Rotating lattice

Arata Yamamoto (RIKEN)

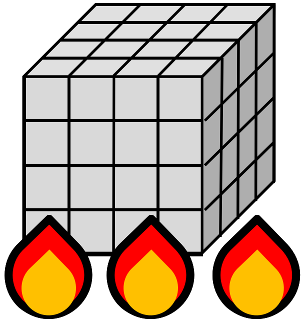
with Yuji Hirono (Tokyo U.)

arXiv:1303.6292

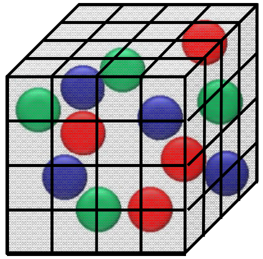
Lattice 2013

Extreme QCD

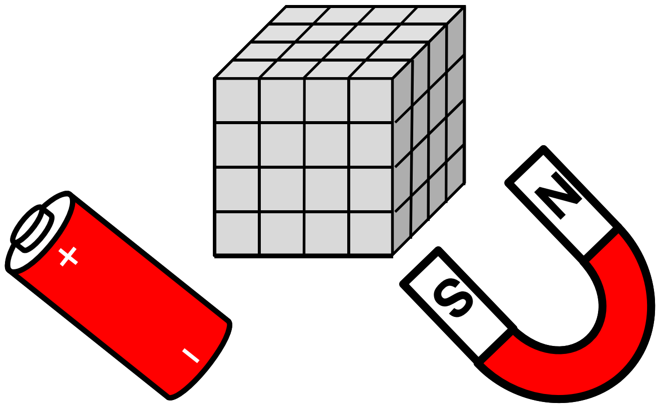
Finite temperature



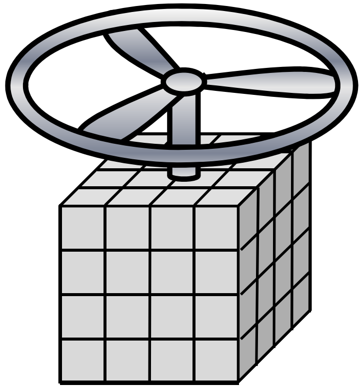
Finite density



External electromagnetic field

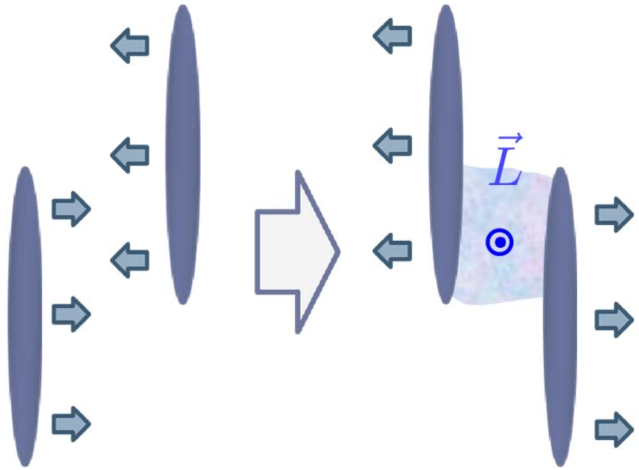


Rotation

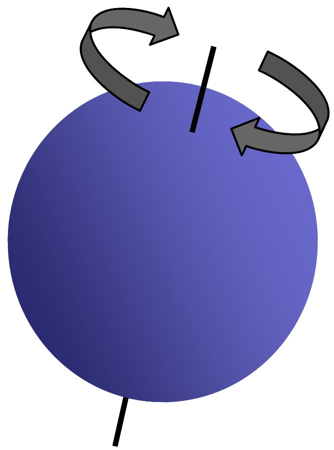


Rotations in QCD

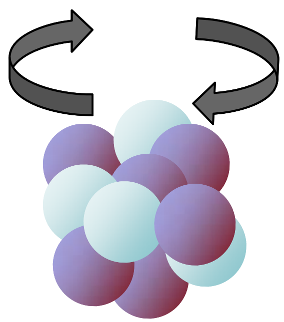
peripheral heavy-ion collision



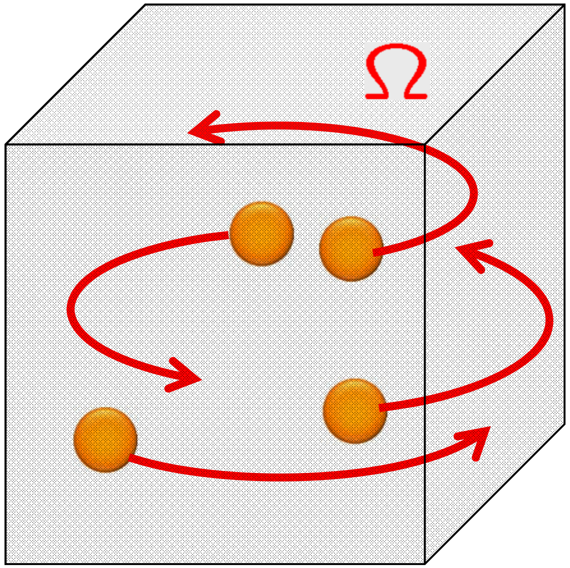
rotating compact star



