On Massive Gauge Theories

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In collaboration with Michele Della Morte, to appear

Introduction

The Higgs has been discovered and the EWSB sector of the SM seems accurately described by the simplest Higgs mechanism

Englert, Brout; Higgs; Guralnik, Hagen, Kibble; Weinberg

$$V(\Phi) = -\frac{m_H^2}{2}(\Phi^\dagger \Phi) + \frac{\lambda}{4}(\Phi^\dagger \Phi)^2$$

$$\langle \Phi \rangle = \left(\begin{array}{c} 0 \\ \frac{v}{\sqrt{2}} \end{array} \right) \to m_W = \frac{1}{2} g v, \quad m_f = \lambda_f \frac{v}{\sqrt{2}}$$

Massive and weakly coupled non-abelian gauge theory:

- where the scale v comes from ?
- Couplings to the Higgs create havoc in the SM: the flavour problem...

Introduction

This naive characterisation of SSB is not at work beyond perturbation theory

In any lattice formulation of gauge theories: Elitzur's theorem

$$\langle \Phi \rangle = 0$$

Elitzur; Frölich et al

A non-perturbative derivation of the weakly-coupled electroweak sector of the SM might shed some light on the hierarchy and flavour problems...

Massive gauge theories on the lattice

Any theory of gauge fields with non-gauge invariant interactions is equivalent to a gauge theory + scalar degrees of freedom

$$\int [dU]\ e^{-S_{g.i.}[U]-S_{n.g.i}[U]} = \int [d\Omega] \int [dU]\ e^{-S_{g.i.}[U]-S_{n.g.i}[U^\Omega]}$$
$$\equiv \int [d\Omega] \int [dU] e^{-\tilde{S}[U,\Omega]}$$

$$\tilde{S}[U,\Omega] = \tilde{S}[U^\Lambda, \Lambda\Omega]$$

In particular Gauge theory with a mass term = Gauged non-linear σ model

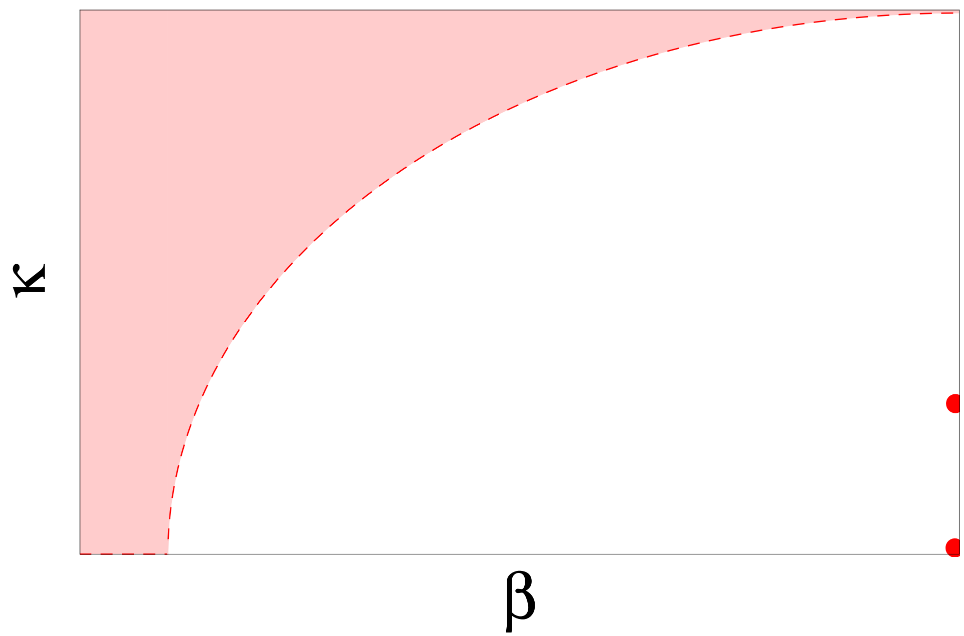
$$S_m[U] = \beta \sum_x \sum_{\mu,\nu} \mathrm{Retr}\left[1 - P(x,\mu,\nu)\right] - \frac{\kappa}{2} \sum_x \sum_\mu \mathrm{tr}\left[U_\mu + U_\mu^\dagger\right],$$

$$\tilde{S}[U,\Omega] \equiv \beta \sum_x \sum_{\mu,\nu} \mathrm{Retr}\left[1 - P(x,\mu,\nu)\right] - \frac{\kappa}{2} \sum_x \sum_\mu \mathrm{tr}\left[\Omega^\dagger(x) U_\mu(x) \Omega(x + a\hat{\mu}) + h.c.\right]$$

