Analytic continuation in two color QCD with clover-improved Wilson fermion

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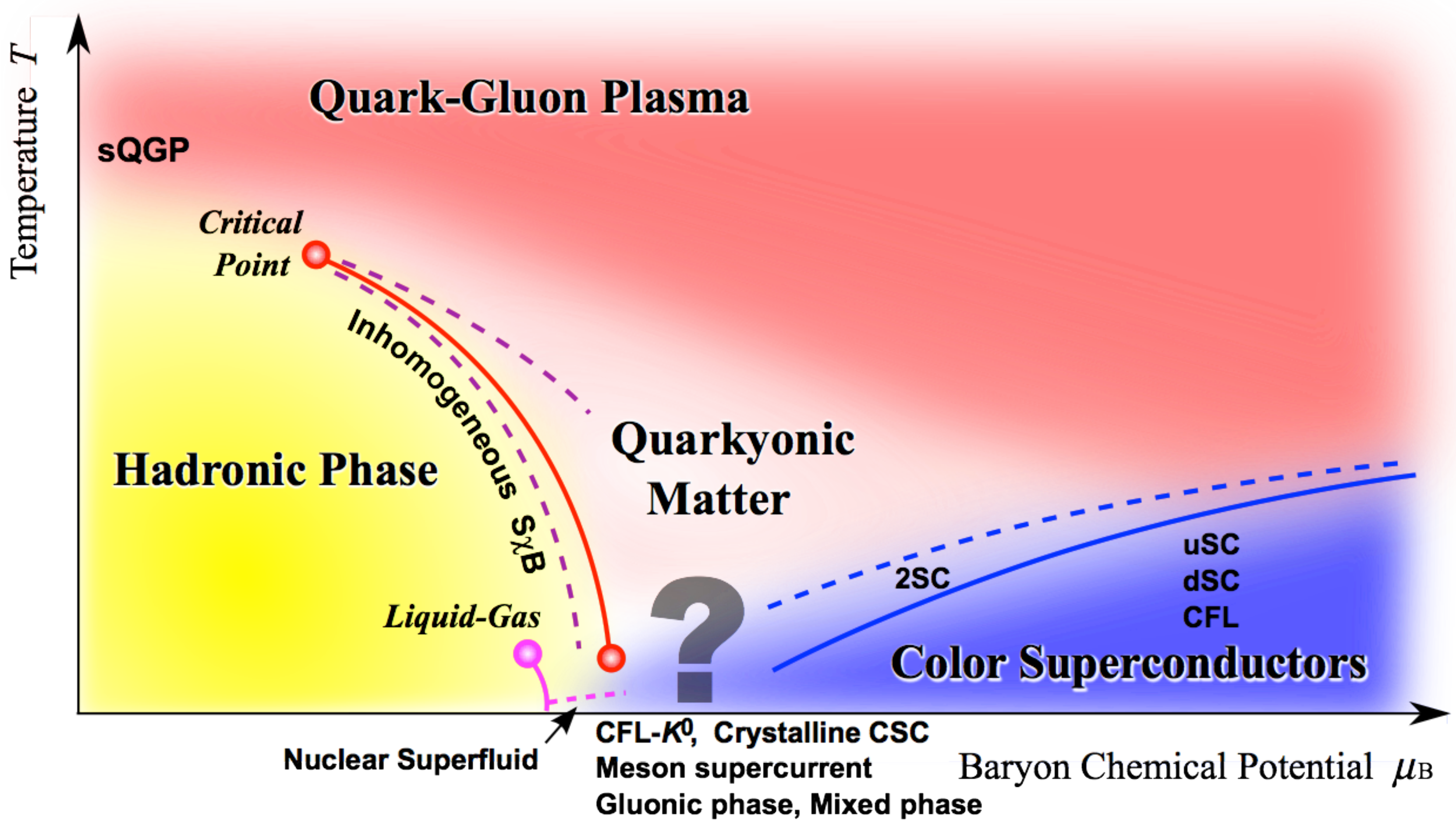
QCD Phase Diagram

Fukushima and Hatsuda, Rept. Prog. Phys. 74



Sign Problem

$$\langle \mathcal{O} \rangle = \int dU e^{-S[U]} \mathcal{O}[U]$$

$U \ (x_\nu, 4, N_c)$

very huge multiple integral

MC with importance sampling

integrand $\quad e^{-S[U]} \sim \det \Delta(\mu) e^{-S_g[U]}$

Finite chemical potential μ (Sign Problem)

