Bottomonium spectral functions at $T > 0$

A signal for the quark-gluon plasma from the lattice

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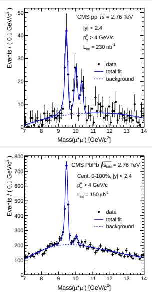
- Dissocation of heavy-quark bound states in a deconfined medium contributes to suppression of quarkonium yield in heavy-ion collions.
- Can suppression patterns provide a thermometer for quark-gluon plasma?
- Competing effects such as statistical recombination less pronounced for heavier quarks.
- Lattice can complement other approaches such as analytical weak-coupling results from effective field theories and potential models.



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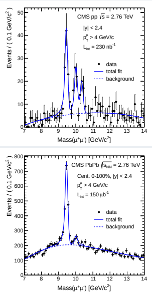

Earlier results from numerical simulations [FASTSUM, 1109.4496] from previous generation of $N_f = 2$ ensembles:

- concluded the survival of the $1\mathrm{S}$ state above the crossover temperature T_c,
- indicated melting of P-wave directly above T_c,

[G. Aarts, this session]

while it was observed from the spectral functions:

- that the $2\mathrm{S}$-states were suppressed at $T \approx 1.7 T_c$.

The road to reality

N_f	2	2+1
Light	Wil/Ham.-Wu	Clover
NRQCD	$O(v^4)$	$O(v^4)$
Gauge	Symanzik	Symanzik
a_s	0.16 fm	0.12 fm
$1/a_\tau$	7.35 GeV	5.67 GeV
a_s/a_τ	6	3.5
m_π/m_ρ	0.55	0.45
L/a_s	12	24
T_{max}	$2\,T_c$	$1.9\,T_c$
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Non-relativistic QCD (NRQCD) on the lattice

- Requires only $m_b \gg T$, cf. weak-coupling approaches which require ordering of other relevant scales.
- ✓ Tuning of $\hat{m}_b$ via dispersion relation due to state-independent energy shift in NRQCD.

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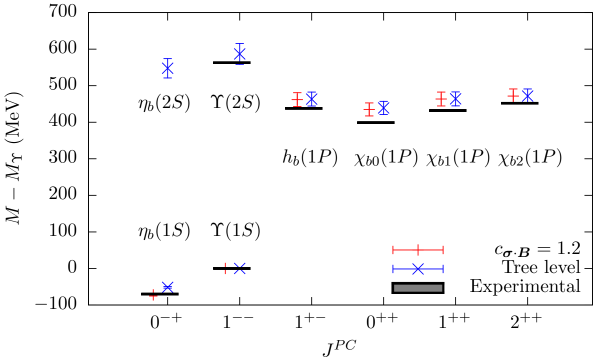

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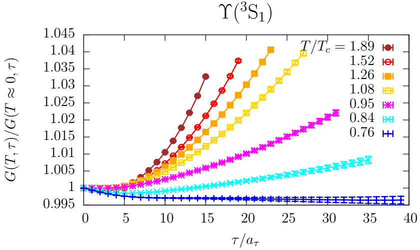
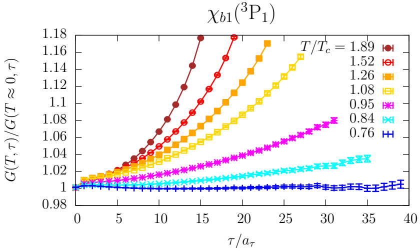
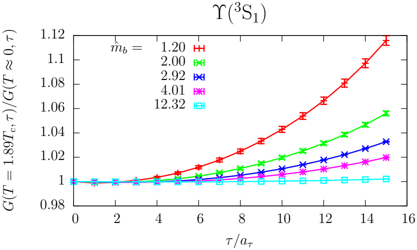
Modification of correlators at $T > 0$

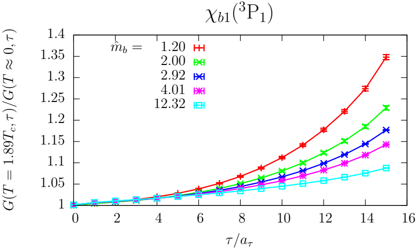




- P-wave correlator exhibits stronger T-dependence.

Dependence on m_b





? Lighter bottomonium more spatially extended and color-Debye screening more effective.

