Heavy quark potential at finite imaginary chemical potential

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Introduction

The heavy quark potential

- a fundamental quantity to understand interactions between quarks
- largely affects the behavior of heavy-quark bound states (J/Ψ, Υ) in QGP created at the center of heavy-ion collisions.

[T. Matsui, H. Satz, Phys. Lett. B 178(1987) 416]

By lattice QCD, the heavy quark potential has been investigated

- for zero chemical potential ($\mu = 0$) ⇐ many studies
- for finite chemical potential (small μ/T) ⇐ a few studies

We focus on the chemical potential dependence of the heavy quark potential.

Imaginary chemical potential region

At real μ : sign problem

At imaginary $\mu = i\mu_{\mathrm{I}}$: no sign problem

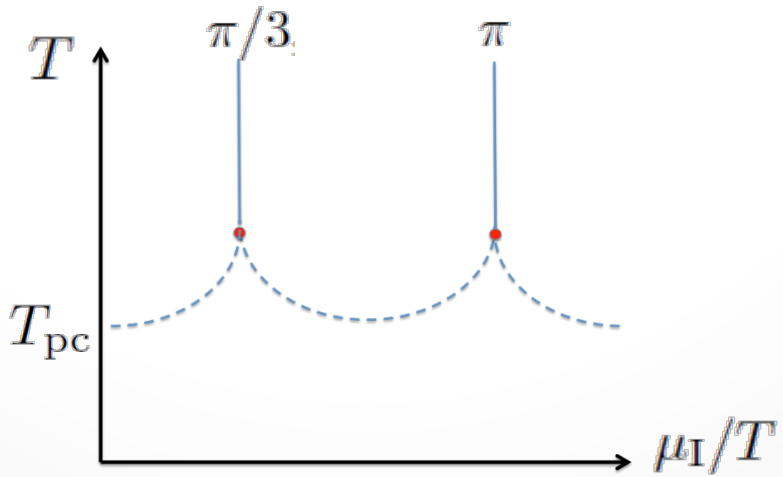
In the imaginary μ, QCD has two properties

- Roberge-Weiss periodicity

$$Z\left(\frac{\mu_{\mathrm{I}}}{T}\right) = Z\left(\frac{\mu_{\mathrm{I}}}{T} + \frac{2\pi}{N_c}\right)$$

- Roberge-Weiss transition

[A. Roberge and N. Weiss, Nucl. Phys. B, 275, 734(1986)]



Lattice set up

Lattice action : renormalization group improved Iwasaki gauge action
and clover improved Wilson fermion action (N_f=2)

Lattice size : $N_s^3 \times N_t = 16^3 \times 4$

$T_{\rm pc} \simeq 171$ MeV at $\mu = 0$

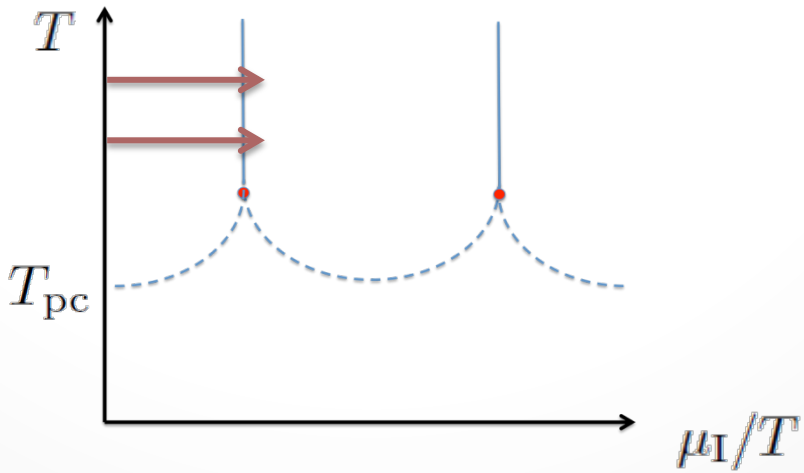
[A. Ali Khan, et al(CP-PACS collaboration), Phys. Rev. D 63, 034502(2000)]