$$(\det \Delta(\mu))^* = \det \Delta(-\mu^*) \in \mathbb{C}$$

NO sign problem

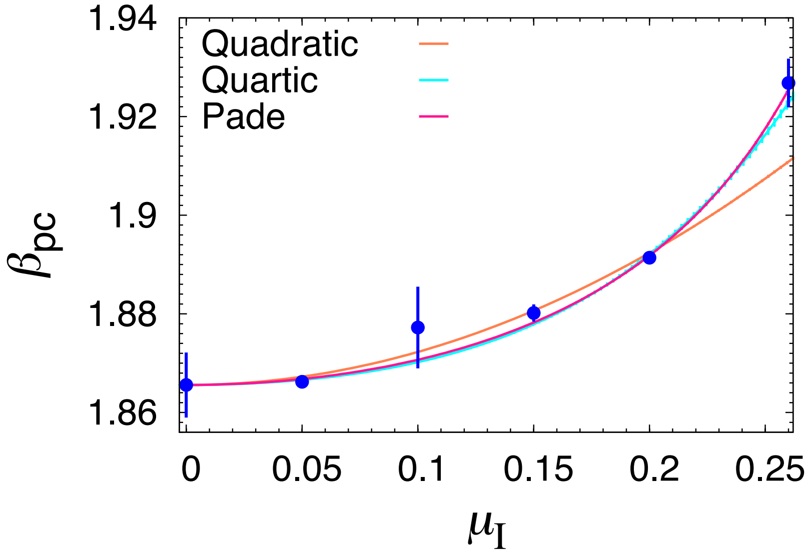
Imaginary μ $\quad (\det \Delta(i\mu))^* = \det \Delta(i\mu) \in \mathbb{R}$

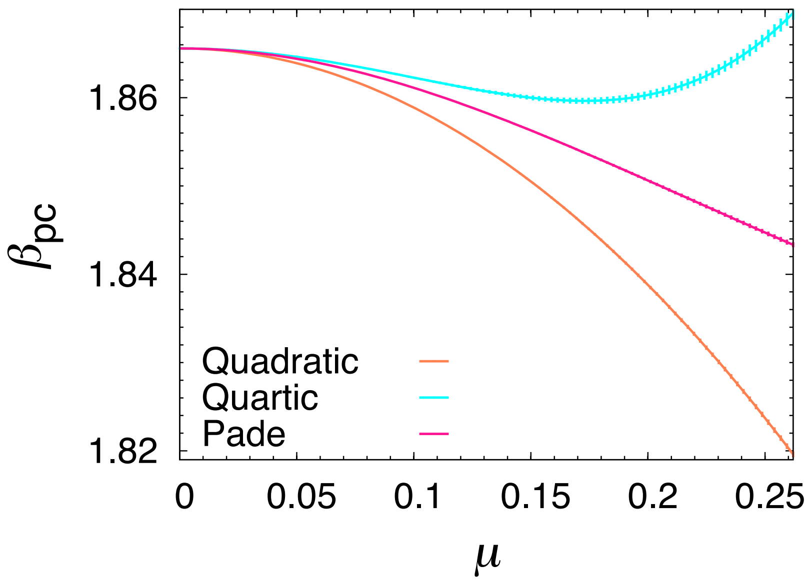
Analytic Continuation from imaginary μ

Lattice data can be made analytic continuation from imaginary to real μ.

such as the phase transition line

Nagata and Nakamura, PRD83





For two color QCD, NO sign problem at both imaginary and real μ.

We can check reliability of the analytic continuation method.

Cea, Cosmai, D'Elia and Papa, PRD77

Outline

We investigated two-color QCD phase diagram
at both imaginary and real μ.

We use the RG improved gauge + Clover improved Wilson fermion
on the line of the constant physics with Mps/Mv=0.80 in $8^3 \times 4$ size.

We calculated
the Polyakov loop, the quark number density, the chiral condensate.

We evaluated the analytic continuation of these quantities.

Polyakov Loop

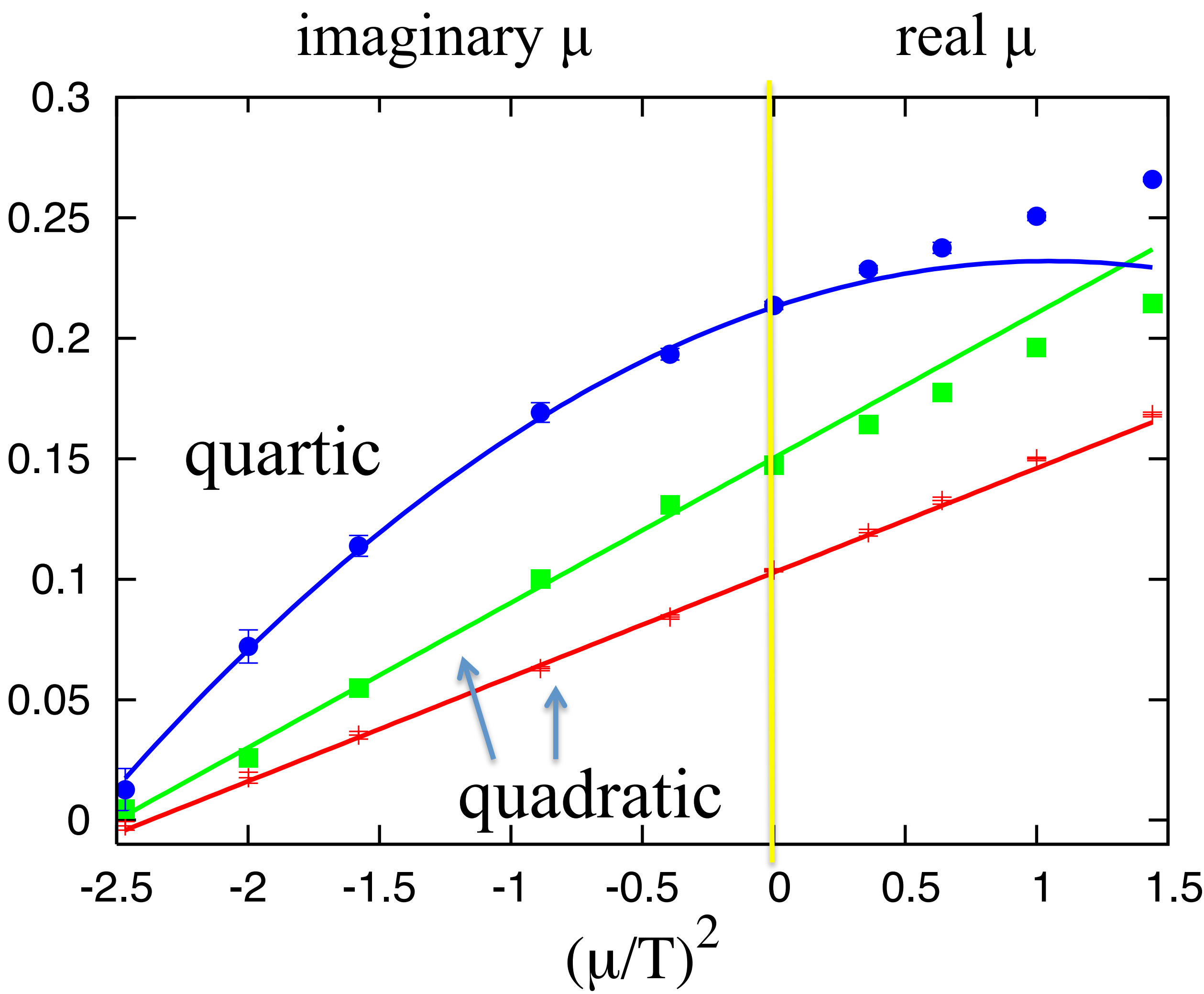
$$L = \frac{1}{N_c} \mathrm{tr_c} \prod_t U_4(\mathbf{x}, t)$$