Non-relativistic quarks at $T > 0$

$$G(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \frac{\cosh(\omega\tau - \omega/2T)}{\sinh(\omega/2T)} \rho(\omega) \longrightarrow$$

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- The continuum hadronic spectral function for free quarks is known:

$$\rho_{\mathrm{free}}(\omega) \propto \omega^\alpha \Theta(\omega) \quad \Longrightarrow \quad G_{\mathrm{free}}(\tau) \propto \frac{e^{-\omega_0\tau}}{\tau^{\alpha+1}}, \qquad \alpha_S = \frac{1}{2},\ \alpha_P = \frac{3}{2}.$$

[Burnier, Laine, Vepsäläinen, '07]

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- We might expect such 'almost-free' behaviour above the dissocation temperatures. To this end, define:

$$\gamma_{\text{eff}}(\tau) \equiv -\tau \frac{G'(\tau)}{G(\tau)} \overset{G=G_{\text{free}}}{=} \omega_0\tau + \alpha + 1.$$

Maximum entropy method (MEM)

Given finite stochastic correlator data the inverse Laplace transform is an ill-posed problem. Resort to Bayesian inference of most plausible spectral function.

[Asakawa, Hatsuda, Nakahara '02]

- Dependence on prior information or default model must be tested.
- Use Bryan's algorithm.

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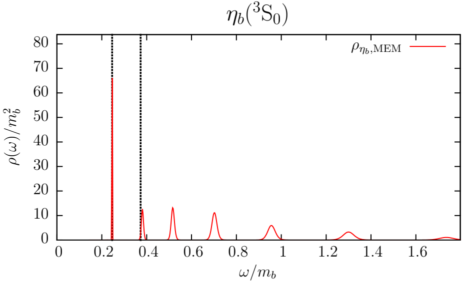

Preliminary zero-T spectral function

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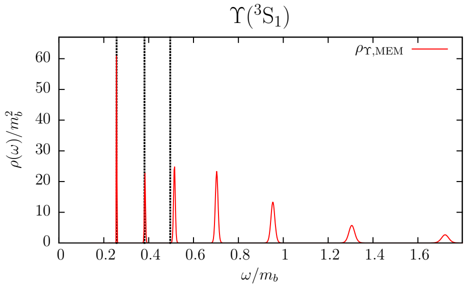

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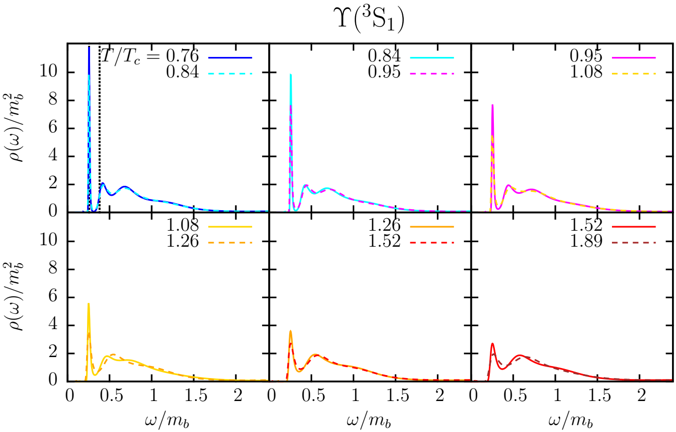
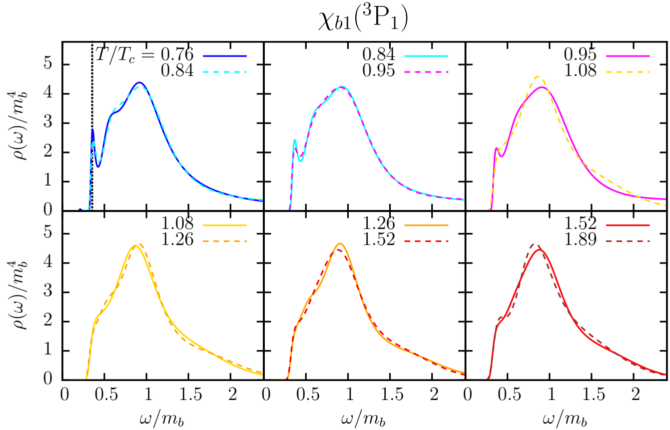
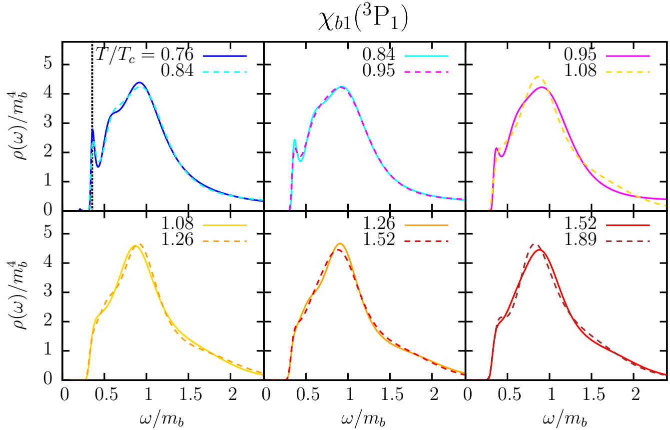
Temperature dependence: S-waves



- $\Upsilon(2\mathrm{S})$ peak is indiscernible at higher T, while ground state peak is still present up to highest temperatures, consistent with previous study.

