Lattice QCD studies of multi-strange baryon-bayon interactions

Kenji Sasaki (*CCS, University of Tsukuba*)

for HAL QCD collaboration

HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration

S. Aoki (*YITP*)

B. Charron (*Univ. of Tokyo*)

T. Doi (*RIKEN*)

F. Etminan (*Univ. of Tsukuba*)

T. Hatsuda (*RIKEN*)

Y. Ikeda (*RIKEN*)

T. Inoue (*Nihon Univ.*)

N. Ishii (*Univ. of Tsukuba*)

K. Murano (*YITP*)

H. Nemura (*Univ. of Tsukuba*)

M. Yamada (*Univ. of Tsukuba*)

Introduction

Baryon-baryon interactions are key to understand nuclear structures and astrophysical phenomena

Inputs for nuclear structure / reaction, astrophysical phenomenon

NN interaction

Properties of BB interactions are not known very well except for NN interaction

Realistic nucleon-nucleon potential is constructed by fitting large amount of NN scattering data

YN / YY interaction

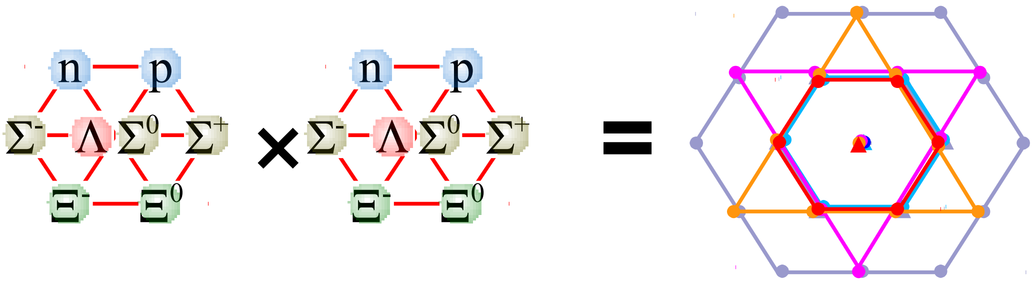
It is important to know structure of hypernucleus and deep inside of neutron star
It is not easy to access the multi-strangeness interaction experimentally.
Experimental data are insufficient to determine parameters in phenomenological YN and YY interaction model.

Lattice QCD results for YN and YY interactions are highly awaited

Introduction

Strangeness brought the deeper understanding of BB interaction.

Three flavor (u,d,s) world : SU(3) symmetric limit

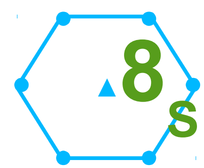


Wide variety of BB interaction

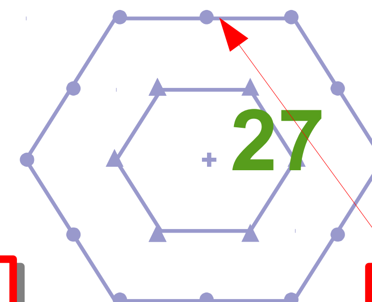
Spin singlet

Flavor symmetric

H-dibaryon state is expected



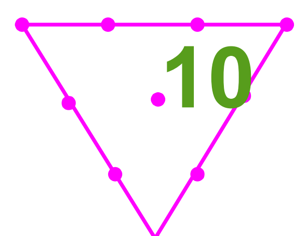
Pauli forbidden state



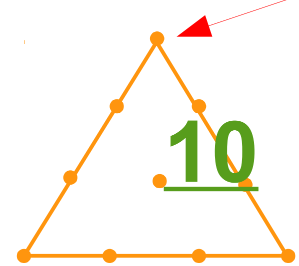
NN sector

Spin triplet

Flavor anti-symmetric

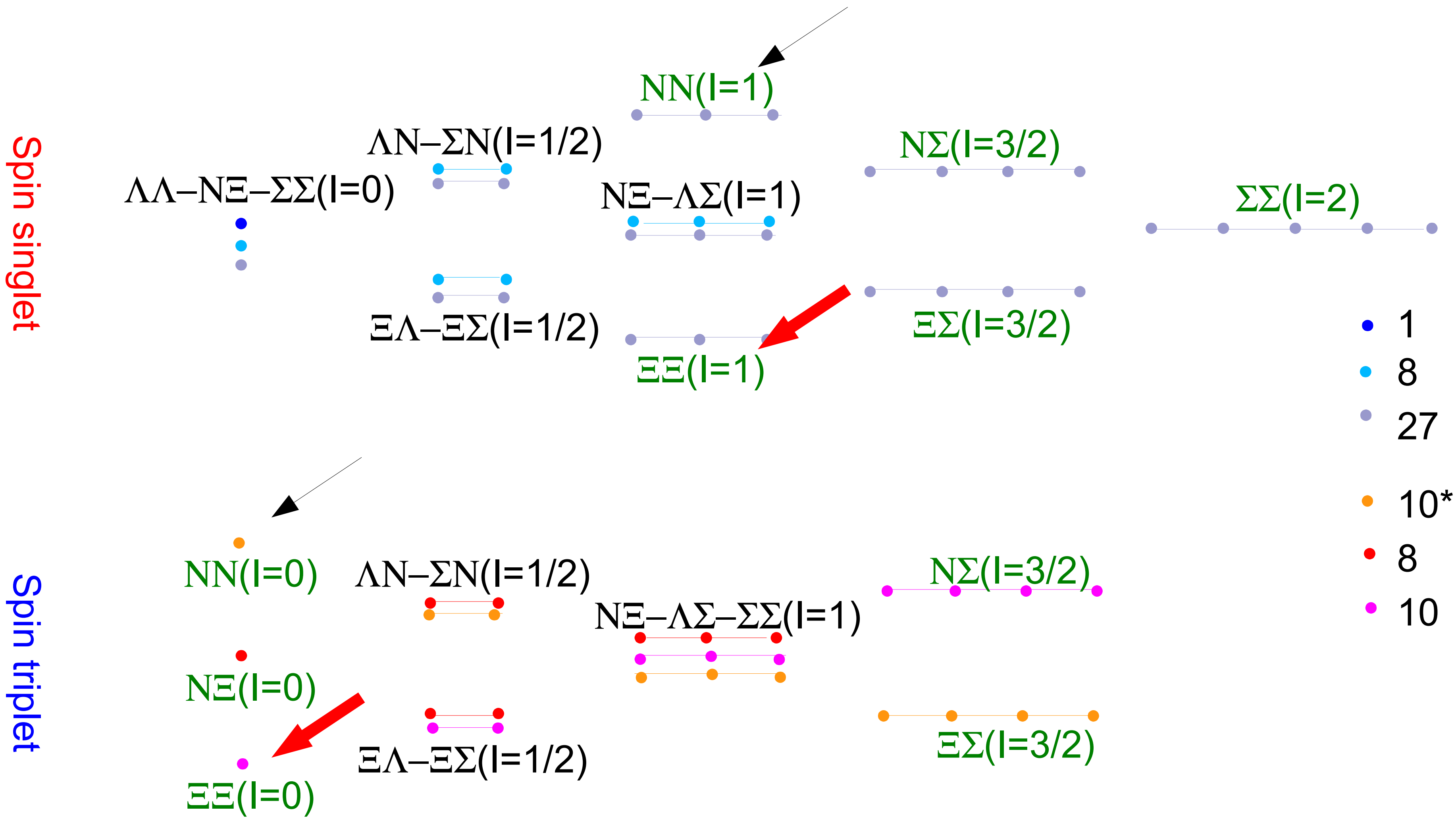


Almost forbidden state



Introduction

Three flavor (u,d,s) world : broken SU(3) symmetry



In this study, we focus on the S=-4 BB interaction, ΞΞ interaction.