$$\langle L \rangle = \sum_{n=0}^{n_{\max}} a_{2n} \mu^{2n}$$

$T/T_c = 0.815$

$T/T_c = 0.95$

$T/T_c = 1.08$



fugacity expansion

$$\theta = \mu/T$$

$$Z(\mu) = \sum_{n=-\infty}^{\infty} \underline{Z_n} e^{n\mu/T} = \sum_{k\geq 0} Z_{2k}\cosh(2k\theta)$$

Z_n canonical partition function,

satisfying $Z_n = Z_{-n}, \quad Z_{n\neq kN_c} = 0$

Number density

$$\langle n \rangle = T\frac{\partial}{\partial\mu}\ln Z(\mu) = \frac{\sum 2kZ_{2k}\sinh(2k\theta)}{\sum Z_{2k}\cosh(2k\theta)}$$

Chiral condensate

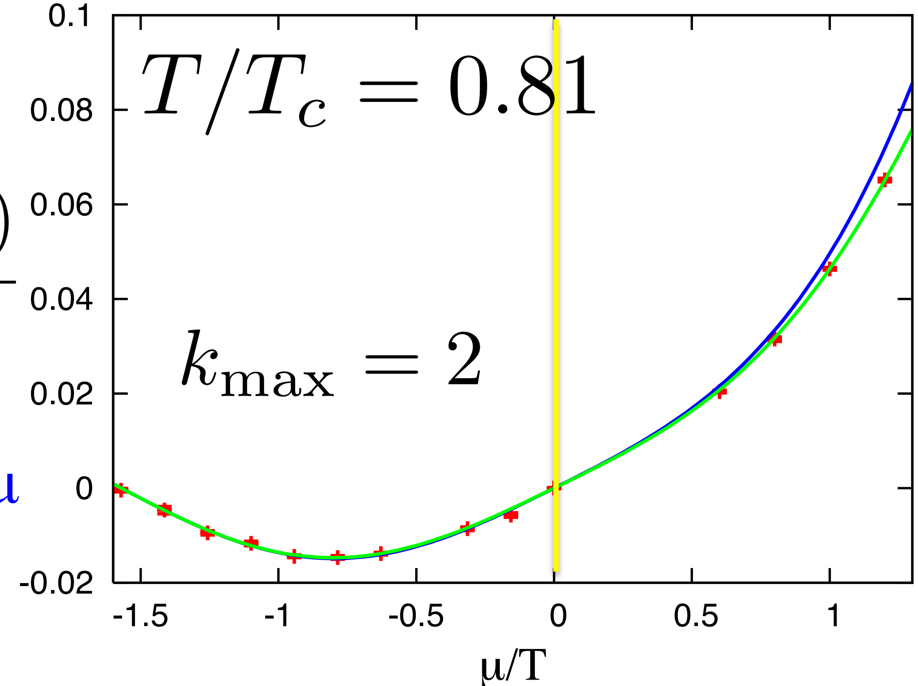
$$\langle \bar{q}q \rangle = T\frac{\partial}{\partial m_q}\ln Z(\mu) = \frac{\sum D_{2k}\cosh(2k\theta)}{\sum Z_{2k}\cosh(2k\theta)}, \quad D_{2k} = T\frac{\partial Z_{2k}}{\partial m_q}$$