$m_{\rm PS}/m_{\rm V} = 0.80$ line of constant physics

[Y. Maezawa, et al(WHOT-QCD collaboration), Phys. Rev. D 75, 074501(2007)]

Temperatures : $T \approx 1.20\,T_{\rm pc}(\beta = 1.95),\ 1.35\,T_{\rm pc}(\beta = 2.00)$

Chemical potential : $\mu_{\rm I}/T$ = 0 ~ 1.20



Heavy quark potential

Polyakov loop $L(\boldsymbol{x}) = \prod_{t=1}^{N_t} U_4(\boldsymbol{x}, t)$

Polyakov loop correlation function

$$e^{-V_{q\bar{q}}/T} = \langle \mathrm{Tr} L(\boldsymbol{x}) \mathrm{Tr} L^\dagger(\boldsymbol{y}) \rangle \ : q\bar{q} \text{ potential}$$
$$e^{-V_{qq}/T} = \langle \mathrm{Tr} L(\boldsymbol{x}) \mathrm{Tr} L(\boldsymbol{y}) \rangle \ : qq \text{ potential}$$

Gauge fixing
(Coulomb gauge)

$$3 \otimes \bar{3} = 1 \oplus 8 \ : q\bar{q} \text{ potential}$$
$$3 \otimes 3 = 6 \oplus 3^* \ : qq \text{ potential}$$

[S. Nadkarni, Phys. Rev. D 34, 3904(1986)]

At imaginary μ,

- $q\bar{q}$ potentials are invariant under charge conjugation. ($\mathcal{C}$-even) $\Rightarrow$ real
- qq potentials are not invariant under charge conjugation. $\Rightarrow$ complex

For all the color channels, we consider the real part of the heavy quark potential at imaginary μ.

Analytic continuation from imaginary to real chemical potential

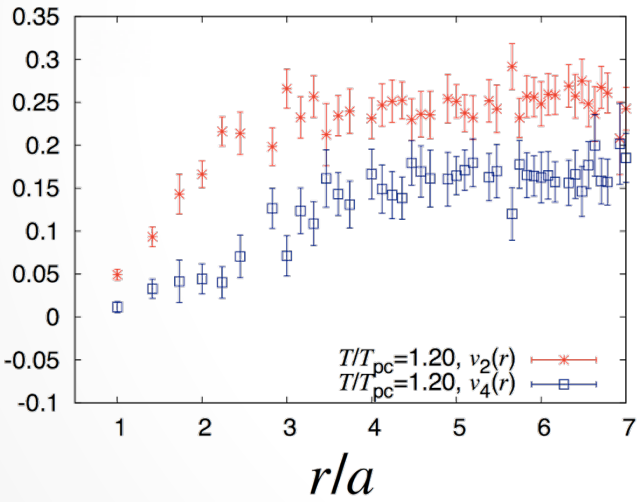
At imaginary chemical potential

$$\frac{V_M(r,T,\mu_{\rm I})}{T} = v_0(r) + v_1(r)\left(i\frac{\mu_{\rm I}}{T}\right) - v_2(r)\left(i\frac{\mu_{\rm I}}{T}\right)^2 - v_3(r)\left(i\frac{\mu_{\rm I}}{T}\right)^3 + v_4(r)\left(i\frac{\mu_{\rm I}}{T}\right)^4$$