For $SU(2)$: $SU(2)$+Higgs in the limit $\lambda \to \infty$

Phase Diagram

Lang, Rebbi, Virasoro; Osterwalder, Seiler; Fradkin, Shenker; Forster, Nielsen, Ninomiya



- $\kappa = 0$: pure gauge theory, critical point at $\beta_c = \infty$

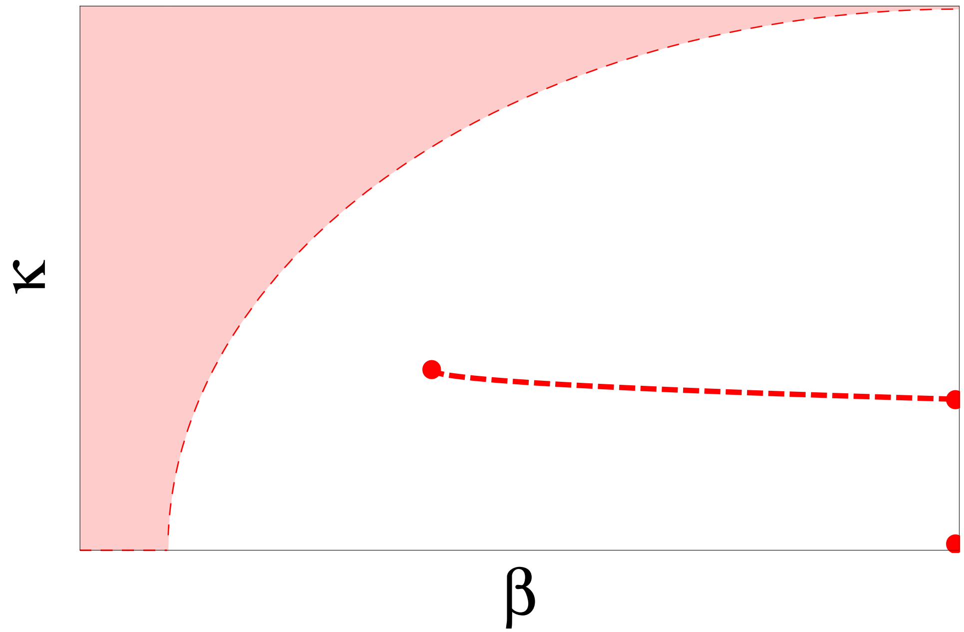
  $\beta = \infty$: σ-model, critical point at κ_c

- $\kappa < \kappa_{\min}$: pure gauge theory universality class (continuum limit at $\beta \to \infty$)

- confinement/Higgs phases: analytically connected in red region

Phase Diagram

Langguth, Montvay, Weisz; Campos; Caudy, Greensite



Recent studies: line of first order phase transition end-point at $\beta_c \simeq 2.7$

Bonati, Cossu, D'Elia, Di Giacomo 2010

Is there a continuum or a scaling region within the Higgs phase ? $\xi_\Omega / a \to \infty$?

If so what Wilsonian effective theory describes it ? What are the light dogs ? Is it renormalizable ?

Strategy in this work: search for lines of constant physics and test scaling

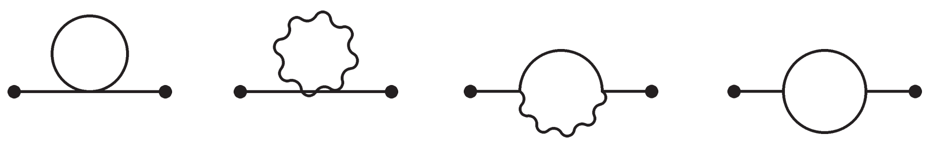
Lattice perturbation Theory

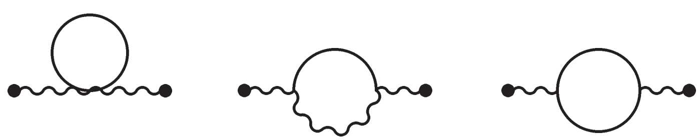
A perturbative expansion is possible as a low-energy expansion $p^2 \ll \frac{\kappa}{a^2} = \frac{2m_W^2}{g_0^2}$ (natural cutoff $\sim 4\pi\frac{\sqrt{\kappa}}{a}$)