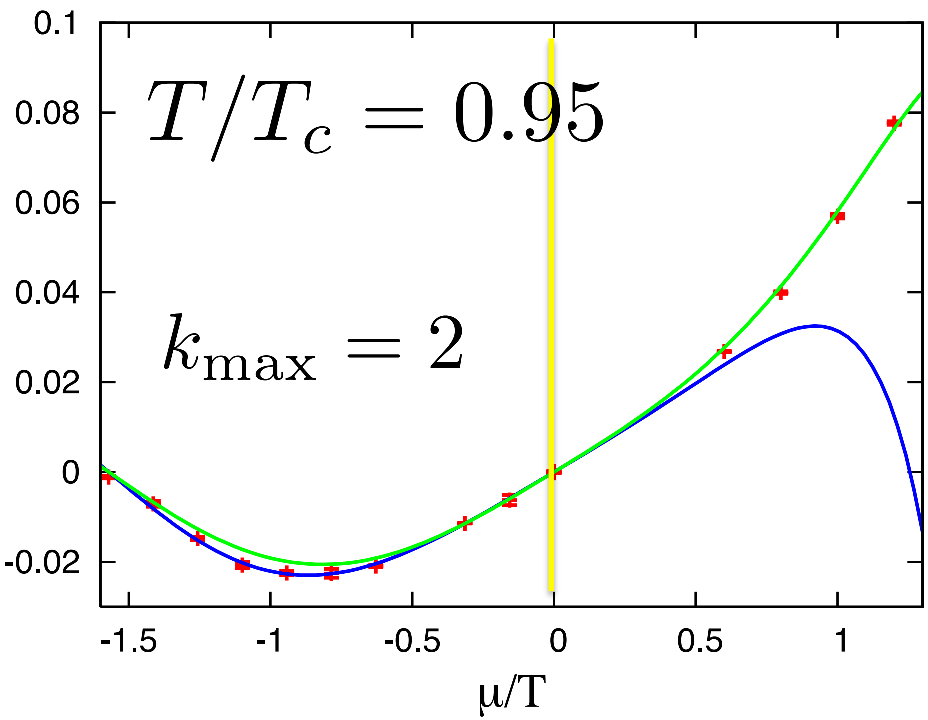
Quark Number Density

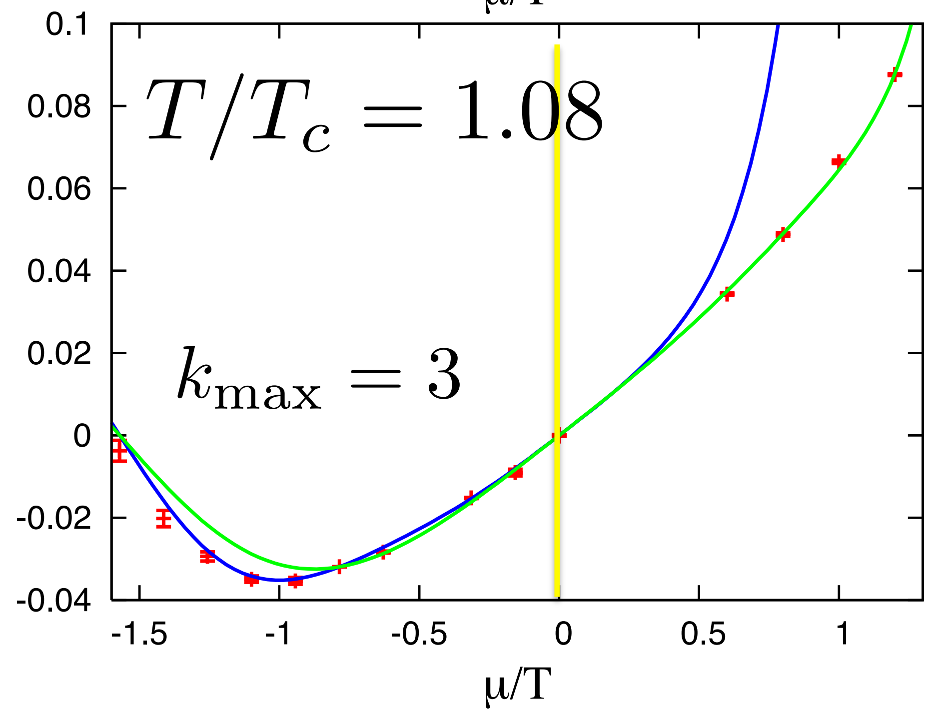
$$\langle n \rangle = \frac{\sum_{k \le k_{\max}} 2k c_{2k} \sinh(2k\theta)}{\sum_{k \le k_{\max}} c_{2k} \cosh(2k\theta)}$$

