Phase transitions in dense 2-colour QCD

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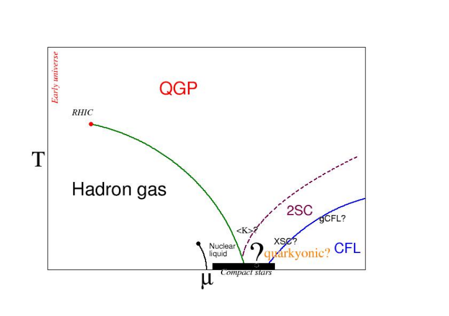
Outline

See talk by Seamus Cotter at 1830 for equation of state

Background



- A plethora of phases at high μ, low T
- Based on models and perturbation theory

Indirect approach

Study QCD-like theories without a sign problem

- Generic features of strongly interacting systems at $\mu \neq 0$
- Check on model calculations, functional methods

QC_2D vs QCD

- Baryons are bosons (diquarks); superfluid ‘nuclear matter’
- Scalar diquark is pseudo-Goldstone (degenerate with pion)
- Onset transition at $\mu_q = m_\pi/2$, not $m_N/3$

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Phase diagram

- Superfluid phase for $\mu > m_\pi/2$: BEC $\longrightarrow$ BCS?
- Exotic phases: quarkyonic, spatially varying?
- Deconfinement at high density, shape of deconfinement line?

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Gluodynamics — SU(2) and SU(3) very similar?

- Effects of deconfinement on gluon propagation?
- Gap equation with effective or one-gluon interaction used to determine superconducting gap $\rightarrow$ more realistic input?

Lattice formulation

We use Wilson fermions:

- Correct symmetry breaking pattern, Goldstone spectrum
- $N_f < 4$ needed to guarantee continuum limit
- No problems with locality, fourth root trick
- Chiral symmetry buried at bottom of Fermi sea

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$$S = \bar{\psi}_1 M(\mu)\psi_1 + \bar{\psi}_2 M(\mu)\psi_2 - J\bar{\psi}_1(C\gamma_5)\tau_2\bar{\psi}_2^T + \bar{J}\psi_2^T(C\gamma_5)\tau_2\psi_1$$

$$\gamma_5 M(\mu)\gamma_5 = M^\dagger(-\mu), \quad C\gamma_5\tau_2 M(\mu) C\gamma_5\tau_2 = -M^*(\mu)$$

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Diquark source $J \equiv \kappa j$ introduced to

- lift low-lying eigenmodes in the superfluid phase
- study diquark condensation without uncontrolled approximations

Simulation parameters

β	κ	a	am_π	m_π/m_ρ
1.9	0.168	0.18fm	0.65	0.80

μ-scans, fixed T

N_s	N_τ	T (MeV)	μa	ja
12	24	47	0.25–1.10	0.02, 0.04 (0.03)
16	24	47	0.30–0.90	0.04
12	16	70	0.30–0.90	0.04
16	12	94	0.20–0.90	0.02, 0.04
16	8	141	0.10–0.90	0.02, 0.04

250–500 trajectories used for each μ.

Simulation parameters

T-scans, fixed μ

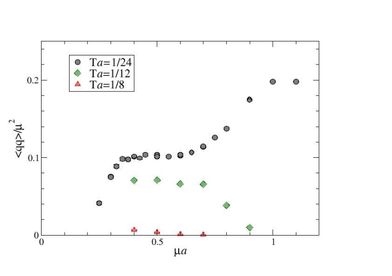
All simulations done on $16^3 \times N_\tau$ lattices

μa	ja	N_τ
0.0	0.0	4–10
0.35	0.02	4–13, 16
	0.04	4–12, 14, 16
0.40	0.02	5–13, 16
	0.04	4–13
0.50	0.02	6–12, 16
	0.04	4–16, 18, 20
0.60	0.02	6–12, 14, 16
	0.04	6–16, 20

In addition, 300 trajectories were generated at $ja = 0.03, 0.05$ for $N_\tau = 9, 10, 11$ at all $\mu > 0$.