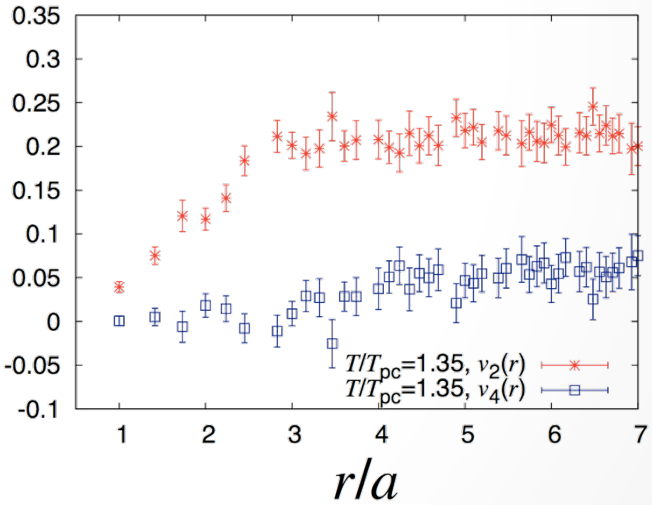
$$M = (1, 8, 3^*, 6)$$

For color-singlet potential

$1.20\,T_{\rm pc}$



$1.35\,T_{\rm pc}$



At real chemical potential

$$\frac{V_M(r,T,\mu_{\rm R})}{T} = v_0(r) + v_1(r)\left(\frac{\mu_{\rm R}}{T}\right) - v_2(r)\left(\frac{\mu_{\rm R}}{T}\right)^2 - v_3(r)\left(\frac{\mu_{\rm R}}{T}\right)^3 + v_4(r)\left(\frac{\mu_{\rm R}}{T}\right)^4$$

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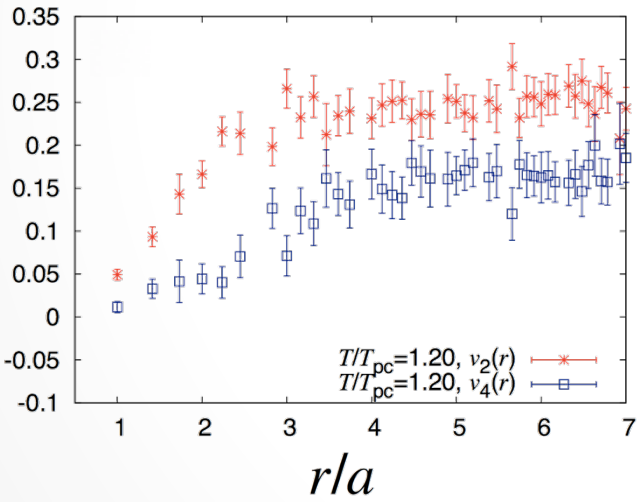

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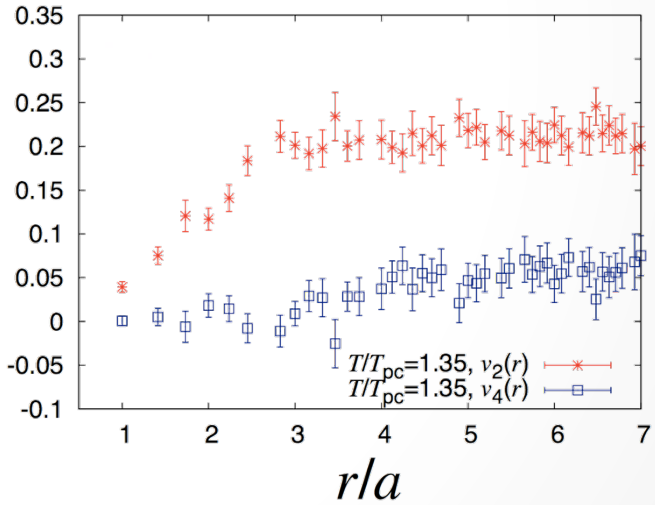

At real chemical potential

$$\frac{V_M(r,T,\mu_{\rm R})}{T} = v_0(r) + v_1(r)\left(\frac{\mu_{\rm R}}{T}\right) - v_2(r)\left(\frac{\mu_{\rm R}}{T}\right)^2$$