Blue : fitted c_{2k} at only imaginary μ

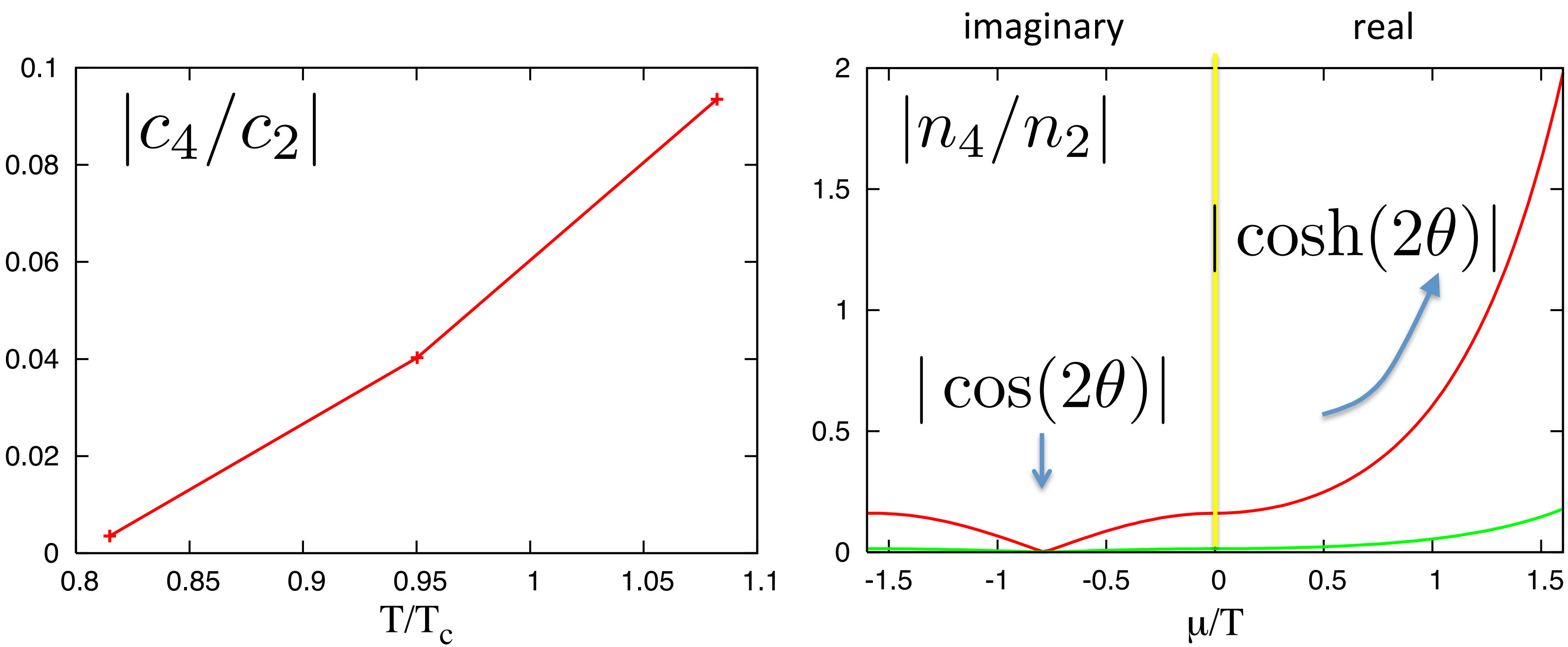
Green : at all μ region







$$\langle n \rangle = \frac{\sum_k n_{2k}}{Z(\mu)}, \quad n_{2k} = 4kc_{2k} \sinh(2k\theta)$$



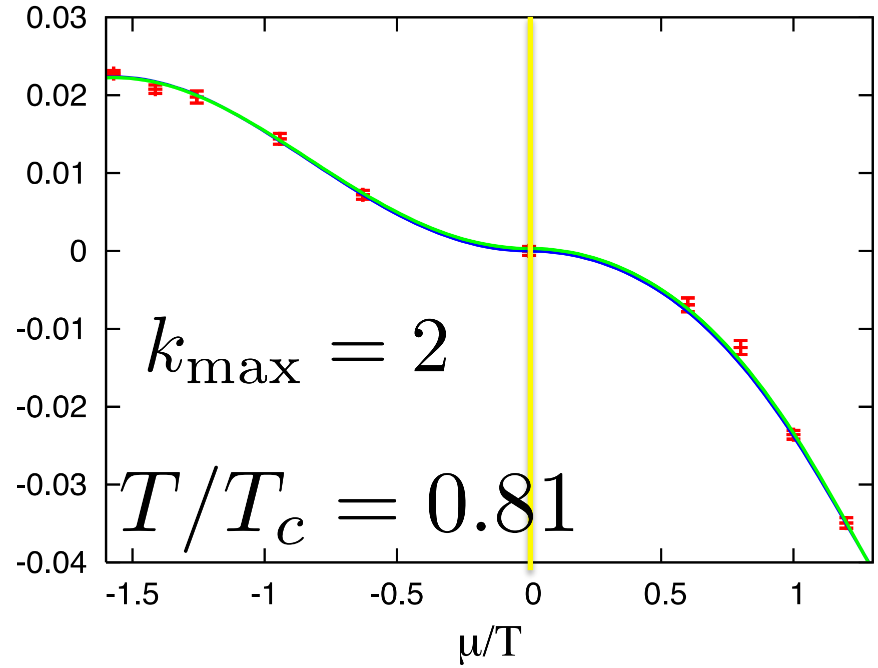
In order to predict large real μ,
we have to evaluate higher order correction at imaginary μ.

