Phase transitions in dense 2-colour QCD

Jon-Ivar Skullerud
with Tamer Boz, Seamus Cotter, Leonard Fister
Pietro Giudice, Simon Hands
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Outline

Background
QC$_2$D vs QCD
Lattice formulation

Phase transitions
Superfluid to normal
Deconfinement

Gluon propagator

Summary

See talk by Seamus Cotter at 1830 for equation of state
Background

▷ A plethora of phases at high $\mu$, low $T$
▷ Based on models and perturbation theory

Indirect approach

Study QCD-like theories without a sign problem

▷ Generic features of strongly interacting systems at $\mu \neq 0$
▷ Check on model calculations, functional methods
QC$_2$D vs QCD

- Baryons are **bosons** (diquarks); superfluid ‘nuclear matter’
- Scalar diquark is pseudo-Goldstone (degenerate with pion)
- Onset transition at $\mu_q = m_\pi/2$, not $m_N/3$
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Phase diagram

- Superfluid phase for $\mu > m_\pi/2$: BEC $\rightarrow$ BCS?
- Exotic phases: quarkyonic, spatially varying?
- Deconfinement at high density, shape of deconfinement line?
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**Phase diagram**

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- Deconfinement at high density, shape of deconfinement line?

**Gluodynamics** — SU(2) and SU(3) very similar?

- Effects of deconfinement on gluon propagation?
- Gap equation with effective or one-gluon interaction used to determine superconducting gap $\rightarrow$ more realistic input?
Lattice formulation

We use **Wilson fermions**:  
- Correct symmetry breaking pattern, Goldstone spectrum  
- $N_f < 4$ needed to guarantee continuum limit  
- No problems with locality, fourth root trick  
- Chiral symmetry buried at bottom of Fermi sea
Lattice formulation

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$$S = \bar{\psi}_1 M(\mu) \psi_1 + \bar{\psi}_2 M(\mu) \psi_2 - J \bar{\psi}_1 (C \gamma_5) \tau_2 \bar{\psi}_2^T + \bar{J} \psi_2^T (C \gamma_5) \tau_2 \psi_1$$

$$\gamma_5 M(\mu) \gamma_5 = M^\dagger(-\mu), \quad C \gamma_5 \tau_2 M(\mu) C \gamma_5 \tau_2 = -M^*(\mu)$$
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\]

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\]

Diquark source $J \equiv \kappa j$ introduced to

- lift low-lying eigenmodes in the superfluid phase
- study diquark condensation without uncontrolled approximations
### Simulation parameters

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\kappa$</th>
<th>$a$</th>
<th>$am_\pi$</th>
<th>$m_\pi/m_\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.9</td>
<td>0.168</td>
<td>0.18fm</td>
<td>0.65</td>
<td>0.80</td>
</tr>
</tbody>
</table>

$\mu$-scans, fixed $T$

<table>
<thead>
<tr>
<th>$N_s$</th>
<th>$N_\tau$</th>
<th>$T$ (MeV)</th>
<th>$\mu a$</th>
<th>$ja$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>24</td>
<td>47</td>
<td>0.25–1.10</td>
<td>0.02, 0.04 (0.03)</td>
</tr>
<tr>
<td>16</td>
<td>24</td>
<td>47</td>
<td>0.30–0.90</td>
<td>0.04</td>
</tr>
<tr>
<td>12</td>
<td>16</td>
<td>70</td>
<td>0.30–0.90</td>
<td>0.04</td>
</tr>
<tr>
<td>16</td>
<td>12</td>
<td>94</td>
<td>0.20–0.90</td>
<td>0.02, 0.04</td>
</tr>
<tr>
<td>16</td>
<td>8</td>
<td>141</td>
<td>0.10–0.90</td>
<td>0.02, 0.04</td>
</tr>
</tbody>
</table>

250–500 trajectories used for each $\mu$. 
## Simulation parameters

*T*-scans, fixed $\mu$

All simulations done on $16^3 \times N_\tau$ lattices

<table>
<thead>
<tr>
<th>$\mu a$</th>
<th>$ja$</th>
<th>$N_\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>4–10</td>
</tr>
<tr>
<td>0.35</td>
<td>0.02</td>
<td>4–13, 16</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>4–12, 14, 16</td>
</tr>
<tr>
<td>0.40</td>
<td>0.02</td>
<td>5–13, 16</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>4–13</td>
</tr>
<tr>
<td>0.50</td>
<td>0.02</td>
<td>6–12, 16</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>4–16, 18, 20</td>
</tr>
<tr>
<td>0.60</td>
<td>0.02</td>
<td>6–12, 14, 16</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>6–16, 20</td>
</tr>
</tbody>
</table>

In addition, 300 trajectories were generated at $ja = 0.03, 0.05$ for $N_\tau = 9, 10, 11$ at all $\mu > 0$. 

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[Note: The table and text are extracted from the provided image, with formatting adjusted for improved readability.]
Diquark condensate — $\mu$-scan

Results shown are for linear extrapolation
Power law $\langle qq \rangle = A j^\alpha$ works for $\mu a \lesssim 0.4$, with $\alpha = 0.85 - 0.5$.

- BCS scaling $\langle qq \rangle \sim \mu^2$ for $0.35 \lesssim \mu a \lesssim 0.7$
- Melted at $T = 141\text{MeV}$ ($N_T = 8$)
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- BCS scaling $\langle qq \rangle \sim \mu^2$ for $0.35 \lesssim \mu a \lesssim 0.7$
- Melted at $T = 141$ MeV ($N_\tau = 8$)
- New transition for $\mu a \gtrsim 0.7$?
- Melting for $N_\tau = 12$, $\mu a \gtrsim 0.7$?