[S. Ejiri, et al(WHOT-QCD collaboration), Phys. Rev. D 82, 014508(2010)]

Analytic continuation from imaginary to real chemical potential

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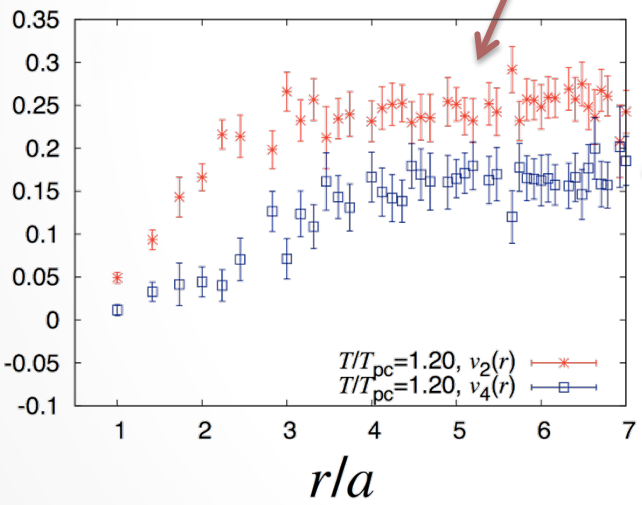

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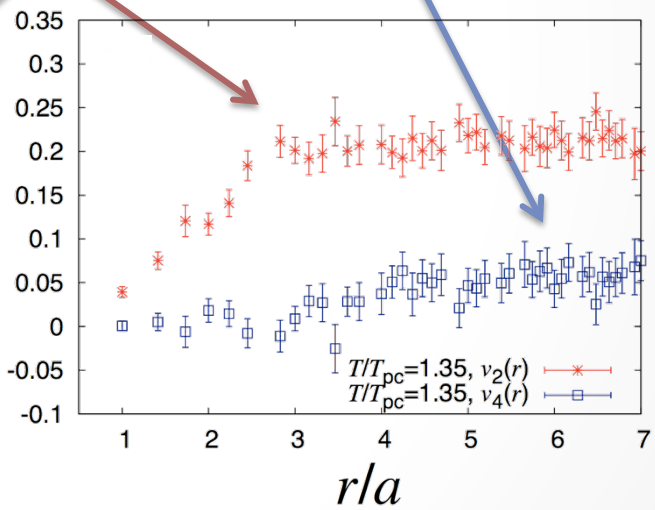