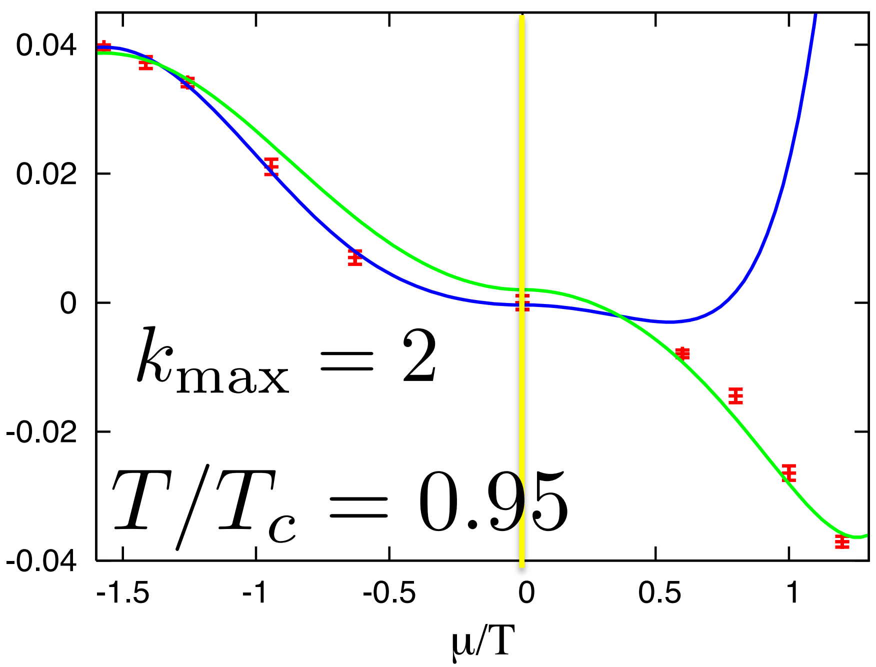
Chiral Condensate

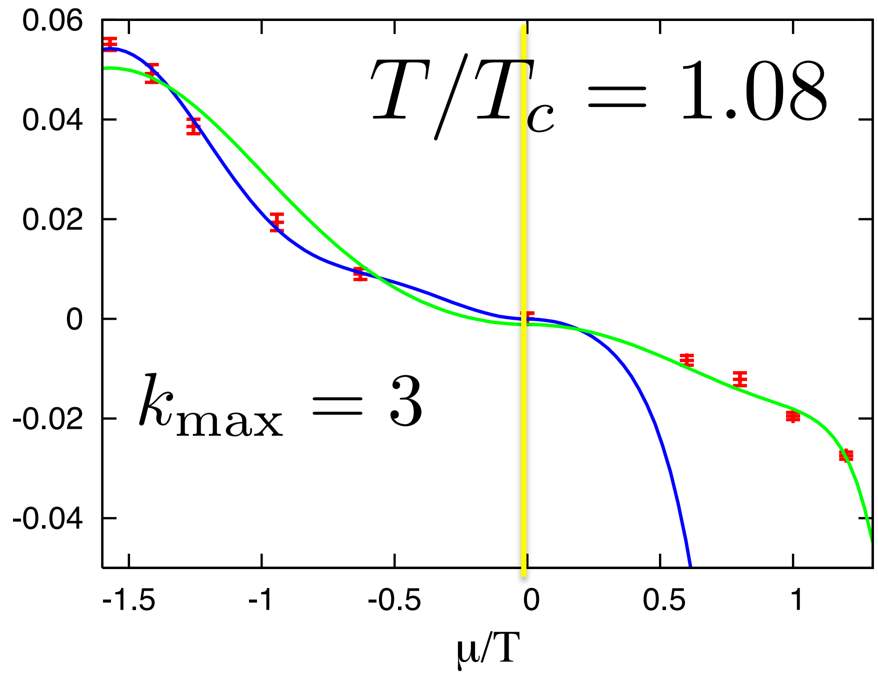
$$\langle \bar{q}q \rangle = \frac{\sum d_{2k} \cosh(2k\theta)}{Z(\mu)}$$