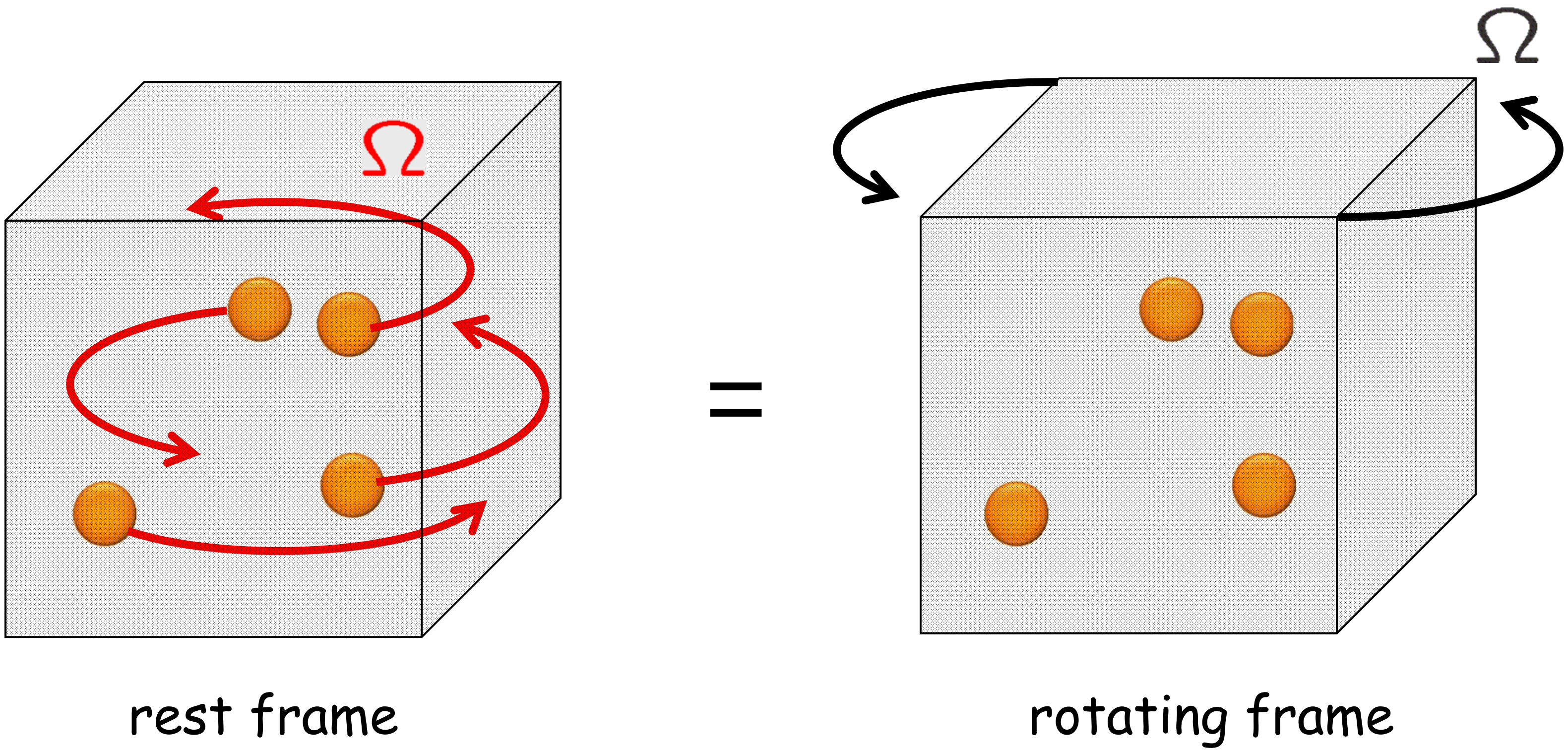
rotating/high-spin nucleus



Simulation of rotating matter



Simulation of rotating matter



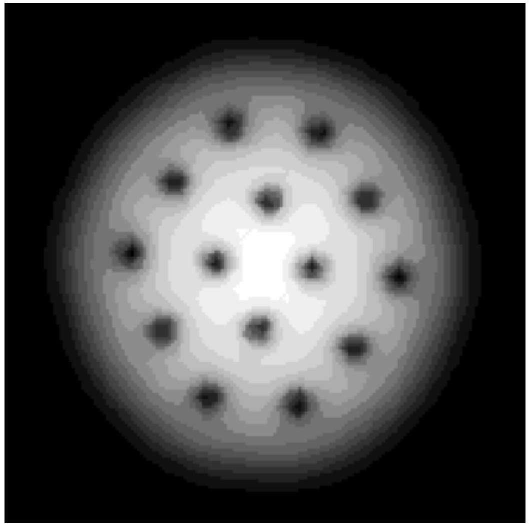
$$d\vec{x}_{\rm rest} = d\vec{x} - \vec{\Omega} \times \vec{x}\, dt$$

Simulation of rotating matter

cf.) rotating BEC in condensed matter physics

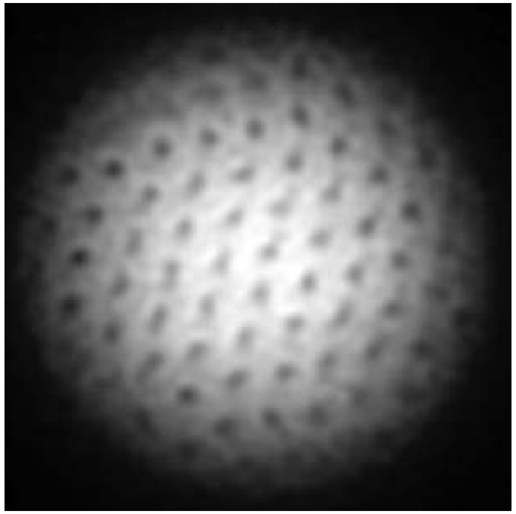
$$H \to H - \vec{L} \cdot \vec{\Omega}$$

simulation



[Kasamatsu, Tsubota, Ueda (2002)]

experiment



[Zwierlein et. al. (2005)]

Rotating lattice QCD

Euclidean rotation:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & y\Omega \\ 0 & 1 & 0 & -x\Omega \\ 0 & 0 & 1 & 0 \\ y\Omega & -x\Omega & 0 & 1 + r^2\Omega^2 \end{pmatrix}$$

QCD action in a rotating frame → lattice Monte Carlo simulation

discretize

Continuum gluon action

quadratic terms

$$S_G = \int d^4x \, \frac{1}{g_{\rm YM}^2} {\rm tr}[(1 + r^2\Omega^2)F_{xy}F_{xy}$$

$$+ (1 + y^2\Omega^2)F_{xz}F_{xz} + (1 + x^2\Omega^2)F_{yz}F_{yz}$$

$$+ F_{x\tau}F_{x\tau} + F_{y\tau}F_{y\tau} + F_{z\tau}F_{z\tau}$$

$$+ 2y\Omega F_{xy}F_{y\tau} - 2x\Omega F_{yx}F_{x\tau}$$

$$+ 2y\Omega F_{xz}F_{z\tau} - 2x\Omega F_{yz}F_{z\tau} + 2xy\Omega^2 F_{xz}F_{zy}]$$