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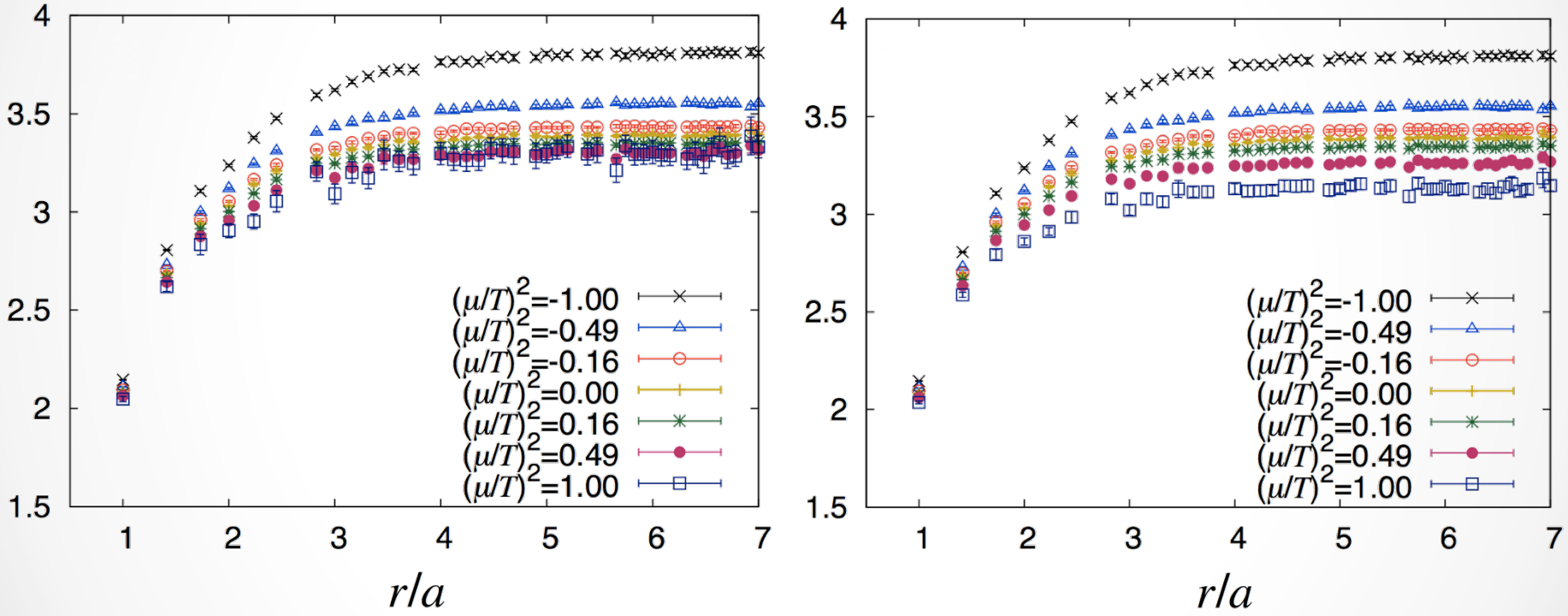
Color-singlet potential

$1.20\,T_{\rm pc}$

$4^{\rm th}$ order term is neglected.



$(\mu/T)^2$dependence of potentials is much weaker at real μ than at imaginary μ .

Color-singlet potential

$1.35\,T_{\rm pc}$

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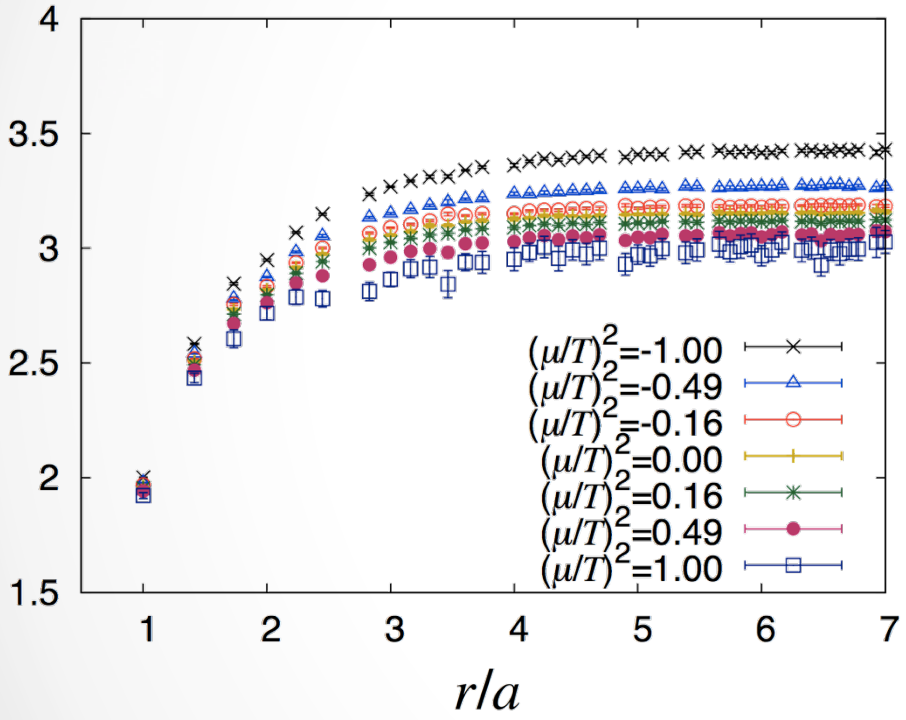