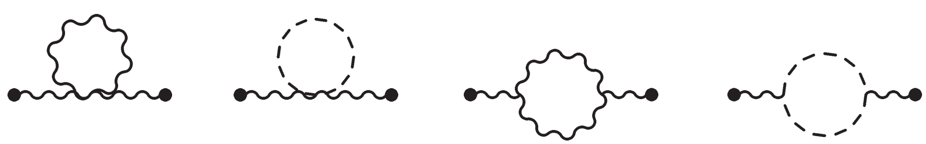
Appelquist, Bernard

In the regime $m_W^2 \ll p^2 \ll \frac{2m_W^2}{g_0^2}$ with background field method (B_μ^a, ω^a) :

Lüscher, Weisz

 $= -\frac{\kappa+\Delta\kappa}{4}\hat{p}^2\omega^a(-p)\omega^a(p)$



 $=$

$$\left\{-\frac{1}{2(g^2+\Delta g^2)}\left[g_{\mu\nu}\hat{p}^2 - \hat{p}_\mu\hat{p}_\nu\right] - \frac{1}{4}(\kappa+\Delta\kappa)g_{\mu\nu}\right\} B_\mu^a(-p)B_\nu^a(p)$$

Up to corrections of $\mathcal{O}(m_W^2/p^2, p^2/\kappa)$ (preliminary):

$$
\begin{aligned}
\left(\frac{\Delta g^2}{g^4}\right) &= \frac{N}{(4\pi)^2}\left(-\frac{29}{8}\ln p^2 + \frac{63}{9}\right) + N\left(\frac{7}{48}P_1 + \frac{29}{8}P_2 + \frac{1}{16}\right) - \frac{1}{8N} \\
\frac{\Delta\kappa}{\kappa} &= \frac{1}{\kappa}\left[\frac{1}{8N} - \frac{N}{16} - \frac{N}{2}P_1\right] \\
&- g^2\left[\left(\frac{5N}{32} - \frac{3}{16N}\right)P_1 + \frac{3N}{4}P_2 - \frac{N}{4}\frac{1}{(4\pi)^2} - \frac{3N}{4(4\pi)^2}\log(m_W^2)\right. \\
&+ \left.\frac{2N}{(4\pi)^2}F\left(\frac{m_W^2}{p^2}\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
P_1 &\equiv \int_{-\pi}^{\pi}\frac{d^4p}{(2\pi)^4}\frac{1}{\hat{p}^2} = 0.15493339... \\
P_2 &\equiv \lim_{\mu\to 0}\left\{\frac{1}{(4\pi)^2}\log(\mu^2) + \int_{-\pi}^{\pi}\frac{d^4p}{(2\pi)^4}\frac{1}{(\hat{p}^2+\mu^2)^2}\right\} = 0.02401318.... \\
F(x) &\equiv 1 - \sqrt{1+4x}\ \mathrm{arccoth}\sqrt{1+4x}
\end{aligned}
$$

Ungauged result $g = 0$: Shushpanov, Smilga

There is asymptotic freedom

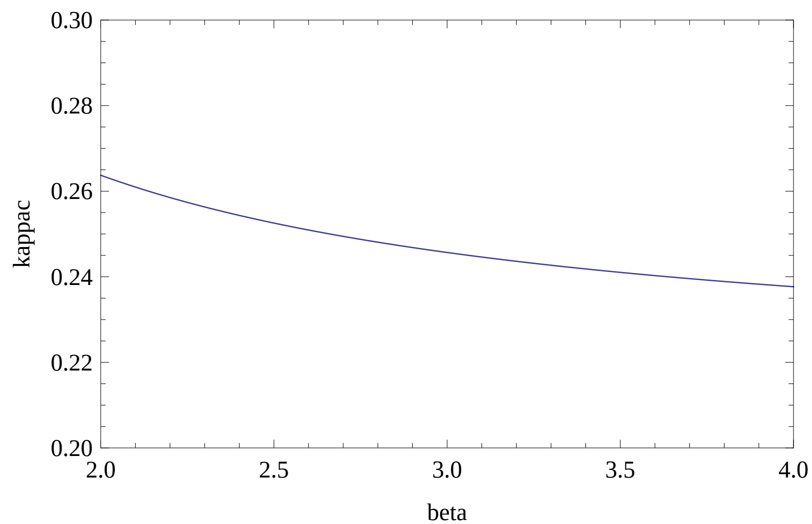
$$\beta(g) = -\frac{29N}{8(4\pi)^2} g^3 + \dots,$$

Fabbrichesi et al

but with a different coefficient as in the pure gauge theory.

