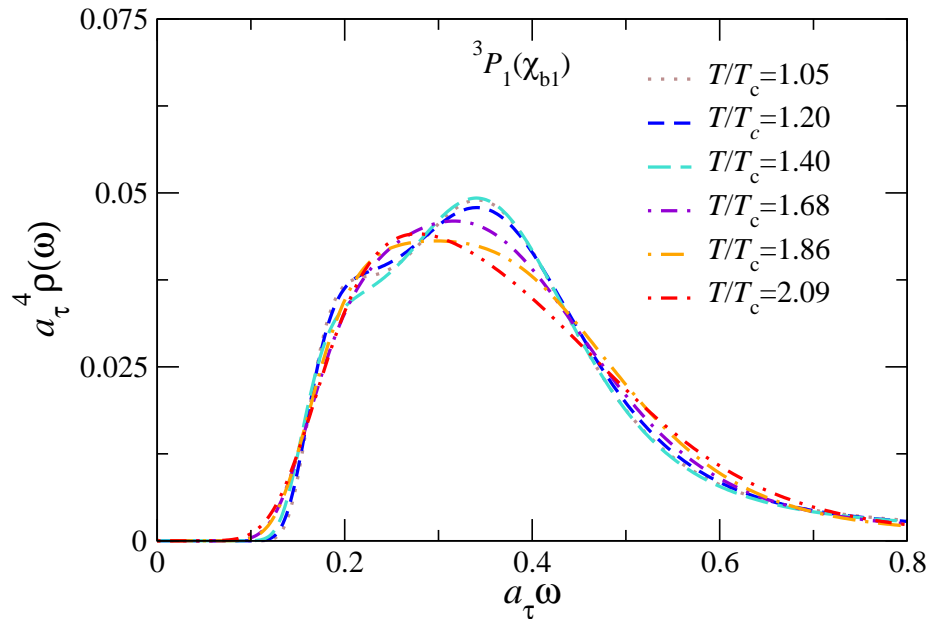


$N_{\tau} = 16$

- exclude $\tau_{\max} = N_{\tau} - 1, -2$
- stability of results under further variation of τ_{\min}, τ_{\max}

P waves: stronger interplay with periodic gluons

Melted P waves



- melting above T_c : a featureless blob ?
- shape is similar to free lattice spectral function

Summary

- bottomonium: NRQCD on QGP background
- S wave ground states survive, at rest and moving
excited states appear suppressed
- P wave states melt immediately above T_c
- use of NRQCD greatly improves reliability of MEM
P waves more sensitive to periodic gluons
- in progress: extension to $N_f = 2 + 1$ on a finer lattice

see talks by Tim Harris, Seyong Kim

Helmholtz Alliance

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Gert Aarts, Swansea University
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