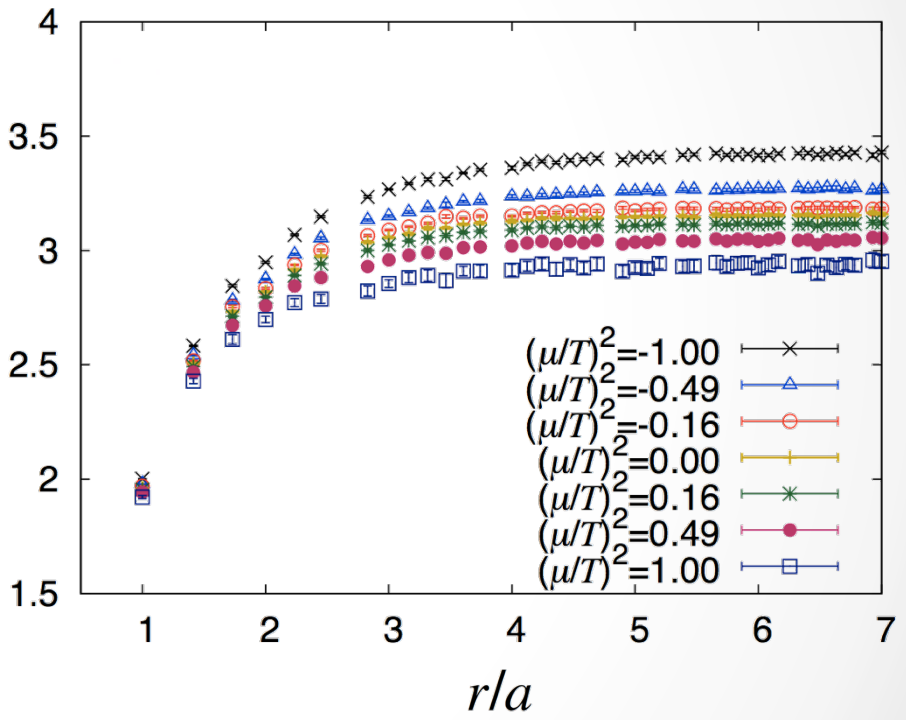

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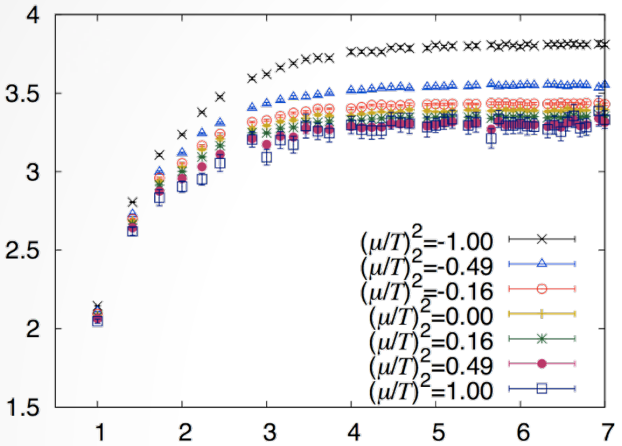
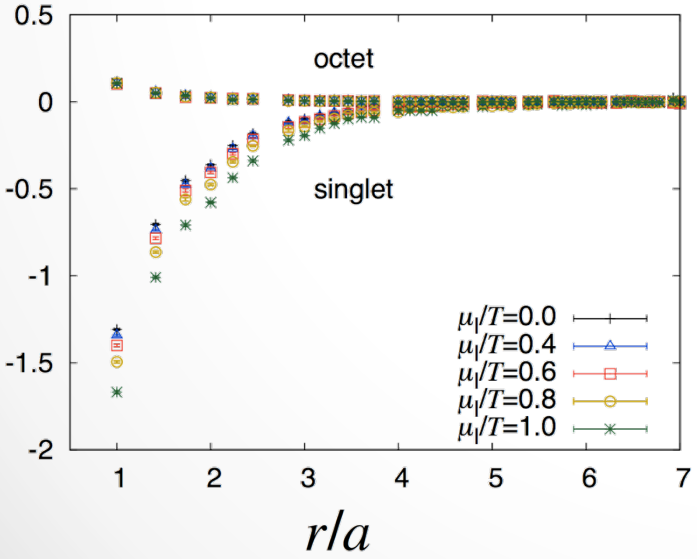
Heavy quark interaction