Diquark condensate — μ-scan

Results shown are for linear extrapolation
Power law $\langle qq\rangle = Aj^{\alpha}$ works for $\mu a \lesssim 0.4$, with $\alpha = 0.85 - 0.5$.



- BCS scaling $\langle qq\rangle \sim \mu^2$ for $0.35 \lesssim \mu a \lesssim 0.7$
- Melted at $T = 141\text{MeV}$ $(N_\tau = 8)$

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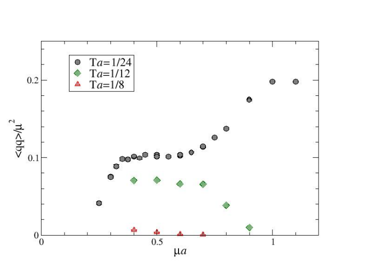
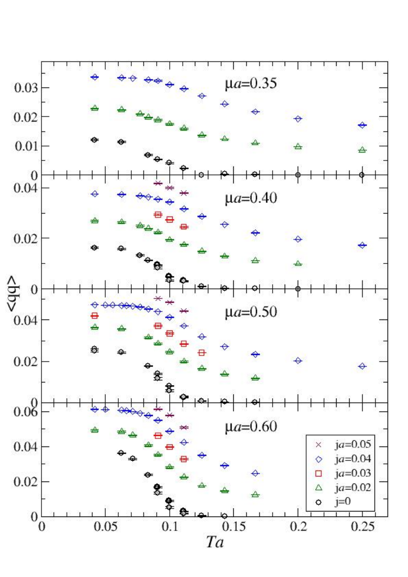


- BCS scaling $\langle qq \rangle \sim \mu^2$ for $0.35 \lesssim \mu a \lesssim 0.7$
- Melted at $T = 141\text{MeV}$ $(N_\tau = 8)$
- New transition for $\mu a \gtrsim 0.7$?
- Melting for $N_\tau = 12, \mu a \gtrsim 0.7$?

$N_\tau = 16$ results are very close to $N_\tau = 24$ results.

Superfluid to normal transition



$j \to 0$ extrapolation not fully under control
Linear form used here

Transition temperatures from inflection points (and $j \to 0$)

$a\mu$	T_s (MeV)
0.35	82(27)
0.40	94(9)
0.50	93(6)
0.60	93(7)

Remarkably constant!

Deconfinement transition

Polyakov loop L requires renormalisation,

$$L_R = e^{-F_q/T} = e^{-(F_0+\Delta F)/T} = Z_L^{N_\tau} L_0$$

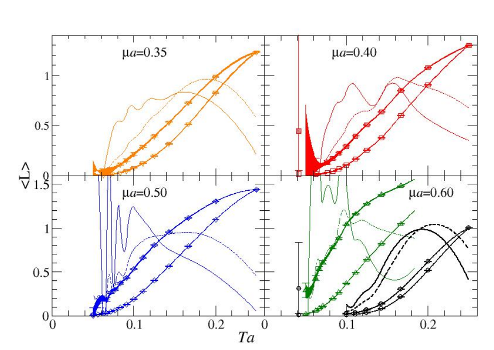
We use two schemes to determine $Z_L = \exp(-a\Delta F)$,

Scheme A $\qquad L_R(T = \frac{1}{4a}, \mu = 0) = 1\,,$

Scheme B $\qquad L_R(T = \frac{1}{4a}, \mu = 0) = 0.5\,.$

We determine the deconfinement temperature (crossover region) from the inflection point (linear region) of L_R

Deconfinement transition



Scheme dependence ⟷
broad crossover?