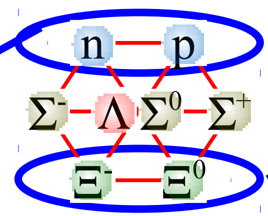
Introduction

Similarity and/or dissimilarity to NN system

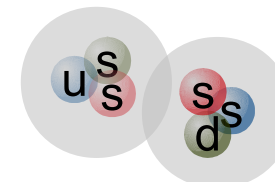
Octet baryon
Iso-doublet

Flavor component

$$p=[u\,d]u=\bar{s}\,u$$
$$n=[u\,d]d=\bar{s}\,d$$

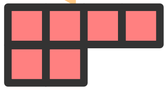


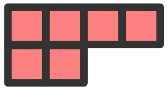
$$\Xi^0=[s\,u]s=\bar{d}\,s$$
$$\Xi\;=[s\,d]s=\bar{u}\,s$$



Conjugate flavor structure

Iso-triplet state

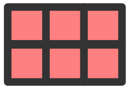


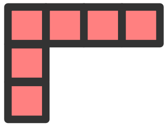


$$pp\;=\;\bar{s}\,u\,\bar{s}\,u$$
$$pn+np=\;\bar{s}\,u\,\bar{s}\,d+\bar{s}\,d\,\bar{s}\,u$$
$$nn\;=\;\bar{s}\,d\,\bar{s}\,d$$

$$\Xi^0\Xi^0\;=\;\bar{d}\,s\,\bar{d}\,s$$
$$\Xi^0\Xi+\Xi\Xi^0=\;\bar{d}\,s\,\bar{u}\,s+\bar{u}\,s\,\bar{d}\,s$$
$$\Xi\Xi\;=\;\bar{u}\,s\,\bar{u}\,s$$

Iso-singlet state



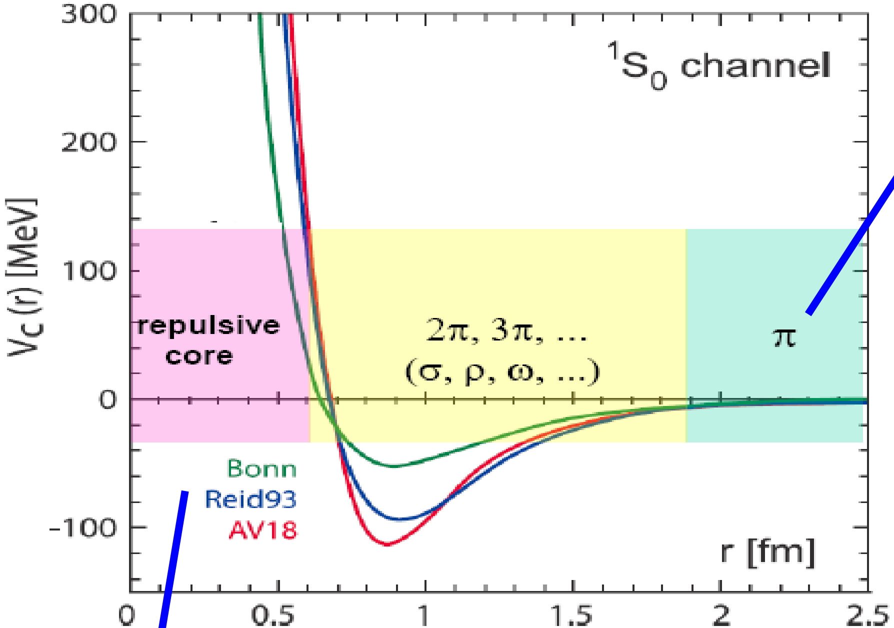


$$pn-np=\;\bar{s}\,u\,\bar{s}\,d-\bar{s}\,d\,\bar{s}\,u$$

$$\Xi^0\Xi-\Xi\Xi^0=\;\bar{d}\,s\,\bar{u}\,s-\bar{u}\,s\,\bar{d}\,s$$

Introduction

Potential



Long range part

Meson exchange contribution is dominant

When meson masses decrease,
range of potential becomes longer.

Decreasing ud-quark masses means
that the potential range extends

Short range part

Otsuki, Tamagaki, Yasuno PTPS (1965)578
Oka, Shimizu and Yazaki NPA464 (1987)

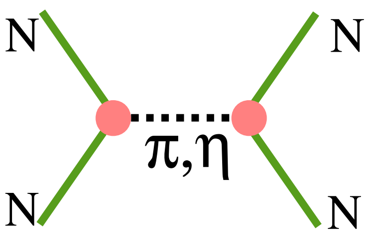
Quark degrees of freedom is important
Quark Pauli principle
Color magnetic interaction (replusive for all BB channels except for H dibaryon channel)

Introduction

Meson exchange interaction

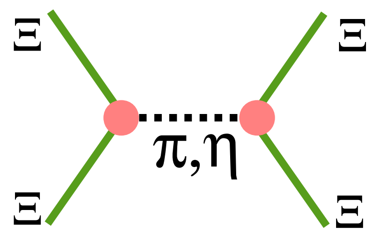
Leading contributions are given by π and η exchange contributions

- Weaker attraction
  - π exchange contribution in $\Xi\Xi$ is much weaker than NN
  - η meson mass is much heavier than the pion mass



$$\pi : g^2 f_\pi(r)$$

$$\eta : \left[\frac{4\alpha - 1}{\sqrt{3}}\right]^2 g^2 f_\eta(r)$$



$$\pi : \left[-(1-2\alpha)\right]^2 g^2 f_\pi(r)$$

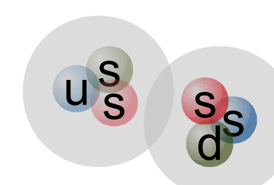
$$\eta : \left[\frac{-1+2\alpha}{\sqrt{3}}\right]^2 g^2 f_\eta(r)$$

$$\alpha = 2/5$$

Color magnetic interaction (CMI) and repulsive core

One gluon exchange ← Dominant contribution at short range region

$$V_{OGE}^{CMI} \propto \frac{1}{m_{q1} m_{q2}} \langle \lambda_1 \cdot \lambda_2 \sigma_1 \cdot \sigma_2 \rangle f(r_{ij})$$



If quark mass decreases, CMI contributions are enhanced.

Short range repulsion could be increased...

Introduction

Iso-singlet channel

We can access the potential of 10 irreducible representation.

Potential of 10 irrep is expected to be repulsive due to the quark Pauli effect.