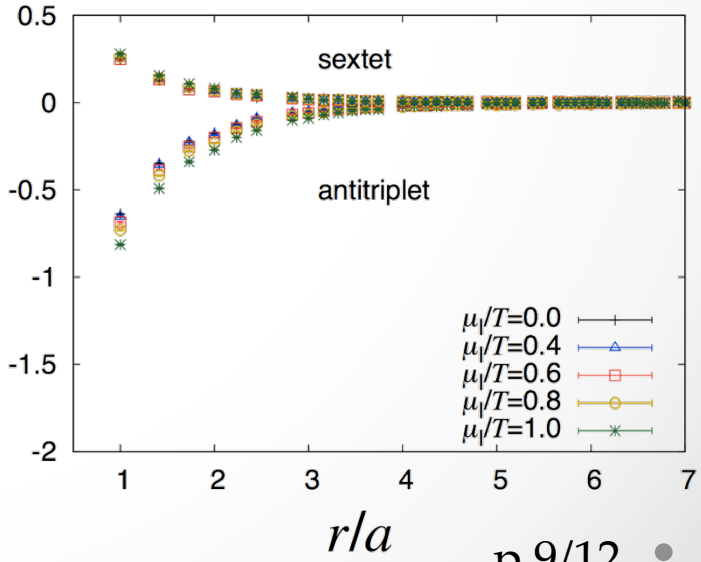


$V_M(r)$ tends to twice the single-quark free energy $2F_q(T,\mu_{\rm I})$.

$2F_q(T,\mu_{\rm I})$ is subtracted from $V_M(r)$.

$1.20\,T_{\rm pc}$





Color-Debye screening mass

Screening masses are obtained from fitting the heavy-quark potential to the screened Coulomb form :

$$V_M(r,T,\mu) = C_M \frac{\alpha_{\rm eff}(T,\mu)}{r} e^{-m_{\rm D}(T,\mu)r}$$

$C_M \equiv \langle \sum_{a=1}^{8} t_1^a \cdot t_2^a \rangle_M$: Casimir factor $\quad C_1 = -\frac{4}{3}$ (singlet channel)

$\alpha_{\rm eff}(T,\mu)$: effective running coupling

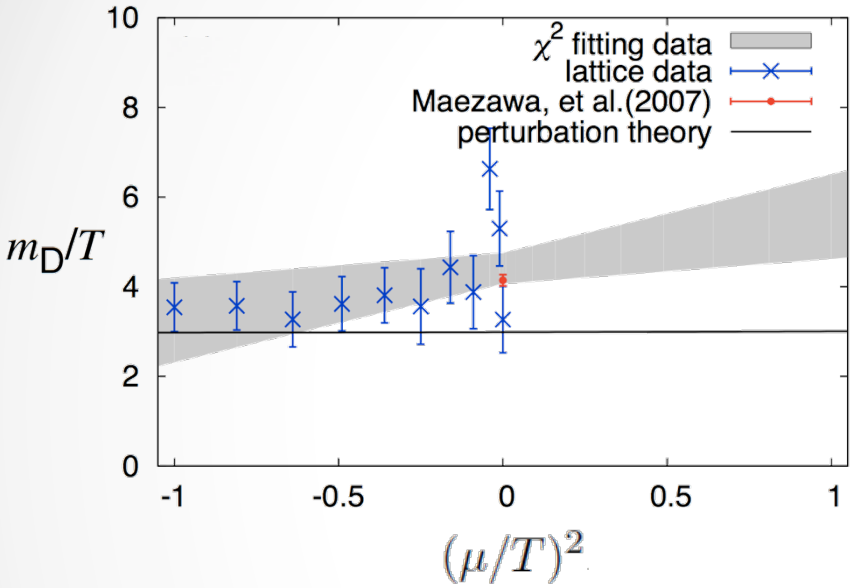
$m_{\rm D}(T,\mu)$: color-Debye screening mass

