The Hadronic Decays of Decuplet Baryons

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Introduction

Maiani Testa no-go theorem

There are two main methods that have been considered for the difficult task of directly extracting resonance decay rates from lattice data.

Analysis of energy levels in a finite box
Lüscher; Meissner

Analysis of the time dependence of Greens functions
Michael ; McNeile

Methods

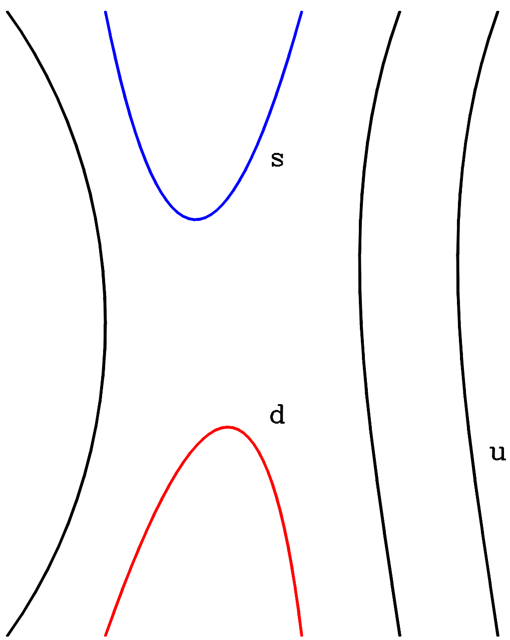
We need very reliable energy level values, both for the ground state and the first few excited states.

Many of the important energy eigenstates look like baryon + meson rather than single baryons.

Methods

Solution: Look at a matrix of correlators, including meson+ baryon operators.
(currently, largest matrix considered is 56×56, for Δ with momentum.)

Efficient meson + baryon states: Use perambulators Peardon



Methods

$2+1$ flavour simulations. $24^3 \times 48$ and $32^3 \times 64$.
$(m_u + m_d + m_s)/3$ held constant (at physical value). Data with all three quark masses equal, and with $m_s > m_u, m_d$.
Start at

$$m_u = m_d = m_s \approx m_s^{phys}/3$$

Move towards physical point by increasing m_s, decreasing m_l.

Methods

$2+1$ flavour simulations.

Concentrate on states with total charge $+2$. Simplifies matters a little - even in the spin $\frac{1}{2}$ sector we don't need to worry about mixing with nucleon states, the only 3-quark states will be spin $\frac{1}{2}$ Δs (which do appear in the PDG listings).

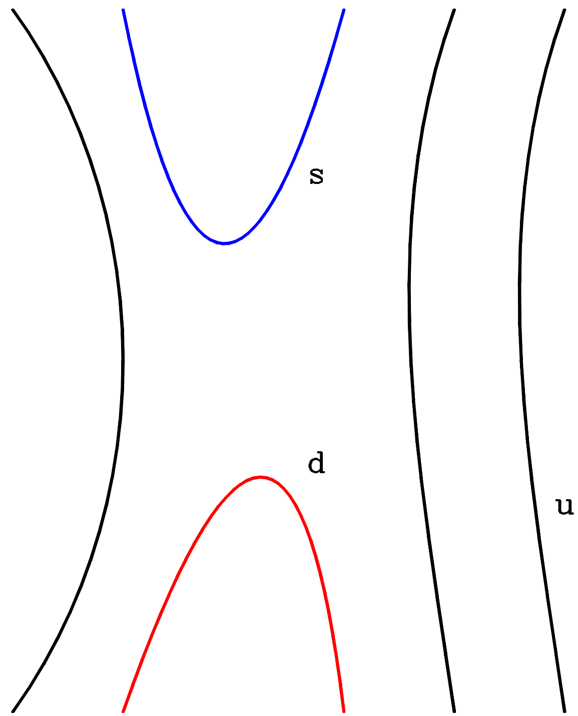
$N\pi \to \Sigma K$

Concentrate on one channel $p\pi^+ \to \Sigma^+ K^+$
S-wave.

At $SU(3)$ symmetric point initial and final states both have equal mass.

$$m_u = m_d = m_s \approx m_s^{phys}/3$$

$N\pi \to \Sigma K$



One of the 6 contributions to $p\pi^+ \to \Sigma^+ K^+$.

$N\pi \to N\pi$ and $\Sigma K \to \Sigma K$ require additional terms.

$N\pi \to \Sigma K$

Concentrate on one channel $p\pi^+ \to \Sigma^+ K^+$

Interaction
$$\begin{pmatrix} E_0 & g \\ g & E_0 \end{pmatrix}$$

Splits the levels
One combination is decuplet $(|p\pi^+\rangle + |\Sigma^+ K^+\rangle)/\sqrt{2}$
Can convert into qqq state (a Δ).

One is 27-plet (pentaquark). $(|p\pi^+\rangle - |\Sigma^+ K^+\rangle)/\sqrt{2}$
Always $qqqq\bar{q}$ (or higher).