It is contrary to NN system where deuteron bound state exist.

Iso-triplet channel

Potential of flavor 27 plet is expected to be strongly attractive

$^1S_0$ in NN system is virtual state

- EFT calculation found that the bound ΞΞ state in 1S0 channel.
  J. Haidenbauer Nucl.Phys.A881(2012)44
- Meson exchange model calculations.
  Bound or unbound...
  M. Yamaguchi PTP105(2001)627
  Y. Fujiwara PPNP 58(2007)439
- Bound ΞΞ state was found by Lattice QCD simulation at m=389MeV
  S.R. Beane PRD85(2012)054511

Search for the bound ΞΞ state is interesting to understand more about BB interaction

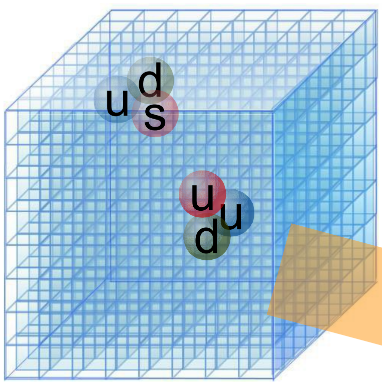
QCD to hadronic interactions

HAL QCD method can derive baryon-baryon potentaial directly from QCD

QCD Lagrangian

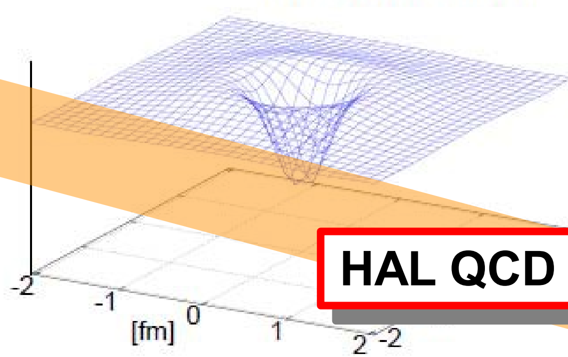
$$L_{QCD} = \bar{q}\left(i\gamma_\mu D^\mu - m\right)q + \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$$

Lattice QCD simulation



1. Measure NBS wave function on the lattice

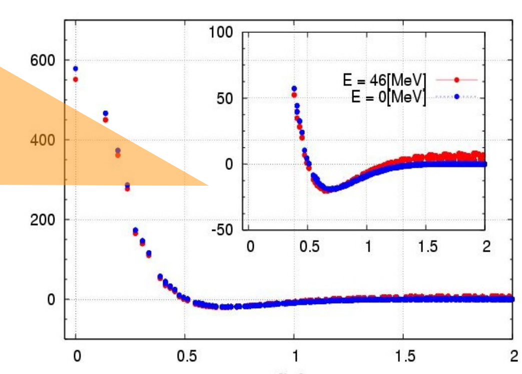
NBS wave function



The potential through our method reproduce to the phase shift by QCD

HAL QCD method

BB interaction (potential)



2. Put NBS wave function in Schroedinger eq

N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett. **99** (2007) 022001

Nambu-Bethe-Salpeter wave function

Definition : equal time NBS w.f.

$$\Psi_\nu(E, t-t_0, \vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(t, \vec{x}+\vec{r}) B_j(t, \vec{x}) | E, \nu, t_0 \rangle$$

E : Total energy of system

ν : other observables which needs to form the complete set

Four point correlator

$$F_{B_1 B_2}(\vec{r}, t) = \langle 0 | T [B_1(\vec{r}, t) B_2(0, t) (\bar{B}_2 \bar{B}_1)_{t_0}] | \rangle = \sum_n A_n \Psi_n e^{-E_n t}$$

Local composite interpolating operators

$$p = udu \quad n = udd \quad \Xi^0 = sus \quad \Xi^- = sds$$

$$\Lambda = \sqrt{\frac{1}{6}} [dsu + sud - 2uds]$$

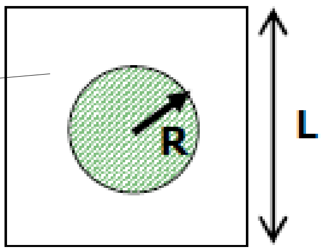
$$B = \epsilon^{abc} (q_a^T C \gamma_5 q_b) q_c$$

$$\Sigma^+ = -usu \quad \Sigma^0 = -\sqrt{\frac{1}{2}} [dsu + usd] \quad \Sigma^- = -dsd$$

NBS wave function has the same asymptotic form with quantum mechanics.

(NBS wave function is characterized from phase shift)

$$\Psi(t - t_{0,} \vec{r}) \simeq A \frac{\sin(pr + \delta(E))}{pr}$$

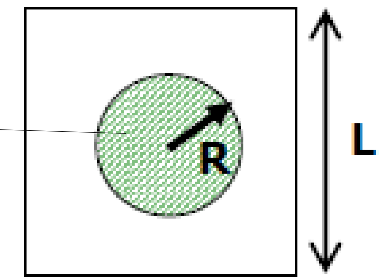


N. Ishii *et al.* [HAL QCD Collab.], Phys. Lett. B **712** (2012) 437

Schrödinger equation

- Define the energy-independent potential in Schrödinger equation (most general form)

$$\left(\frac{k^2}{2\mu}-H_0\right)\Psi(\vec{x})=\int U(\vec{x},\vec{y})\Psi(\vec{y})\,d^3y$$



- Recent development : Time dependent method.

  We replace ψ to *R* defined below

$$\partial_t R_\alpha(\vec{x},E) \;\equiv\; \partial_t\left(\frac{A\,\Psi_\alpha(\vec{x},E)\,e^{-Et}}{e^{-m_A t}\,e^{-m_B t}}\right) \;\propto\; -\frac{p_\alpha^2}{2\mu_\alpha}R_\alpha(\vec{x},E)$$

- Performing the derivative expansion for the interaction kernel

$$\left(-\frac{\partial}{\partial t}-H_0\right)R(\vec{x})=\int U(\vec{x},\vec{y})\,R(\vec{y})\,d^3y$$

- Taking the leading order of derivative expansion of non-local potential

$$U(\vec{x},\vec{y}) \;\simeq\; V_0(\vec{x})\delta(\vec{x}-\vec{y}) \;+\; V_1(\vec{x},\nabla)\delta(\vec{x}-\vec{y}) \;\cdots$$

- Finally local potential was obtained as

$$V(\vec{x}) \;=\; -\frac{\partial_t R(\vec{r})}{R(\vec{v})}+\frac{1}{2\mu}\frac{\nabla^2 R(\vec{x})}{R(\vec{x})}$$