Fitting range : $\sqrt{11} \leq r/a \leq 6.0$

[Y. Maezawa, et al(WHOT-QCD collaboration), Phys. Rev. D 75, 074501(2007)]

Color-Debye screening mass

$1.20\,T_{\rm pc}$



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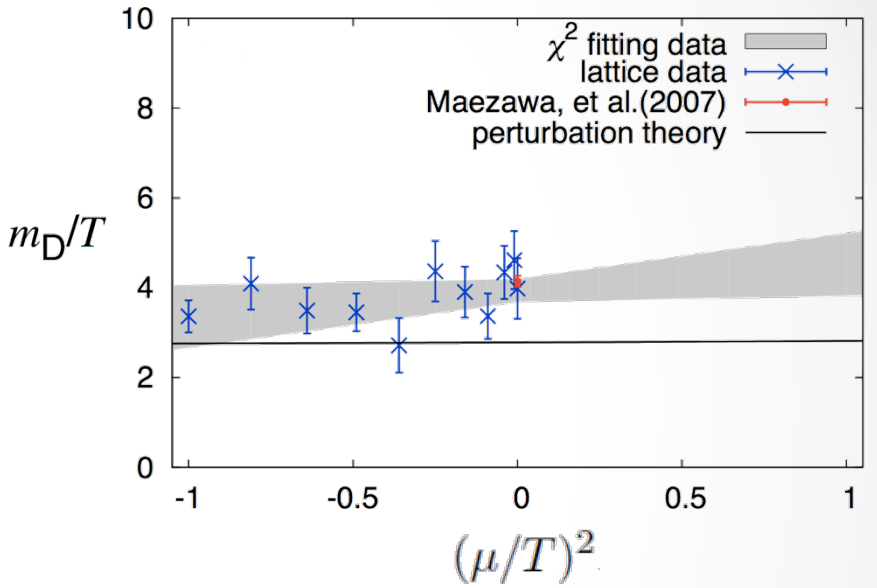

Extrapolation : $\dfrac{m_{\rm D}}{T} = a_0(T) + a_2(T)\left(\dfrac{\mu}{T}\right)^2$

Leading-order thermal perturbation theory : $\dfrac{m_{\rm D}(T,\mu)}{T} = g_{21}(\nu)\sqrt{\left(1+\dfrac{N_f}{6}\right)+\dfrac{N_f}{2\pi^2}\left(\dfrac{\mu}{T}\right)^2}$

$g_{21}(\nu)$: 2-loop running coupling

ν : renormalization point

Summary

- The heavy quark potential at imaginary μ is expanded into a power series of $i\mu_I/T$ up to 4^{th} order.
- At real μ, the 4^{th}-order term weakens μ dependence of the heavy quark potential.
- At long distance, all the color-channel potentials tend to twice the single-quark free energy.
- The color-singlet and -antitriplet interactions are attractive, but the color-octet and -sextet interactions are repulsive. The attractive interaction depends on μ, but the repulsive interaction do not.
- The color-Debye screening mass has stronger μ dependence at both imaginary and real μ than the prediction of the leading thermal perturbation theory.