Temperature dependence: P-waves





- Ground state $\chi_{b1}(1\mathrm{P})$ peak disappears directly above T_c.

Temperature dependence: P-waves

T/T_c	Single exponential χ^2/dof
0.76	0.82
0.84	0.80
0.95	5.36
1.08	20.2
1.26	122
1.52	$\mathcal{O}(10^5)$

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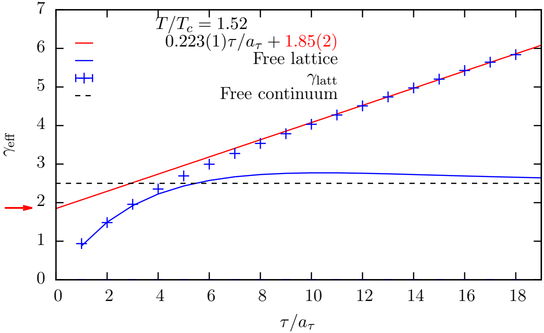

- Ground state $\chi_{b1}(1\text{P})$ peak disappears directly above T_c.
- Direct analysis of correlators supports interpretation of ‘nearly-free’ quark dynamics.

Conclusions and further work

We have seen. . .

- MEM reproducing zero-temperature S-wave energies.
- Spectral functions at finite T signal of $1\mathrm{P}$ above T_c.
- Direct analysis of correlators suggest consistent picture.
- Survival of $1\mathrm{S}$ peak up to at least $1.9T_c$, while $2\mathrm{S}$ peak broadens and is suppressed.
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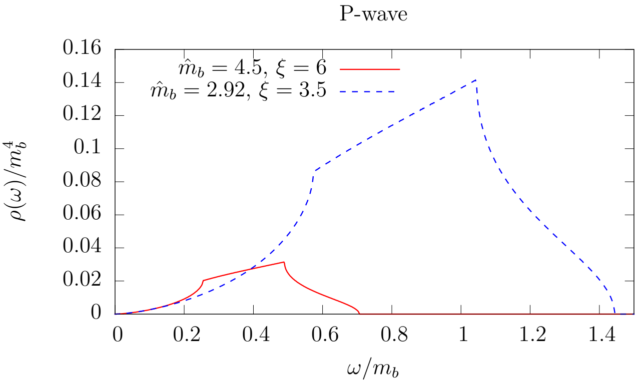
To do:

- Thorough investigation of default model dependence.
- Examine momentum-dependence of peak positions.
- Investigate new approaches for constructing spectral function: extended search space...
- Tuning new generation of ensembles with $\xi = 7$.

Backup slides

Free lattice spectral functions



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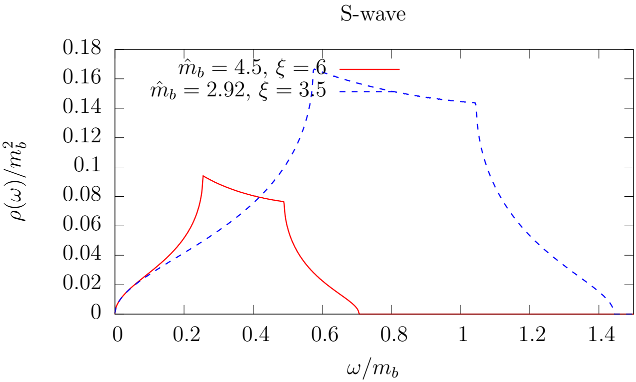

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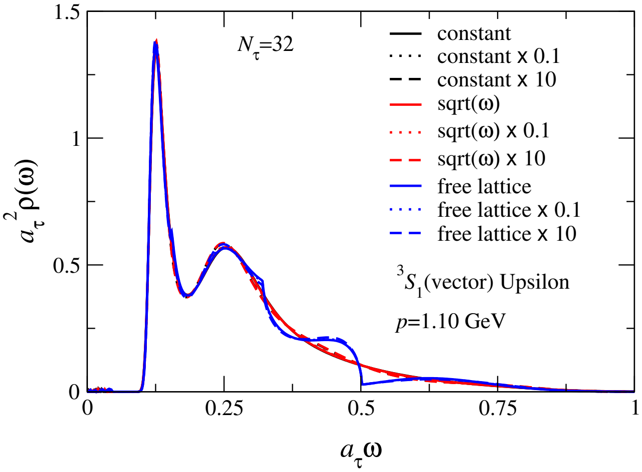
Dependence on default model (First generation ensembles)



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