Staggered operator with topological SU(2) backgrounds at nonzero chemical potential

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02.08.2013

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Outline

- Continuum - Instantons and zero modes at $\mu, T > 0$.
- Test of staggered Dirac operator at $\mu, T > 0$.

More information in **“Topological zero modes at nonzero chemical potential”, arXiv:1305.1241** (PRD in press)

The KvBLL caloron with SU(2) gauge group

Caloron: (anti) self dual solution of YM-Theory $T > 0$.
Explicit construction: KvBLL (Kraan, van Baal, Lee and Lu hep-th/9805168, hep-th/9802108).

Important properties:

- Action density is equal to its topological density (self dual).
- Two monopole constituents.
- Novel holonomy parameter ω.

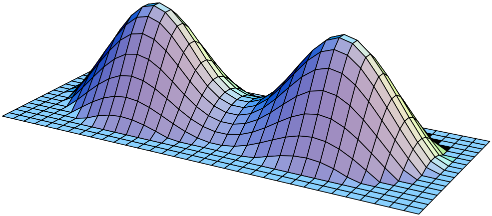


Figure: Action density of a SU(2) KvBLL caloron ($\omega = 1/4$, $\rho = 1\beta$) (Taken from hep-th/9805168).

Continuum theory - Setup

Dirac operator in the fund. rep. with μ

$$\not{D}(\mu) = \gamma_\nu D_\nu - \gamma_0 \mu \,, \quad D_\nu = \partial_\nu + i A_\nu .$$

Eigenproblem: $\not{D}(\mu)\psi_n(x;\mu) = \lambda_n(\mu)\psi_n(x;\mu)$
Density and an inner product (biorthonormalization):

$$\rho_{mn}(x;\mu) \equiv \psi_m^\dagger(x;-\mu)\psi_n(x;\mu), \qquad \int d^4x \, \rho_{mn}(x;\mu) = \delta_{mn} .$$

What happens if A_ν is a charge one SU(2) caloron?
(F. Bruckmann, R. Rödl, T. Sulejmanpasic, '13)

- Density $\rho_{mn}(x;\mu)$ is real (SU(2) property).
- Zero eigenvalue $\lambda_0(\mu) = 0$ exists, for instantons see[1].
- $\rho_{00}(x;\mu)$ has negative regions at $\mu > 0$.
- The localization of the zero mode is still on top of a caloron lump.

[1] (C. Aragao de Carvalho '81, A. Abrikosov '83, M. Cristoforetti '11)

Zero mode density (Zmd) in a background of a caloron ($\mu = 4T$ and $\omega = 1/4$) together with the caloron action density.

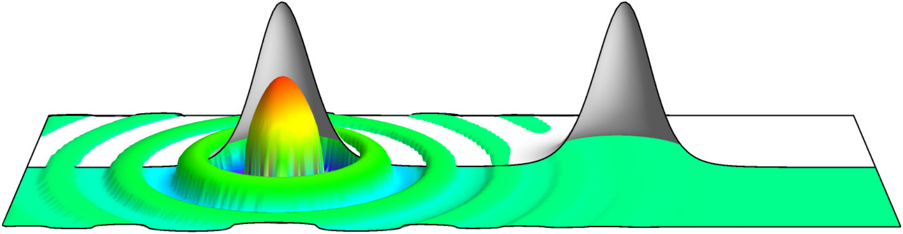


Figure: Colored magnitude is rescaled.

- Zmd localization is not accidentally on top of one lump. (M. Garcia-Perez et al. '99)
- Application of the Index-Theorem[1] at μ and T.

[1] (T. Kanazawa, T. Wettig, N. Yamamoto '11), (R. Gavai, S. Sharma '10)

Lattice - Setup and results

Strategy of the comparison - Lattice vs. continuum

Central question:

Is the staggered operator suitable to reproduce topological zero modes at non zero chemical potential and temperature or not?

Investigate:

- Spectrum of the operator, especially the (quasi-)zero eigenvalues.
- The (quasi-)zero mode density constructed from the staggered modes.