Estimates from Scheme B,
encompassing Scheme A

μa	$T_d a$	T_d (MeV)
0.0	0.193(20)	217(23)
0.35	0.140–0.220	157–247
0.40	0.108–0.200	121–225
0.50	0.080–0.200	90–225
0.60	0.060–0.135	67–152

Gluon propagator

- Essential ingredient in gap equation

$$S^{-1}(p) = S_0^{-1}(p) + Z_2 \int d^4q \Gamma_\mu(p,q) D_{\mu\nu}(q) S(p-q)\gamma_\nu$$

  used to determine dynamical fermion mass and superfluid/superconducting gap
- Link to functional methods (DSE, FRG)
- Electric gluon may signal deconfinement transition

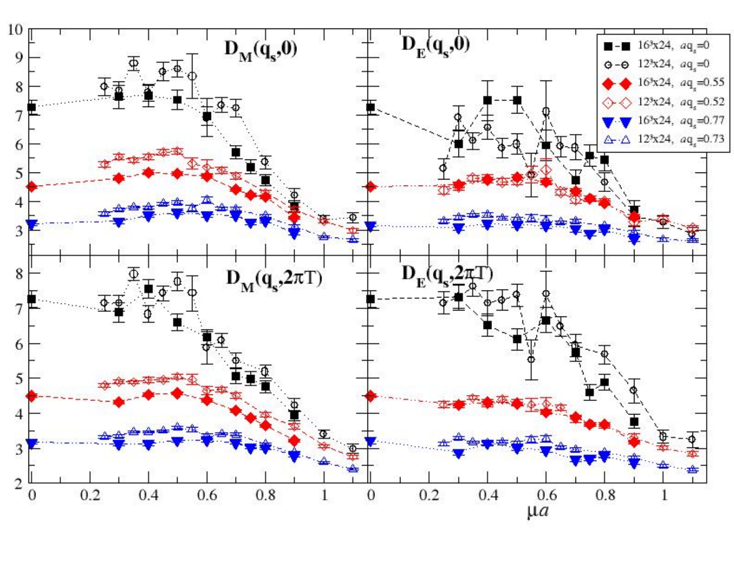
Tensor structure in medium

$$D_{\mu\nu}(\vec{q}, q_0) = P^T_{\mu\nu} D_M(\vec{q}^{\,2}, q_0^2) + P^E_{\mu\nu} D_E(\vec{q}^{\,2}, q_0^2) + \xi \frac{q_\mu q_\nu}{(q^2)^2}$$

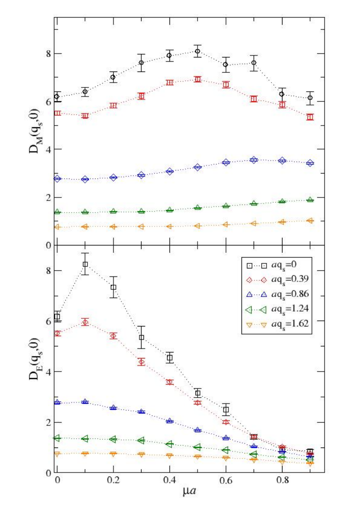
$$P^M_{\mu\nu}(\vec{q}, q_0) = (1-\delta_{0\mu})(1-\delta_{0\nu})(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{\vec{q}^{\,2}}),$$

$$P^E_{\mu\nu}(q_0, \vec{q}) = (\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) - P^M_{\mu\nu}(q_0, \vec{q}).$$

Gluon propagator: μ-scans, $N_\tau = 24$

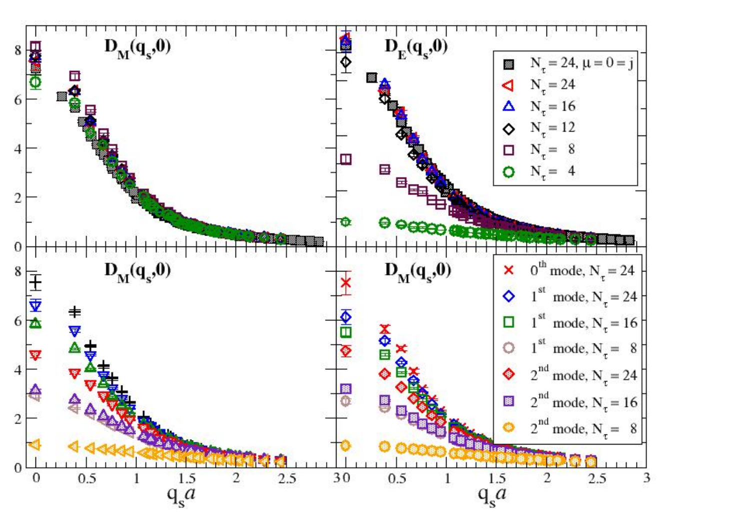


Gluon propagator: μ-scans, $N_\tau = 8$



- Very little volume dependence
- No j-dependence observed
- All modes screened at high μ, low T:
  Weak-coupling theory says static D_M is unscreened
- Static magnetic gluon not screened at high T