$N\pi \to \Sigma K$

Find g from splitting distance
Both states have energy higher than $M_N + M_\pi$ - implies repulsive force.

$$
\begin{aligned}
M_N + M_\pi &= 0.659(1) \\
E_{10} &= 0.673(3) \\
E_{27} &= 0.686(3)
\end{aligned}
$$

The splitting between the two levels looks very robust.

$N\pi \to \Sigma K$

Find g from splitting distance
Both states have energy higher than $M_N + M_\pi$ - implies repulsive force.

$$
\begin{aligned}
M_N + M_\pi &= 0.659(1) \\
E_{10} &= 0.673(3) \\
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\end{aligned}
$$

Without a matrix of correlators, it would be very difficult to split two close levels, or even to be sure that there were two levels, not just one.

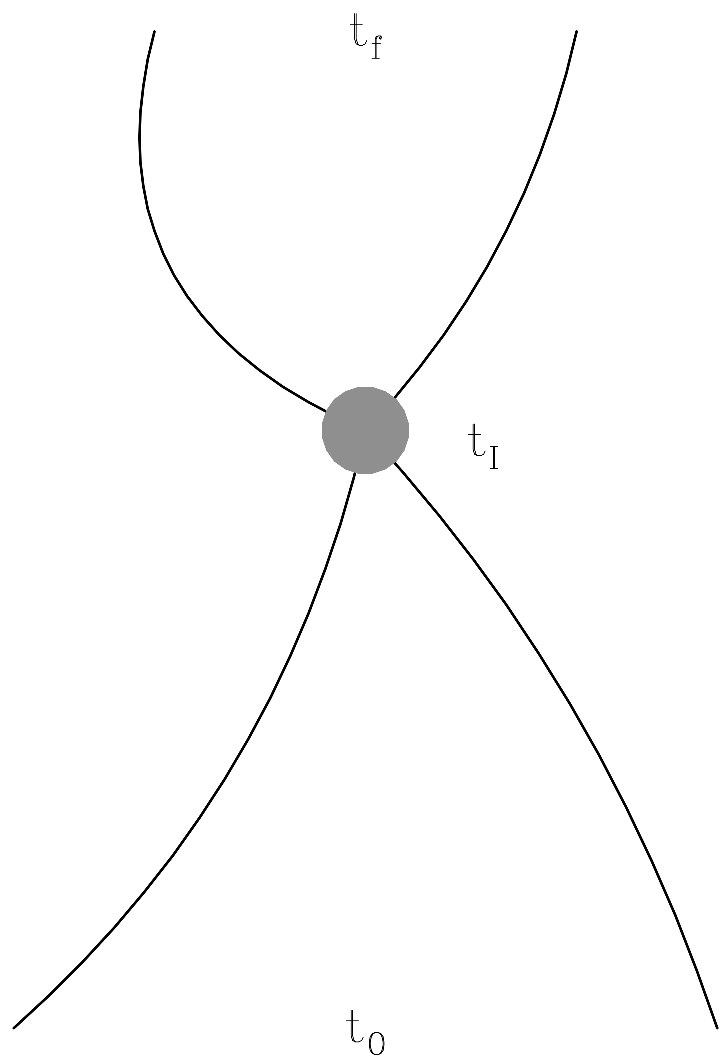
$N\pi \to \Sigma K$

Alternatively, coupling term from transition rate.
Chris Michael, Craig McNeile
Coupling constant for transition $A \to B$, find ratio

$$\frac{C_{AB}(t)}{\sqrt{C_{AA}(t)C_{BB}(t)}}$$

(requires that A and B have similar energies).
$p\pi^+$ at source, $\Sigma^+ K^+$ at sink.