evaluated in n

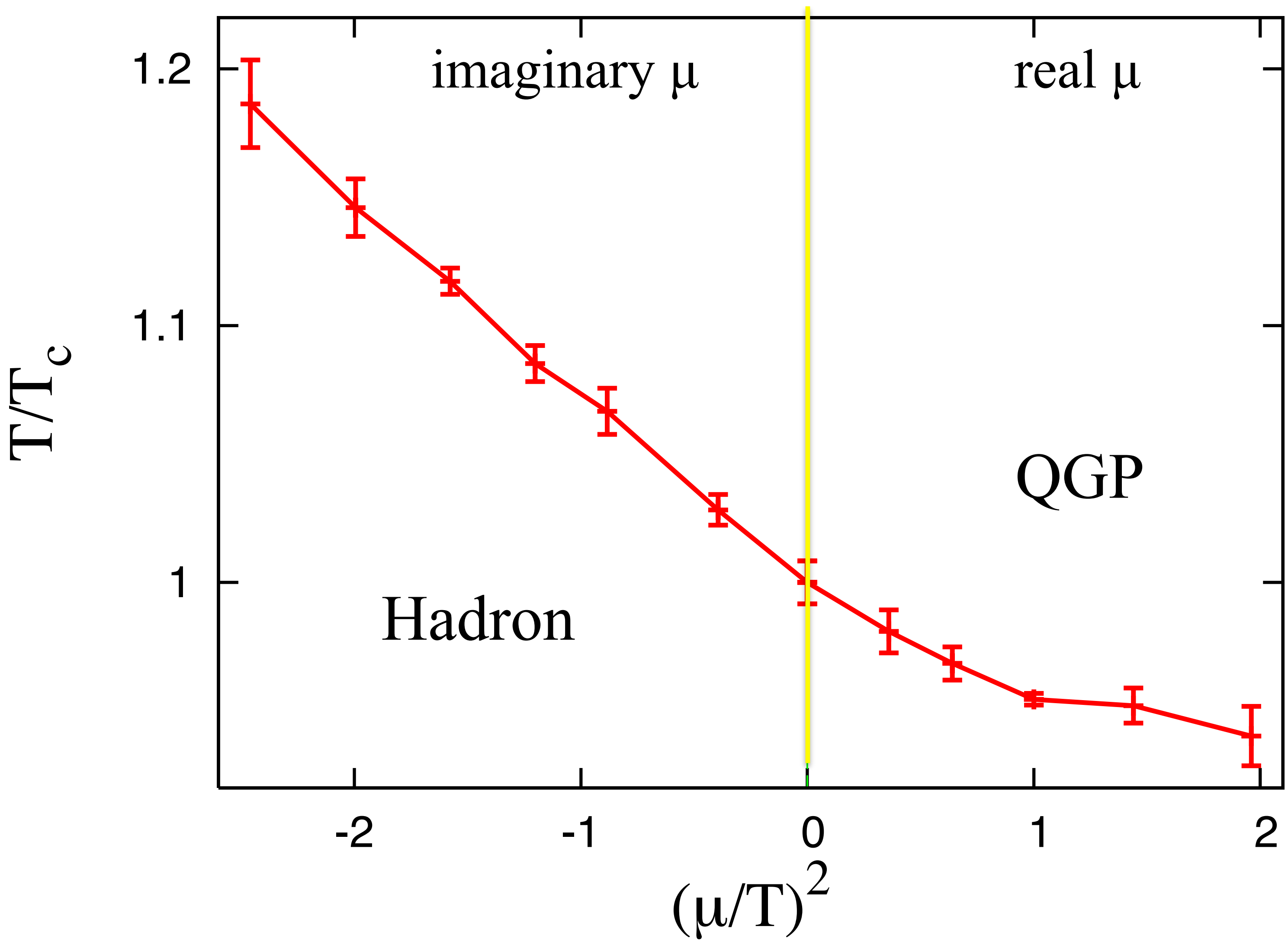
Blue : fitted d_{2k} at only imaginary μ
Green : at all μ region