$N_\tau = 16$ results are very close to $N_\tau = 24$ results.
Superfluid to normal transition

\[ j \to 0 \text{ extrapolation not fully under control} \]

Linear form used here

Transition temperatures from inflection points (and \( j \to 0 \))

<table>
<thead>
<tr>
<th>(a\mu)</th>
<th>(T_s) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>82(27)</td>
</tr>
<tr>
<td>0.40</td>
<td>94(9)</td>
</tr>
<tr>
<td>0.50</td>
<td>93(6)</td>
</tr>
<tr>
<td>0.60</td>
<td>93(7)</td>
</tr>
</tbody>
</table>

Remarkably constant!
Deconfinement transition

Polyakov loop $L$ requires renormalisation,

$$L_R = e^{-F_q/T} = e^{- (F_0 + \Delta F)/T} = L_0^{N_T}$$

We use two schemes to determine $Z_L = \exp(-a\Delta F)$,

**Scheme A**

$$L_R(T = \frac{1}{4a}, \mu = 0) = 1,$$

**Scheme B**

$$L_R(T = \frac{1}{4a}, \mu = 0) = 0.5.$$ 

We determine the deconfinement temperature (crossover region) from the inflection point (linear region) of $L_R$. 
Deconfinement transition

Estimates from Scheme B, encompassing Scheme A

<table>
<thead>
<tr>
<th>$\mu a$</th>
<th>$T_d a$</th>
<th>$T_d$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.193(20)</td>
<td>217(23)</td>
</tr>
<tr>
<td>0.35</td>
<td>0.140–0.220</td>
<td>157–247</td>
</tr>
<tr>
<td>0.40</td>
<td>0.108–0.200</td>
<td>121–225</td>
</tr>
<tr>
<td>0.50</td>
<td>0.080–0.200</td>
<td>90–225</td>
</tr>
<tr>
<td>0.60</td>
<td>0.060–0.135</td>
<td>67–152</td>
</tr>
</tbody>
</table>

Scheme dependence $\leftrightarrow$ broad crossover?
Gluon propagator

- Essential ingredient in gap equation
  \[ S^{-1}(p) = S_0^{-1}(p) + Z_2 \int d^4q \Gamma_\mu(p, q)D_{\mu\nu}(q)S(p - q)\gamma_\nu \]
  used to determine dynamical fermion mass and superfluid/superconducting gap
- Link to functional methods (DSE, FRG)
- Electric gluon may signal deconfinement transition

Tensor structure in medium

\[ D_{\mu\nu}(\vec{q}, q_0) = P^T_{\mu\nu}D_M(\vec{q}^2, q_0^2) + P^E_{\mu\nu}D_E(\vec{q}^2, q_0^2) + \xi \frac{q_\mu q_\nu}{(q^2)^2} \]

\[ P^M_{\mu\nu}(\vec{q}, q_0) = (1 - \delta_{0\mu})(1 - \delta_{0\nu})(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) , \]

\[ P^E_{\mu\nu}(q_0, \vec{q}) = (\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) - P^M_{\mu\nu}(q_0, \vec{q}) . \]
Gluon propagator: $\mu$-scans, $N_T = 24$
Gluon propagator: $\mu$-scans, $N_T = 8$

- Very little volume dependence
- No $j$-dependence observed
- All modes screened at high $\mu$, low $T$:
  Weak-coupling theory says static $D_M$ is unscreened
- Static magnetic gluon not screened at high $T$
Gluon propagator: $T$-scan, $\mu a = 0.5$
Gluon propagator fits

2-parameter fit:

\[ D_k^{\text{fit}}(q^2) = \frac{\Lambda^2(q^2 + \Lambda^2 a_k)^{-b_k}}{(q^2 + \Lambda^2)^2} \]

with \( k = M, E \) and
\( \Lambda a = 0.999(3) \) from \( \mu = j = 0 \) fit

\( \mu \)-scans

T-scan
Summary

The diagram illustrates the phase transitions in a system with varying temperature ($T$) and chemical potential ($\mu$). The region labeled QGP (Quark Gluon Plasma) is highlighted, indicating a phase transition at high temperatures and chemical potentials. The critical points and phase boundaries are demarcated with markers and lines.

- Hadronic phase is shown at lower temperatures and chemical potentials.
- Quarkyonic phase appears at higher temperatures and chemical potentials.

The diagram also includes a section marked BCS, possibly referring to a Bose-Einstein Condensation phase transition, although its exact relevance is unclear from the image alone.
Summary

Evidence for three phases/regions

- Vacuum/hadronic phase below $\mu_o = m_\pi/2$, low $T$
- BCS/quarkyonic for intermediate $\mu$, low $T$
- Deconfined/QGP matter at high $T$
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- Situation at very high $\mu$ unclear
- Possible deconfined superfluid region?
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- Both electric and magnetic gluon screened at high $\mu$, low $T$
- Static magnetic gluon not screened at high $T$
Outlook

- Attempt O(2) scaling fit for superfluid to normal transition?
  → Requires several larger lattice volumes
- Additional fit functions for gluon propagator in progress
- Quark propagators in progress
- Smaller quark mass at fixed lattice spacing in progress
- Finer lattice at same quark mass in progress

Energy density / EOS: See talk by Seamus Cotter at 1830