Classical lattice gauge links via gauge transporter:

$$\mathrm{A}_\mu(x) = \frac{1}{2}\,\overline{\eta}_{3\mu\nu}\,\partial_\nu\Big(\log\Big[\Phi(x)\Big]\Big)\,\tau^3 + \ldots,$$

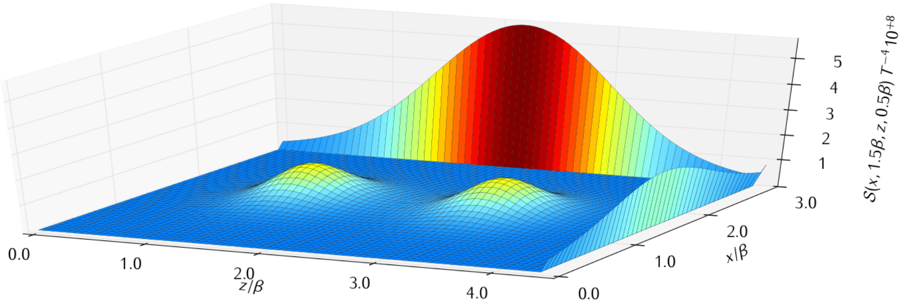
$$\mathrm{U}_\nu(an) = \hat{\mathrm{P}}\exp\left(\mathrm{i}\int_0^1 \vec{\mathrm{A}}_\nu(an + as\hat{\nu})\cdot\vec{\tau}\,a\,\mathrm{d}s\right)$$

$$= \lim_{\mathcal{N}\to\infty}\prod_{k=1}^{\mathcal{N}}\exp\left(\mathrm{i}a\,\mathcal{N}^{-1}\,\vec{\mathrm{A}}_\nu(an + a\frac{k}{\mathcal{N}}\hat{\nu})\cdot\vec{\tau}\right).$$

Standard staggered operator:

$$2a\mathrm{D}(\mu) = \sum_\nu \eta_\nu(n)\Big(\mathrm{U}_\nu(n)\,\delta_{n+\hat{\nu},\,m}e^{a\mu\delta_{\nu,4}} - \mathrm{U}^\dagger_\nu(n-\hat{\nu})\,\delta_{n-\hat{\nu},\,m}e^{-a\mu\delta_{\nu,4}}\Big).$$

Discrete action density and boundary artifacts



- Wilson action density suffers from boundary artifacts due to the periodicity requirement of the gauge links.
- APE smearing reduces this cutoff artifacts α_{APE}=0.45.
  (M. Falcioni et al. '85), (T. DeGrand, A. Hasenfratz, T. G. Kovacs '97)

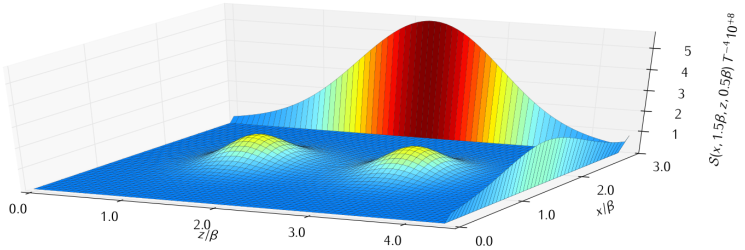


Figure: Action density of SU(2) caloron before smearing.

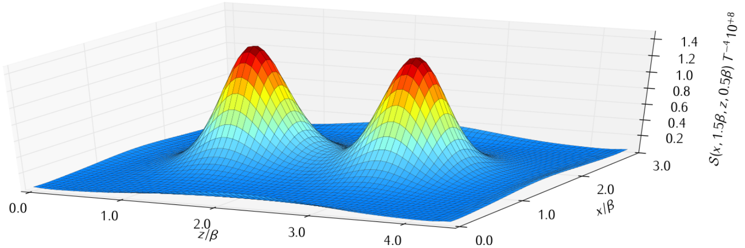
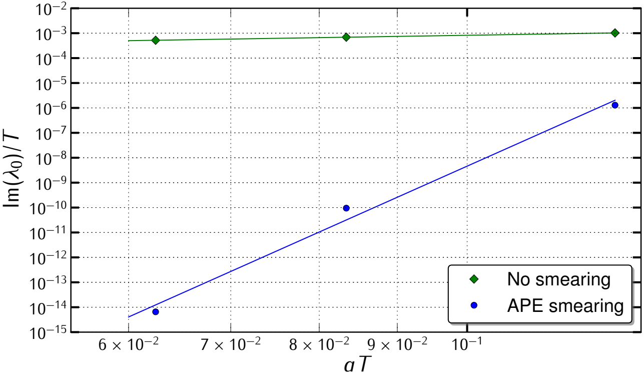


Figure: Action density after 40 APE smearing steps.