A continuum limit ? At this order requires:

$$\kappa + \Delta\kappa = 0$$



and a tunning order by order...

Wilsonian Effective Theory

Let us assume that a scaling region exists where $m_{\rm phys} a \to 0$ with the following properties:

- Asymptotic states are gauge singlets: confinement
- The lightest state has the quantum numbers of the W_μ^a boson and is weakly coupled

Eg: interpolating field

$$V_\mu^a \equiv i\frac{\kappa}{2}\mathrm{Tr}[\Omega(x)^\dagger U_\mu(x)\Omega(x+\hat{\mu})T^a] + h.c.$$

Then the Wilsonian effective theory is itself a massive gauge theory up to effects of higher excited states

Exact global symmetry: Custodial Symmetry

The lattice action preserves an exact $SU(2)$ global symmetry:

$$\Omega(x) \quad \rightarrow \quad \Omega(x)\Lambda, \;\; \Lambda \in SU(N)$$

The corresponding conserved Noether currents are:

$$V_\mu^a \equiv i\frac{\kappa}{2}\mathrm{Tr}[\Omega(x)^\dagger U_\mu(x)\Omega(x+\hat{\mu})T^a] + h.c.$$

$$\hat{\partial}_\mu V_\mu^a = 0,$$

where $\hat{\partial}_\mu \Omega(x) \equiv \Omega(x+\hat{\mu}) - \Omega(x)$

The effective theory must preserve the global symmetry:

$$\begin{aligned}\mathcal{L}_{eff}(W) \quad &= \quad -\frac{1}{4}Z_W \; \partial_{[\mu,}W_{\nu]} \cdot \partial_{[\mu,}W_{\nu]} - \alpha \; W_\mu \times W_\nu \cdot \partial_\mu W_\nu \\ &- \quad Z_W m_W^2 \; W_\mu \cdot W_\mu + \lambda(W_\mu \cdot W_\mu)^2 + \mu(W_\mu \cdot W_\nu)^2,\end{aligned}$$

Imposing that W is the conserved current of the global symmetry in the effective theory:

$$\frac{\partial \mathcal{L}_{eff}}{\partial \partial_\mu \epsilon^a(x)} \propto W_\mu^a(x),$$

then

$$\alpha = -4\lambda = 4\mu = Z_W.$$

while m_W^2 is not constrained.

After canonically normalization:

$$\begin{aligned}\mathcal{L}_W &= -\frac{1}{4}\partial_{[\mu,}W_{\nu]} \cdot \partial_{[\mu,}W_{\nu]} - g\ W_\mu \times W_\nu \cdot \partial_\mu W_\nu - m_W^2\ W_\mu \cdot W_\mu \\ &- \frac{g^2}{4}\left[(W_\mu \cdot W_\mu)^2 - (W_\mu \cdot W_\nu)^2\right],\end{aligned}$$

with $g \equiv Z_W^{-1/2}$. This is a massive Yang-Mills theory!

Exact global symmetry: Ward Identities

For any operator O and a local infinitesimal rotation, $\Lambda(x) = e^{iT^a \epsilon^a(x)}$:

$$\langle -\delta S \; O \rangle + \langle \delta O \rangle = 0,$$

with

$$\delta_\epsilon S[U,\Omega] = -\sum_{x,\mu,a} V_\mu^a(x) \hat{\partial}_\mu \epsilon^a(x),$$

Case I: $O(y,z) \equiv V_\mu^a(y) V_\nu^b(z)$

$$\delta_\epsilon V_\mu^a(x) = -\epsilon^{abc} \left[\epsilon^b(x) \; V_\mu^c(x) - \frac{1}{2} \hat{\partial}_\mu \epsilon^b(x) \; V_\mu^c(x) \right] + \frac{1}{4} V_\mu^0(x) \hat{\partial}_\mu \epsilon^a(x),$$

where V_μ^0 is a singlet under the global symmetry:

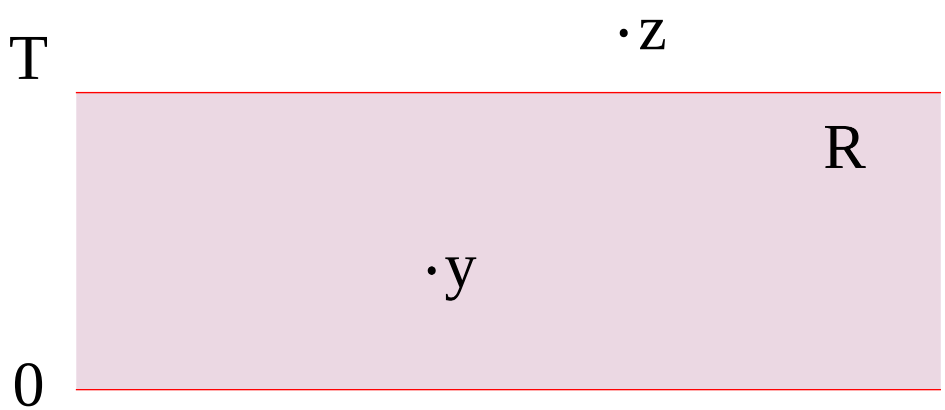
$$V_\mu^0(x) \equiv \frac{\kappa}{2} \mathrm{Tr}[\Omega(x)^\dagger U_\mu(x) \Omega(x+\hat{\mu}) + h.c.]$$

We consider

$$\epsilon^a(x) = \begin{cases} \epsilon^a, & x \in R \\ 0, & x \notin R \end{cases}$$

$y \in R$ (ie. $0 < y_0 < T$) while $z \notin R$, for example $z_0 > T$

Lüscher



The lattice WI implies

$$\epsilon_{abc} \sum_{\mathbf{x}} \langle \left(V_0^c(T, \mathbf{x}) - V_0^c(0, \mathbf{x}) \right) V_\mu^a(y) V_\nu^b(z) \rangle = 2 \langle V_\mu^d(y) V_\nu^d(z) \rangle,$$

Matching to the effective theory

$$V_\mu^a = Z_W^{1/2} W_\mu^a \equiv m_W F_W W_\mu^a$$

and evaluating the three-point function to LO in perturbation theory:

$$\frac{1}{\sqrt{Z_W}} = \frac{g}{m_W^2} \to g = \frac{m_W}{F_W}.$$

Case II: $O \to V_\mu^b(y) V_\nu^c(z) V_\sigma^d(u)$

with $a = b = c = d$ and $y, z, u \notin R$

$$\lambda = -\mu.$$

Case III: $O \to V_\mu^b(y) V_\nu^c(z) V_\sigma^d(u)$ with $a = b \neq c = d$

$$\lambda Z_W^{-2} = \frac{-g^2}{4} \to .$$

Global invariance $\Rightarrow$ local invariance in the effective theory of conserved currents

Wilsonian effective theory of massive gauge bosons + Higgs ?

Possibly other light particles will exist in the spectrum. In particular a light scalar could unitarize the theory if it has the properties of the Higgs...

$$H(x) \leftrightarrow V^0_\mu(x) \equiv \frac{\kappa}{2}\mathrm{Tr}[\Omega(x)^\dagger U_\mu(x)\Omega(x+\hat{\mu}) + h.c.]$$

If such a light particle remains in the spectrum how is the effective theory modified ?

Global symmetry allows the following couplings:

$$\mathcal{L}_H = \frac{1}{2}\partial_\mu H \partial_\mu H - V(H) - \lambda_{HWW} H W_\mu \cdot W_\mu - \lambda_{HHWW} H^2 W_\mu \cdot W_\mu,$$

The WI in this case implies the following matching:

$$V^a_\mu \to W^a_\mu + 2\frac{\lambda_{HWW}}{m_W^2} H W^a_\mu + ...$$

Symmetry does not seem to require the couplings to be those in the SM (i.e.: only ones that make the model perturbatively renormalizable)...

Conclusions

- A gauged non-linear sigma model might be the simplest model for dynamical EWSB
- This is a non-trivial strongly coupled model that needs to be understood non-perturbatively
- Global symmetries indicate that if a continuum limit/scaling region exists it could look very similar to a SSB gauge theory
- Old studies can be very significantly improved with new methods and algorithms developed for QCD