Numerical setup

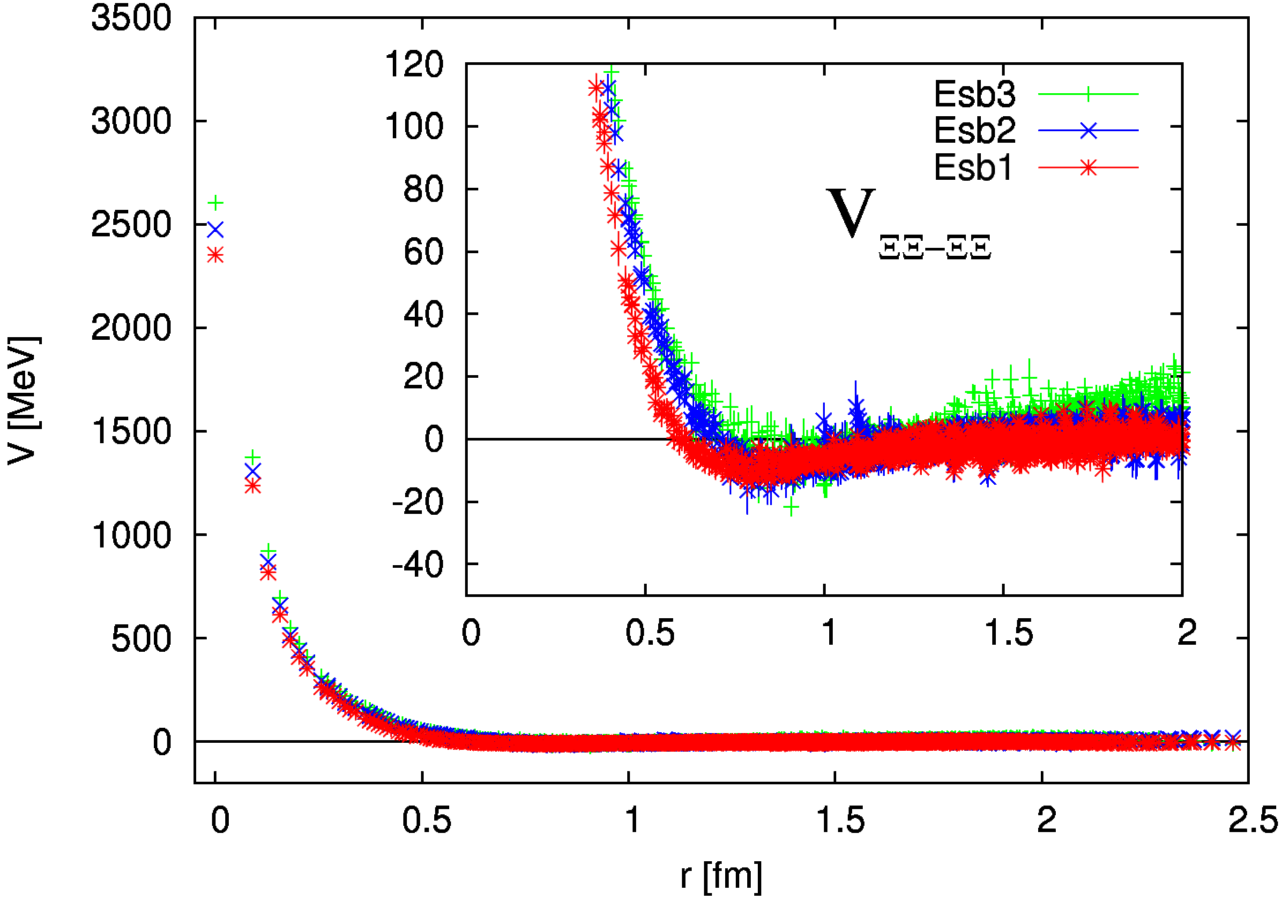
- 2+1 flavor gauge configurations by PACS-CS collaboration.
  - RG improved gauge action & O(a) improved Wilson-clover quark action
  - β = 1.90, a^{-1} = 2.176 [GeV], 32^3x64 lattice, L = 2.902 [fm].
  - κ_s = 0.13640 is fixed, κ_{ud} = 0.13700, 0.13727 and 0.13754 are chosen.
- Flat wall source is considered to produce S-wave B-B state.
- The KEK computer system A & B resources are used.

In unit of MeV	Esb 1	Esb 2	Esb 3
π	701±1	570±2	411±2
K	789±1	713±2	635±2
m_π/m_K	0.89	0.80	0.65
N	1585±5	1411±12	1215±12
Λ	1644±5	1504±10	1351± 8
Σ	1660±4	1531±11	1400±10
Ξ	1710±5	1610± 9	1503± 7