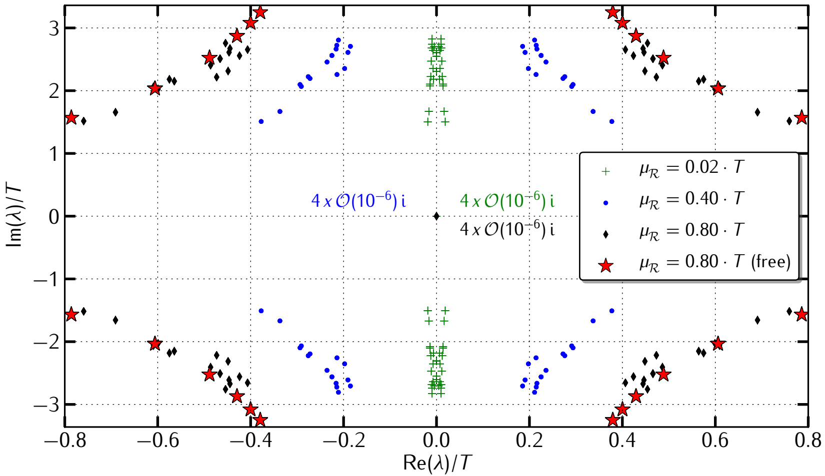
Staggered operator with caloron background and $\mu = 0$

- $\mathrm{D}(\mu = 0)$ is anti-hermitian $\leftrightarrow$ purely imaginary eigenvalues.
- 4 tastes $\leftrightarrow$ 4 (quasi-)zero eigenvalues.



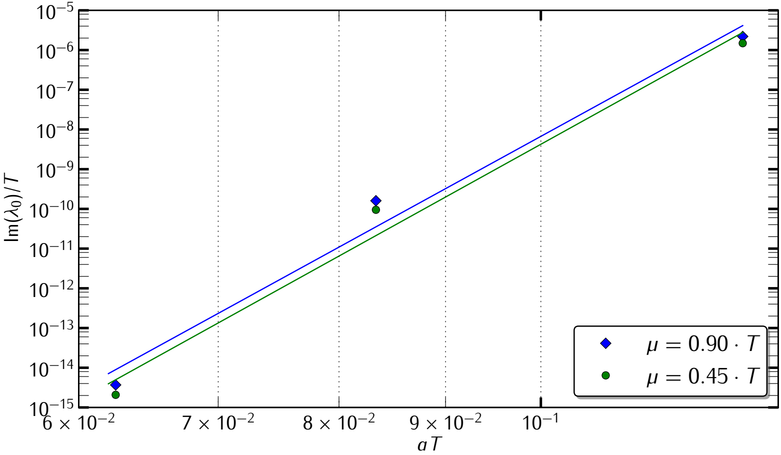
Conclusion: Exact zero modes in the continuum limit.

Spectrum of the staggered operator with non zero μ



- Eigenvalues became complex.
- Quartet due to $\{\eta_5, \mathrm{D}(\mu)\} = 0$ and SU(2) symmetry.
- Exited caloron spectrum similar to the free case.
- (Quasi-)zero eigenvalues at real chemical potential!

Zero eigenvalues at non zero μ



- Continuum limit of $\mathrm{Im}(\lambda_0)/T$ with different μ.
- APE smeared caloron backgrounds with $\omega = 1/4$.

Conclusion: Exact zero modes even with μ (continuum).