Gluon propagator: T-scan, $\mu a = 0.5$



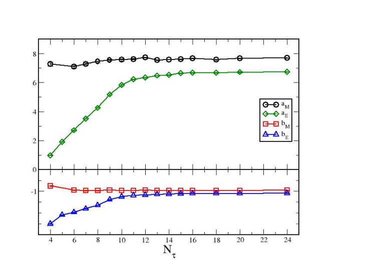
Gluon propagator fits

2-parameter fit:

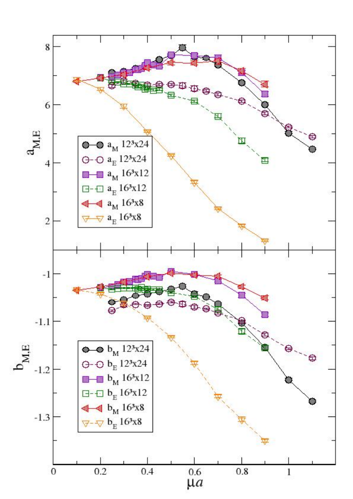
$$D_k^{\mathrm{fit}}(q^2) \;=\; \frac{\Lambda^2(q^2+\Lambda^2 a_k)^{-b_k}}{(q^2+\Lambda^2)^2}$$

with $k = M, E$ and
$\Lambda a = 0.999(3)$ from $\mu = j = 0$ fit

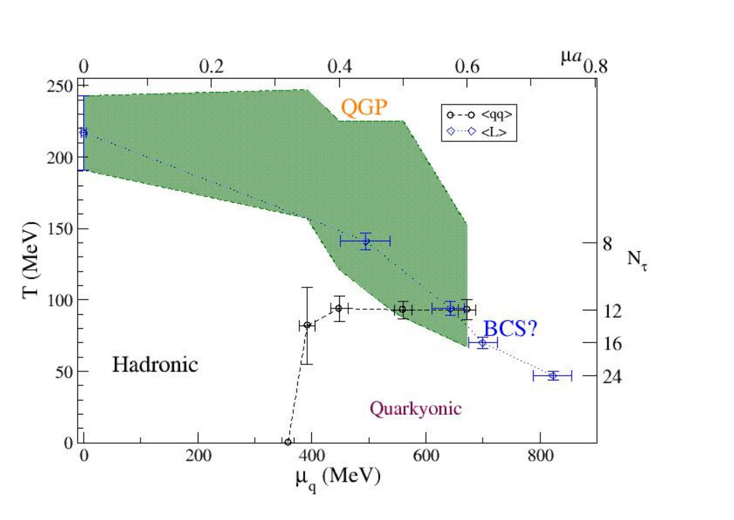
T-scan



μ-scans



Summary


0
0.2
0.4
0.6
μa 0.8
250
200
150
100
50
0
T (MeV)
QGP
<qq>
<L>
8
12
16
24
N_τ
BCS?
Hadronic
Quarkyonic
0
200
400
600
800
μ_q (MeV)

Summary

Evidence for three phases/regions

- Vacuum/hadronic phase below $\mu_o = m_\pi/2$, low T
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- Situation at very high μ unclear
- Possible deconfined superfluid region?

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Outlook

- Attempt O(2) scaling fit for superfluid to normal transition?
  - $\rightarrow$ Requires several larger lattice volumes
- Additional fit functions for gluon propagator in progress
- Quark propagators in progress
- Smaller quark mass at fixed lattice spacing in progress
- Finer lattice at same quark mass in progress

Energy density / EOS: See talk by Seamus Cotter at 1830