u,d quark masses lighter →

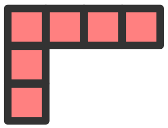
S=-4 channels

$\Xi\Xi$ channel $^3S_1$ I=0 preliminary



Esb1 : mπ= 701 MeV
Esb2 : mπ= 570 MeV
Esb3 : mπ= 411 MeV

Flavor decuplet

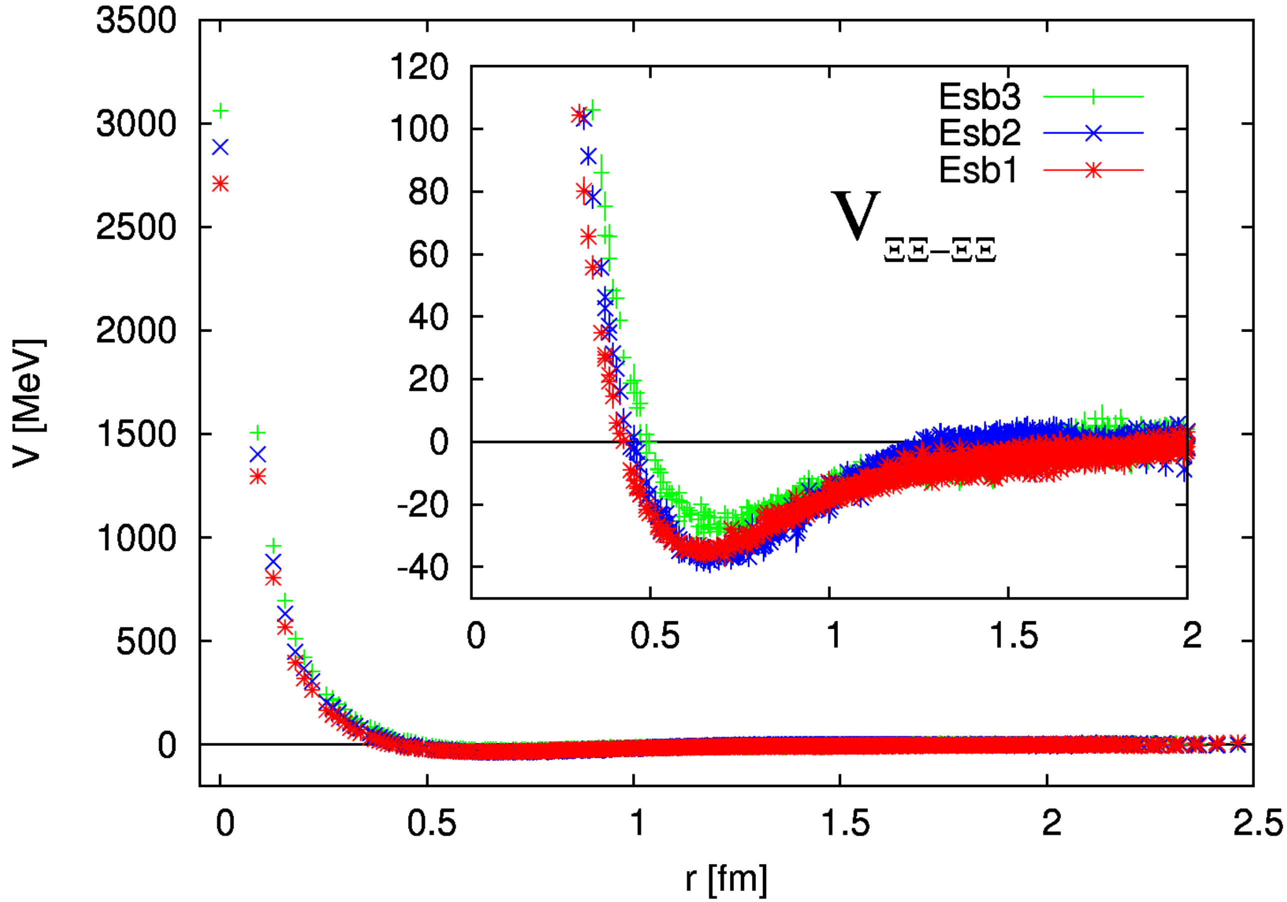


Belong to Decuplet (10) in SU(3) limit

almost forbidden state → Repulsive potential

Quark mass dependence is small

ΞΞ channel $^{1}S_{0}$ I=1 preliminary



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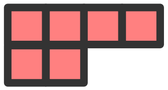
Flavor 27plet



Belong to 27 plet in SU(3) limit

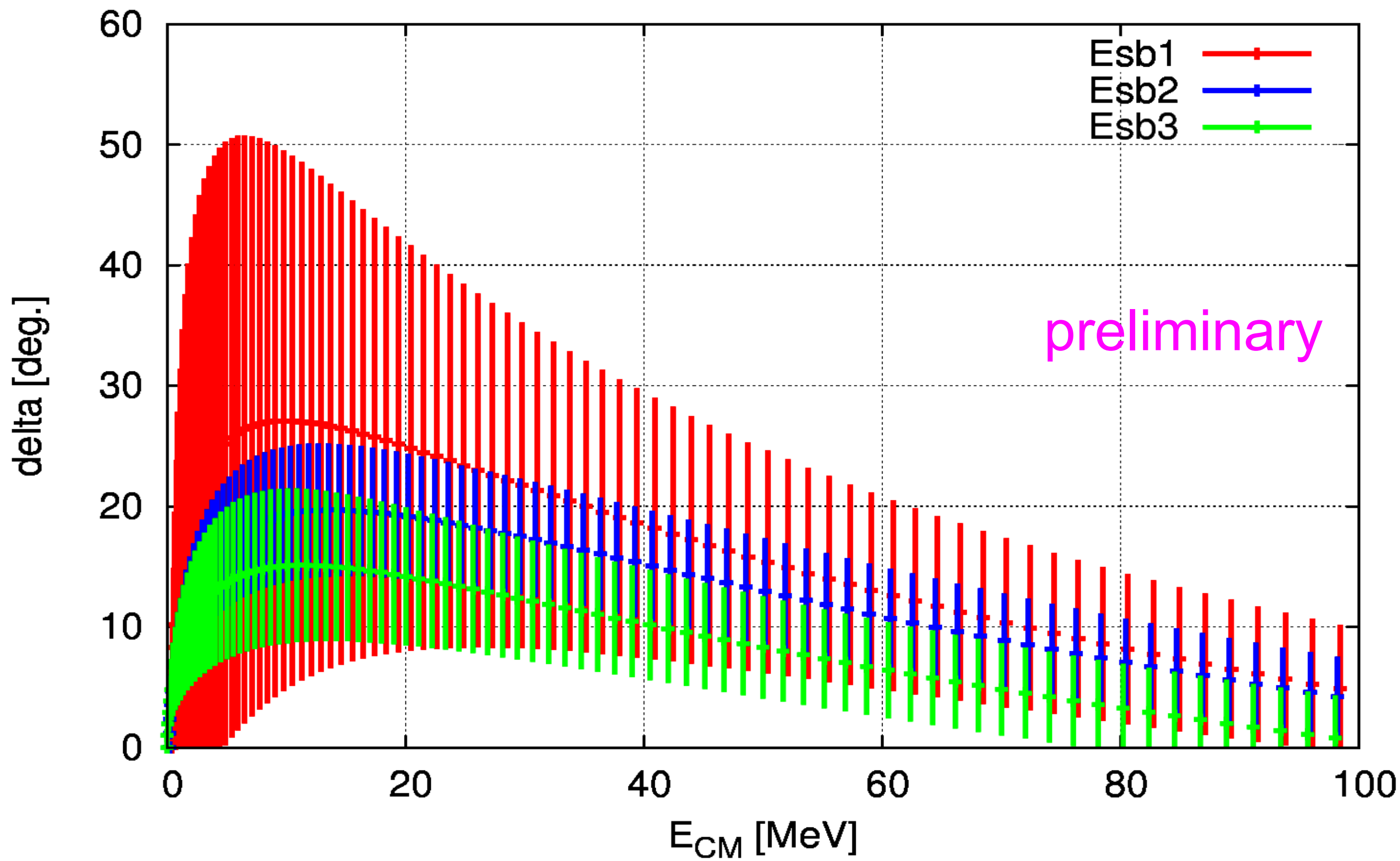
Qualitative behavior is similar to NN potential

Short range repulsion is increasing,
but no clear difference between potentials measured in each configurations

ΞΞ channel $^1S_0$ I=1 : phase shifts

Fit function : $f(r) = A\frac{e^{-mr}}{r}(1-e^{-Br^2}) + \sum_i C_i e^{-D_i r^2}$

m is fixed to the measured pion mass



Phase shift shows an attractive interaction
Attraction becomes weaker as decreasing light quark mass

Summary

- We showed preliminary results of S= -4 BB potentials with L=3fm.
- Qualitatively, potentials are not so much different from

  the potential in SU(3) limit reported by Prof. Inoue.
- We can see quark mass dependence of potentials
  - Enhancement of short range core.
  - Potential range is not clearly extended.