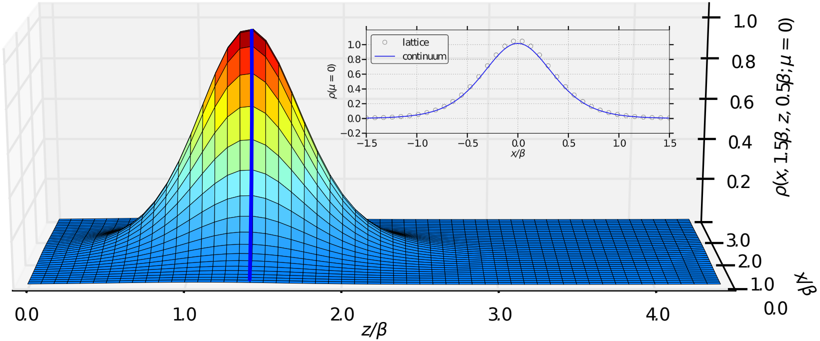
So far ...

- The staggered Dirac operator with APE smeared caloron background has four zero modes (in the continuum limit).
- This is true even at non zero real chemical potential.

... and now

- We can investigate zero mode densities constructed from staggered eigenvectors.

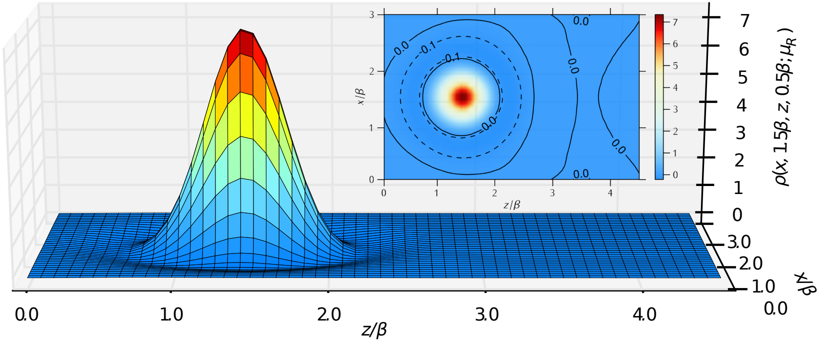
Staggered zero mode density with $\mu = 0$



- Compare one slice with the continuum.
- Zmd of the staggered operator agrees with its continuum counterpart.

Remark: Take one of 4 possible zmd of the staggered operator.

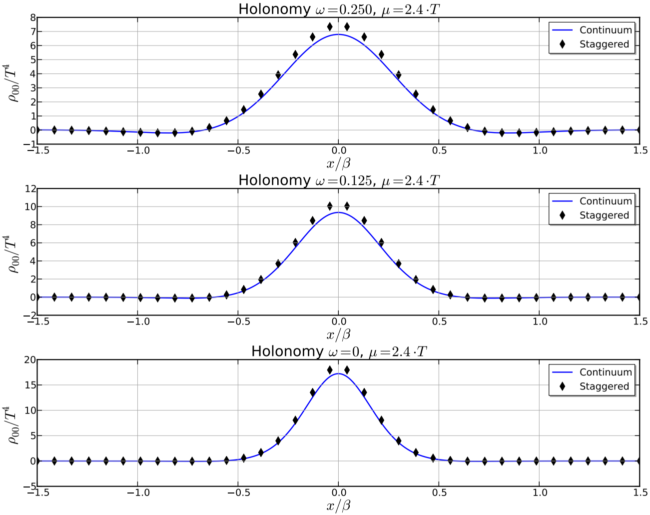
Staggered zero mode density with $\mu = 2.4 \cdot T$



- Zmd is still localized (stronger!) to the same caloron lump.
- Staggered version has characteristic negative regions.

Remark: This zmd is constructed from 2 staggered densities (degenerate eigenvalues).

Comparison of zero mode densities - Lattice vs. continuum



- Good agreement, small discrepancy due to cutoff effects.
- Larger $\mu \leftrightarrow$ numerically hard.

Conclusion

- The staggered operator is suitable to reproduce topological zero modes at non zero chemical potential and temperature.

Outlook

- Extension to SU(3) gauge group (complex density).
- Continuum expressions of the zero mode density for the semiclassical approach to QCD at $T,\ \mu > 0$ (overlap matrix elements).

We use

- ARPACK
- Boost
- Armadillo

Thank you!