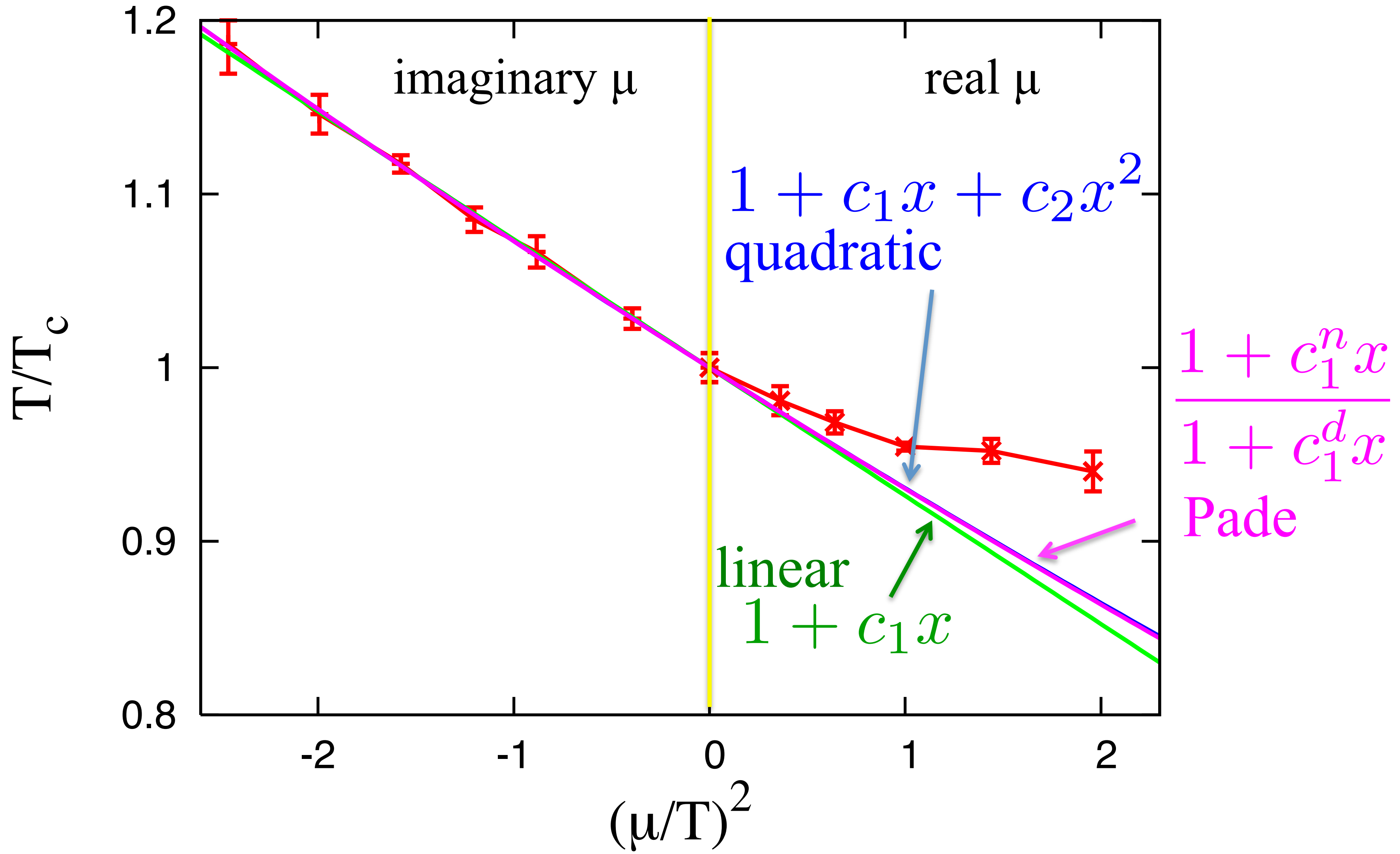


Phase Transition Line

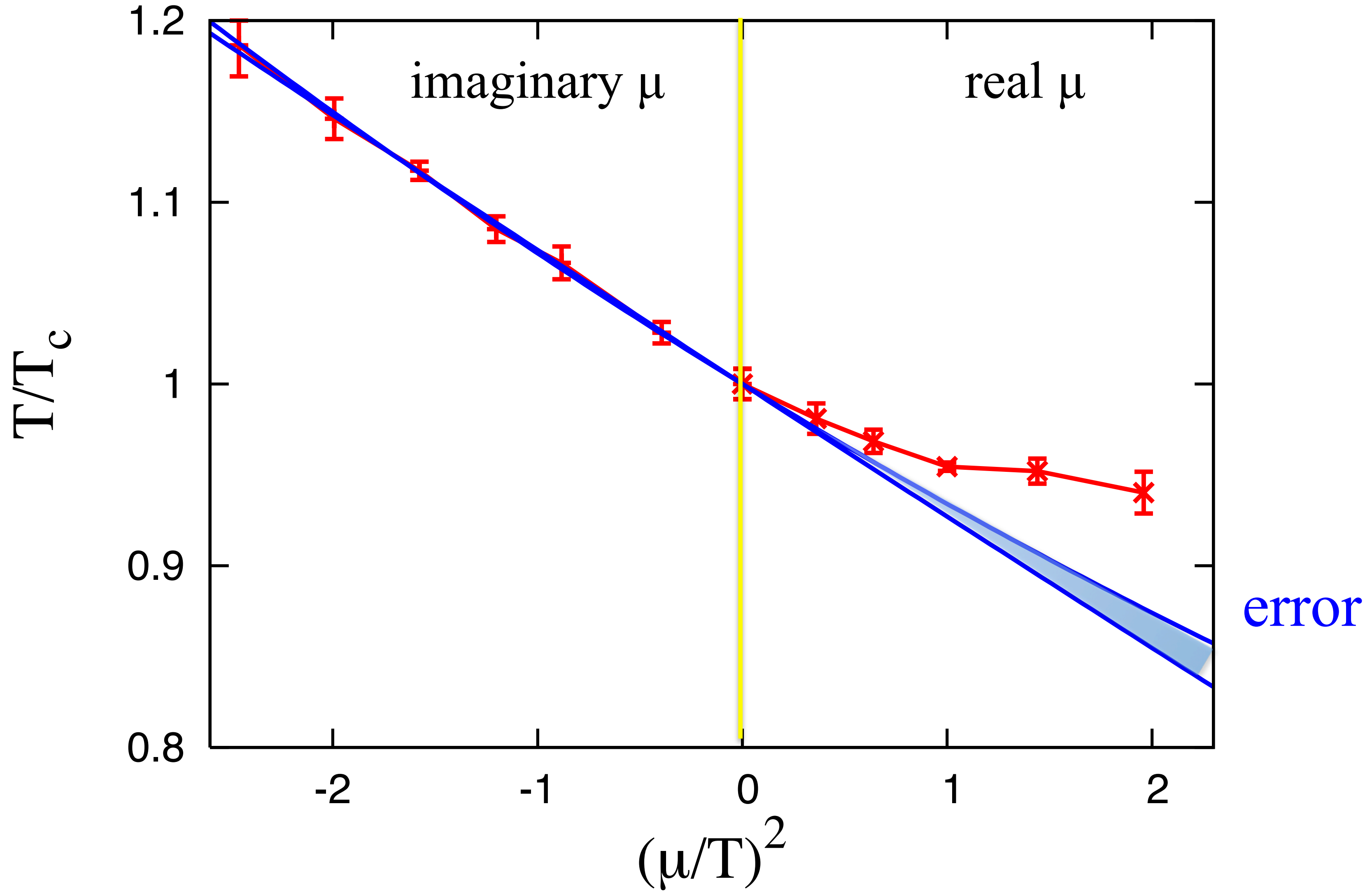


Phase Transition Line

$x = (\mu/T)^2$



Phase Transition Line



Summary

We calculated 2-color QCD phase diagram
with RG improved gauge + clover improved Wilson fermion.

We checked analytic continuation from imaginary to real μ.

For some observables, the fitted functions at imaginary μ
can reproduce the exact results well below Tc,
but the agreement becomes worse above Tc.

For the phase transition line,
the data at imaginary μ is almost on a linear function,
but the function deviates from the exact data at real μ.