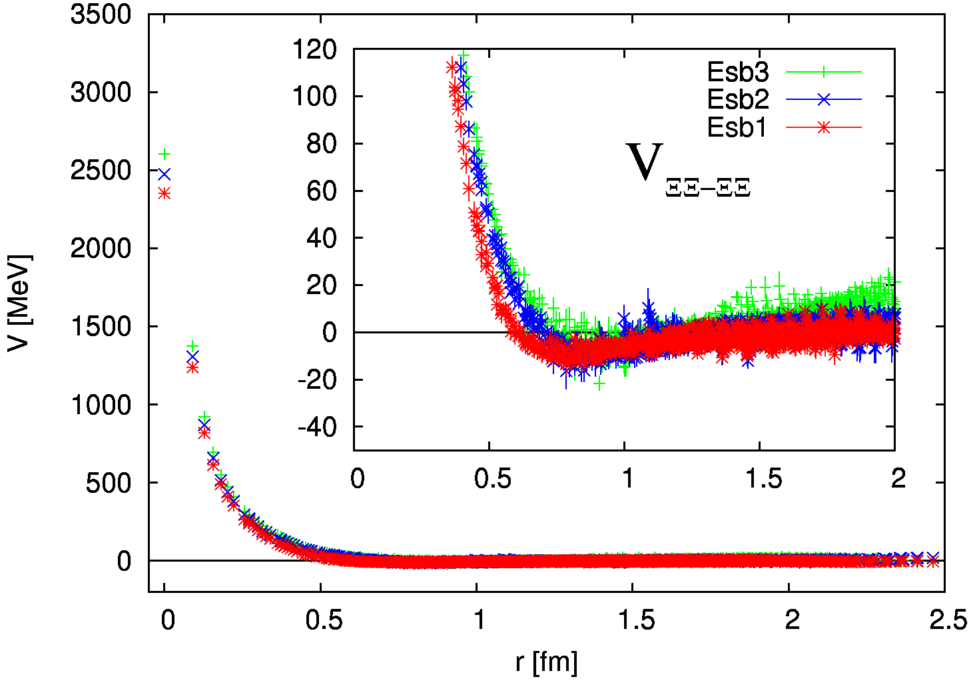
Future works

- Increase statistics
- Separation of tensor potential in spin triplet channel
- Try to find whether $^1S_0$ ΞΞ bound state is exist or not.

backups

ΞΞ channel $^3S_1$ I=0

preliminary



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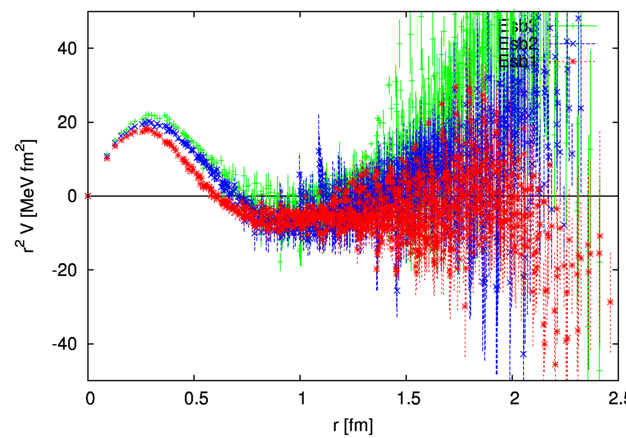
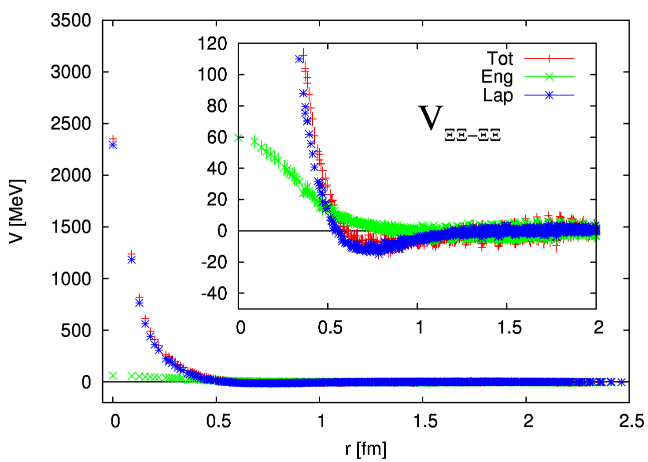
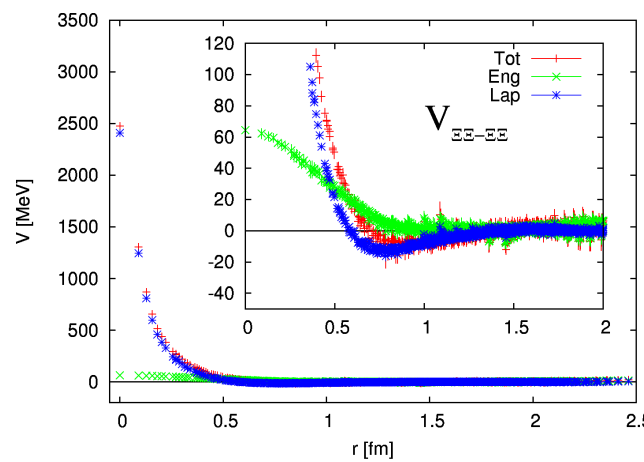