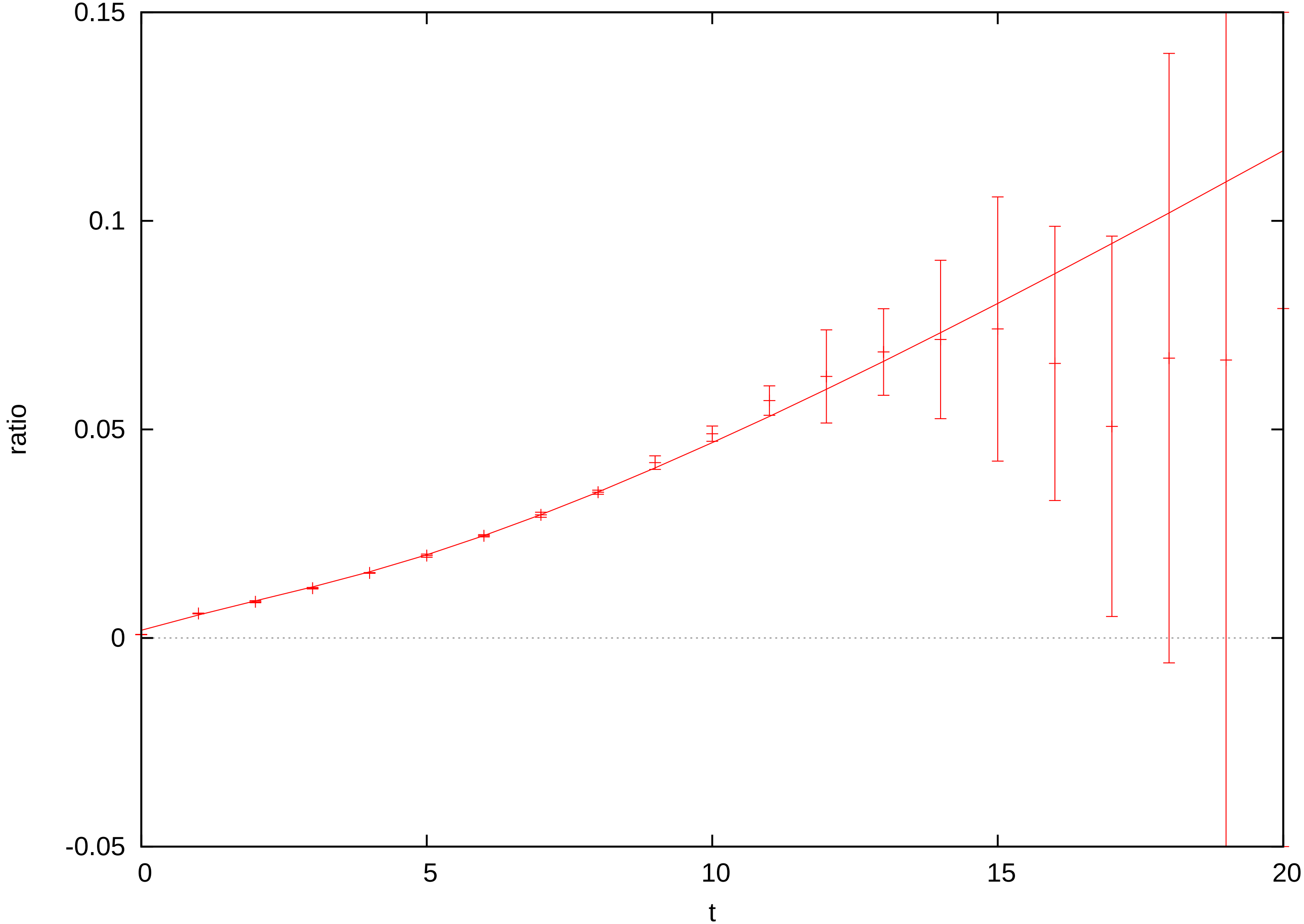
$N\pi \to \Sigma K$



$N\pi \to \Sigma K$

Alternatively, coupling from transition rate.
$p\pi^+$ at source, $\Sigma^+ K^+$ at sink.
Integrate over interaction time t_I, amplitude $\sim g(t_f - t_0)$
Amplitude growing linearly, slope proportional to g. Agrees with value from level splitting
$ag = 0.008(2)$ for interaction term in Hamiltonian.

$N\pi \to \Sigma K$



$N\pi \to \Sigma K$

What happened to the Maiani Testa theorem?
In this case, the splitting between the ground state and the first excited state is so small that we would need to trace the Greens function out to times ~ 200 lattice spacings before the excited state dies away!

$N\pi \to \Sigma K$

Fermi's Golden rule - use transition rate in small box to estimate transition rate in infinite box: gives cross section.

$$\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2} \frac{|\mathcal{M}|^2}{(E_N + E_\pi)^2} \frac{|p_f|}{|p_i|}$$

$N\pi \to \Sigma K$

$$\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2} \frac{|\mathcal{M}|^2}{(E_N + E_\pi)^2} \frac{|p_f|}{|p_i|}$$

Putting in the masses, the value of $\mathcal{M}$ (derived from the off-diagonal term g in the Hamiltonian), we get a value for the cross-section, near threshold, at our $SU(3)$ symmetric point:

$$\sigma = 66a^2 = 0.4(2)\mathrm{fm}^2$$

$N\pi \to \Sigma K$

$$\sigma = 66a^2 = 0.4(2)\mathrm{fm}^2$$

At symmetric point, daughter particles have same masses as parent particles - phases space factors very simple.
At physical point, Σ, K heavier than N, π, will lead to suppression near the threshold.

Conclusions

Have demonstrated that we can detect the finite volume shifts in energy levels, and use them to estimate interaction strengths.

The alternative method, looking at the growth of the transition Greens function $C_{AB}(t)$

For the example we looked at in detail, S-wave scattering, $p\pi^+ \to \Sigma^+ K^+$ both methods agreed well.

We have a lot of data concerning P-wave channels, and are in the process of analysing this.