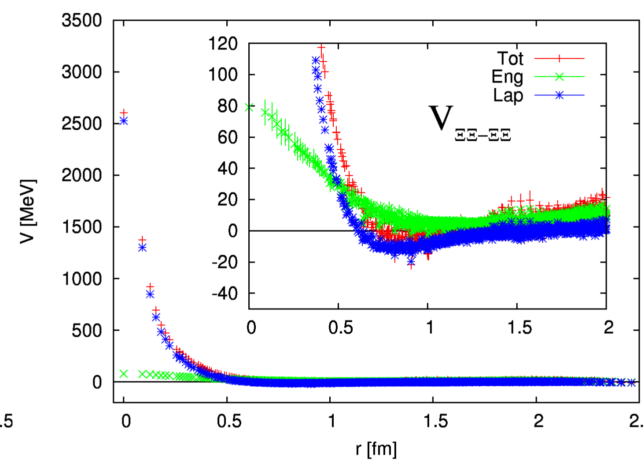
Esb 1



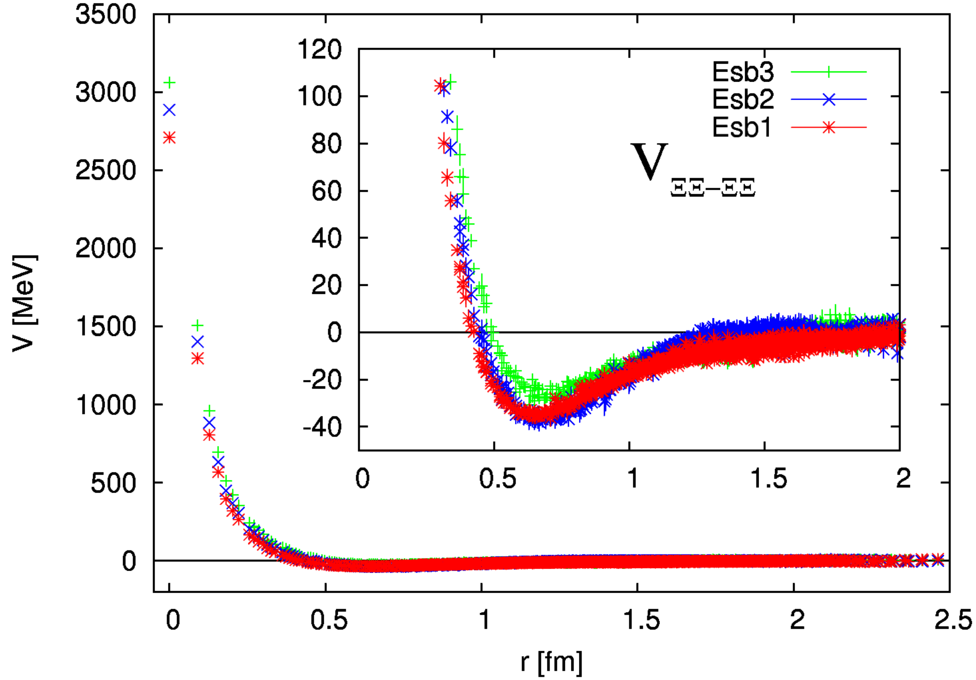
Esb 2



Esb 3



ΞΞ channel $^1S_0$ I=1 preliminary



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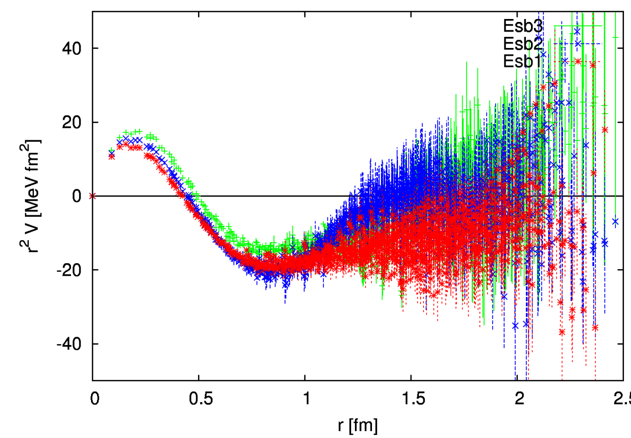

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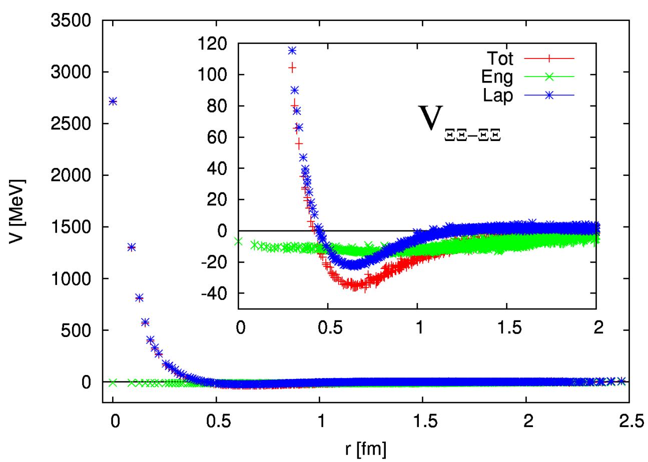
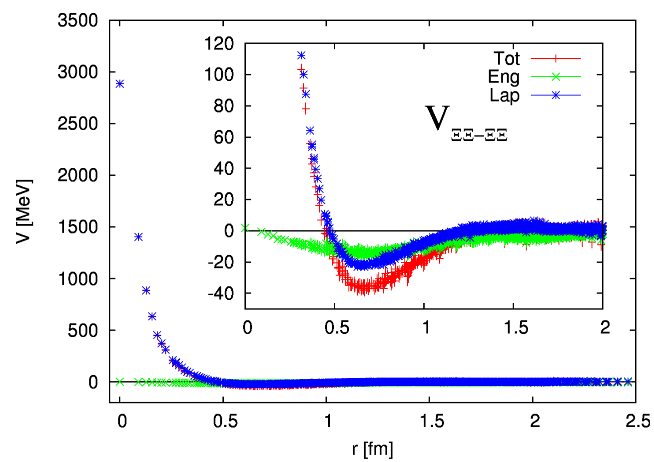
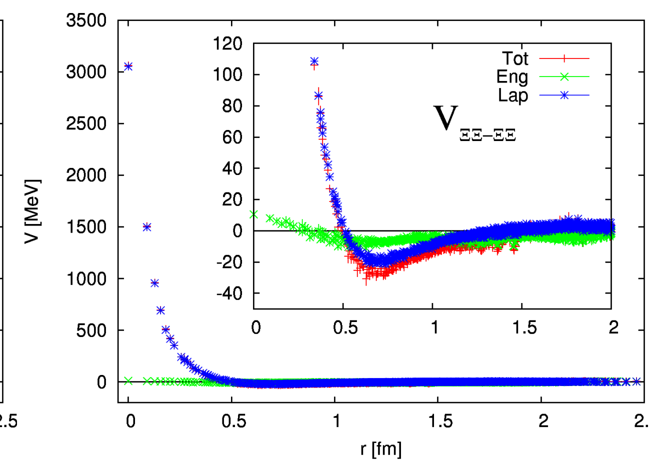


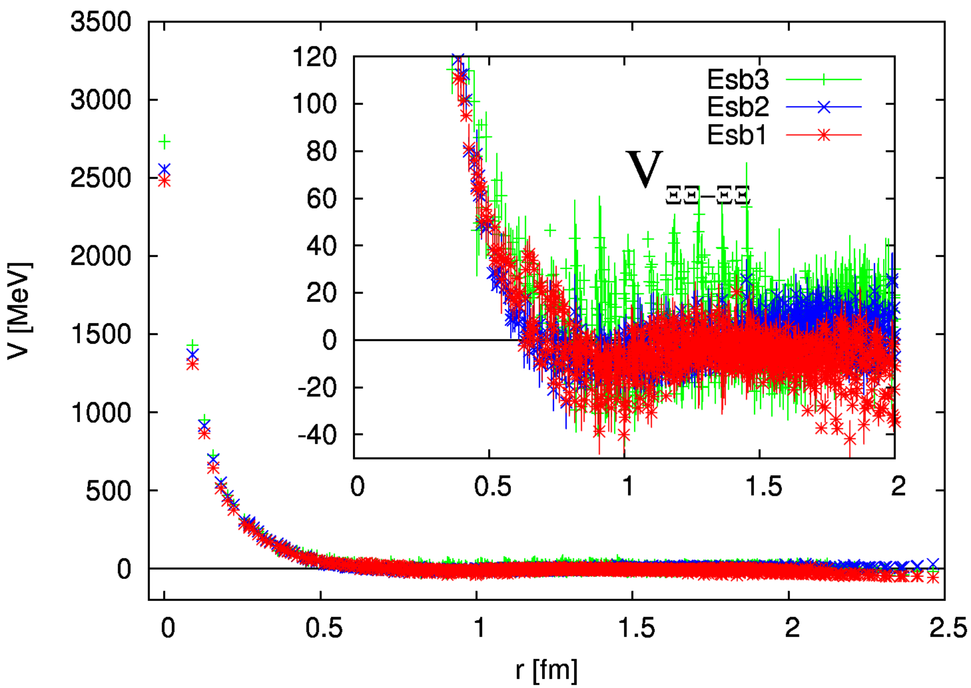
Esb 2



Esb 3



ΞΞ channel ${}^3S_1$ I=0



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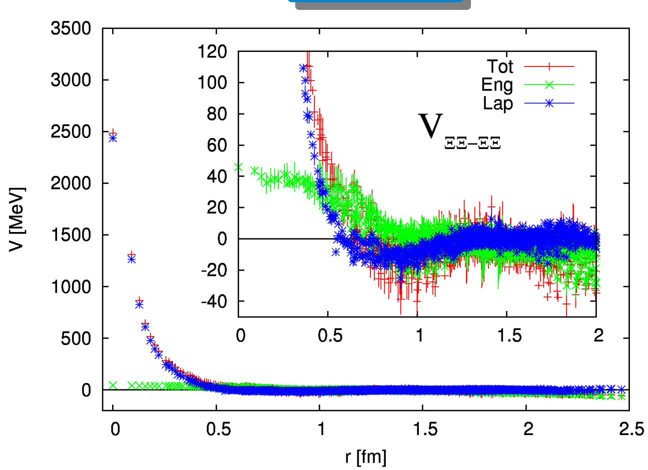

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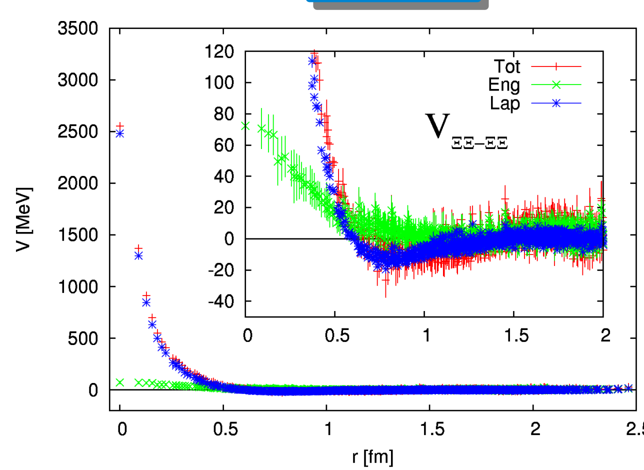

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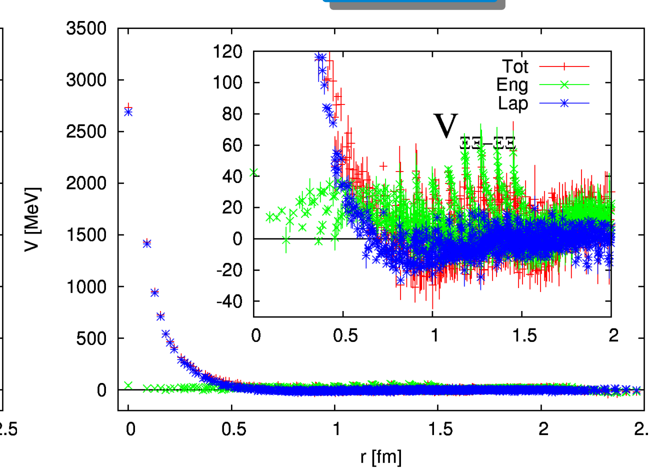
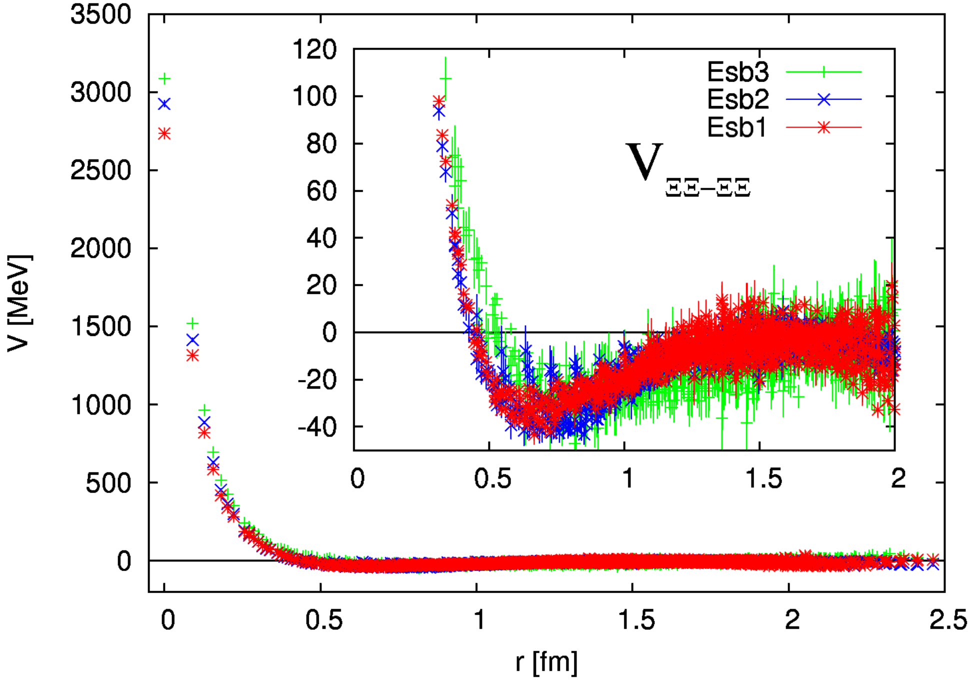


ΞΞ channel $^1S_0$ I=1



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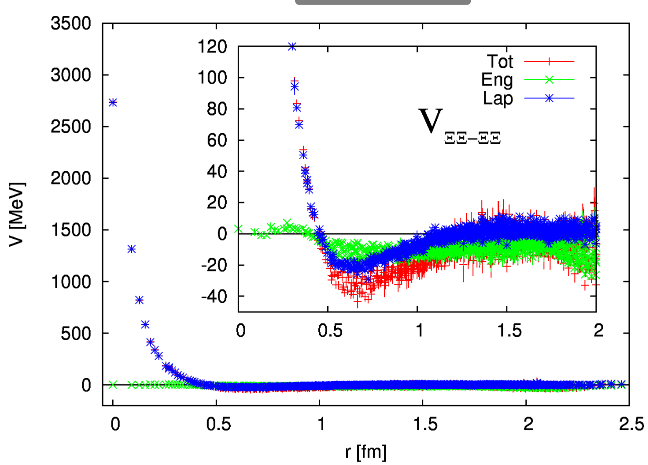

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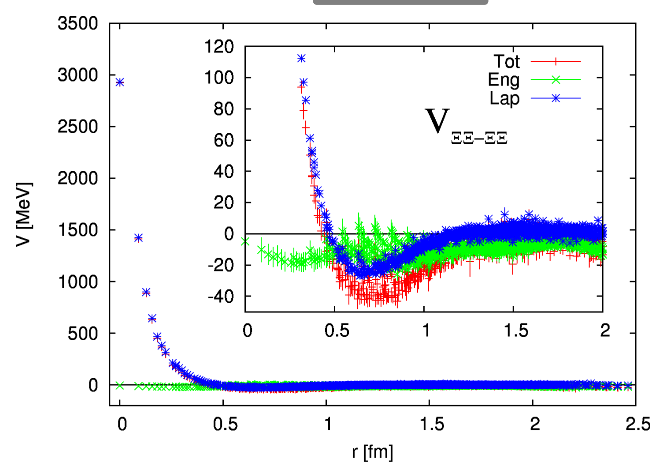

Esb 3