cross terms

Lattice gluon action

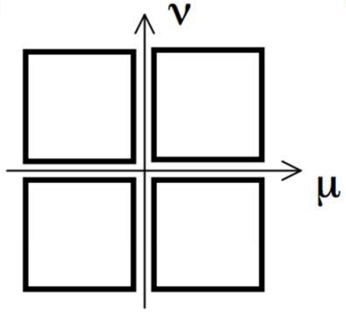
$$S_G = \sum_x \beta \Big[(1 + r^2\Omega^2)\left(1 - \frac{1}{N_c}\mathrm{Re\,tr}\bar{U}_{xy}\right) + (1 + y^2\Omega^2)\left(1 - \frac{1}{N_c}\mathrm{Re\,tr}\bar{U}_{xz}\right)$$

$$+ (1 + x^2\Omega^2)\left(1 - \frac{1}{N_c}\mathrm{Re\,tr}\bar{U}_{yz}\right) + 3 - \frac{1}{N_c}\mathrm{Re\,tr}\left(\bar{U}_{x\tau} + \bar{U}_{y\tau} + \bar{U}_{z\tau}\right)$$

$$- \frac{1}{N_c}\mathrm{Re\,tr}\left(y\Omega\bar{V}_{xy\tau} - x\Omega\bar{V}_{yx\tau} + y\Omega\bar{V}_{xz\tau} - x\Omega\bar{V}_{yz\tau} + xy\Omega^2\bar{V}_{xzy}\right)\Big]$$

Lattice gluon action

plaquettes $\bar{U}_{\mu\nu} = \frac{1}{4}\Bigg($



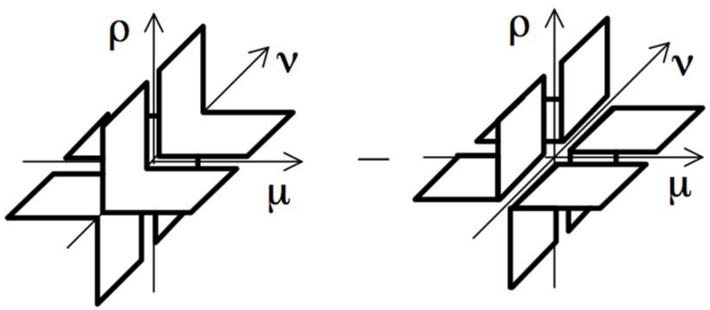
$\Bigg)$

$$S_G = \sum_x \beta \Bigg[(1 + r^2\Omega^2)\left(1 - \frac{1}{N_c}\mathrm{Re\,tr}\bar{U}_{xy}\right) + (1 + y^2\Omega^2)\left(1 - \frac{1}{N_c}\mathrm{Re\,tr}\bar{U}_{xz}\right)$$

$$+ (1 + x^2\Omega^2)\left(1 - \frac{1}{N_c}\mathrm{Re\,tr}\bar{U}_{yz}\right) + 3 - \frac{1}{N_c}\mathrm{Re\,tr}\left(\bar{U}_{x\tau} + \bar{U}_{y\tau} + \bar{U}_{z\tau}\right)$$

$$- \frac{1}{N_c}\mathrm{Re\,tr}\left(y\Omega\bar{V}_{xy\tau} - x\Omega\bar{V}_{yx\tau} + y\Omega\bar{V}_{xz\tau} - x\Omega\bar{V}_{yz\tau} + xy\Omega^2\bar{V}_{xzy}\right)\Bigg]$$

chair-type loops $\bar{V}_{\mu\nu\rho} = \frac{1}{8}\Bigg($



$\Bigg)$

Continuum fermion action

$$S_F = \int d^4x \, \bar{\psi} \Big[\gamma^1 D_x + \gamma^2 D_y + \gamma^3 D_z + \gamma^4 D_\tau + \boxed{\gamma^4 \Omega (x D_y - y D_x)} + \boxed{\gamma^4 i \Omega \frac{\sigma^{12}}{2}} \Big] \psi$$

orbit-rotation coupling spin-rotation coupling

Lattice fermion action

$$S_F = \sum_{x_1,x_2} \bar{\psi}(x_1)\Big[\delta_{x_1,x_2}$$

orbit-rotation coupling

$$- \kappa\Big\{(1-\gamma^1 + y\Omega\gamma^4)T_{x+} + (1+\gamma^1 - y\Omega\gamma^4)T_{x-}$$

$$+ (1-\gamma^2 - x\Omega\gamma^4)T_{y+} + (1+\gamma^2 + x\Omega\gamma^4)T_{y-}$$

$$+ (1-\gamma^3)T_{z+} + (1+\gamma^3)T_{z-}$$

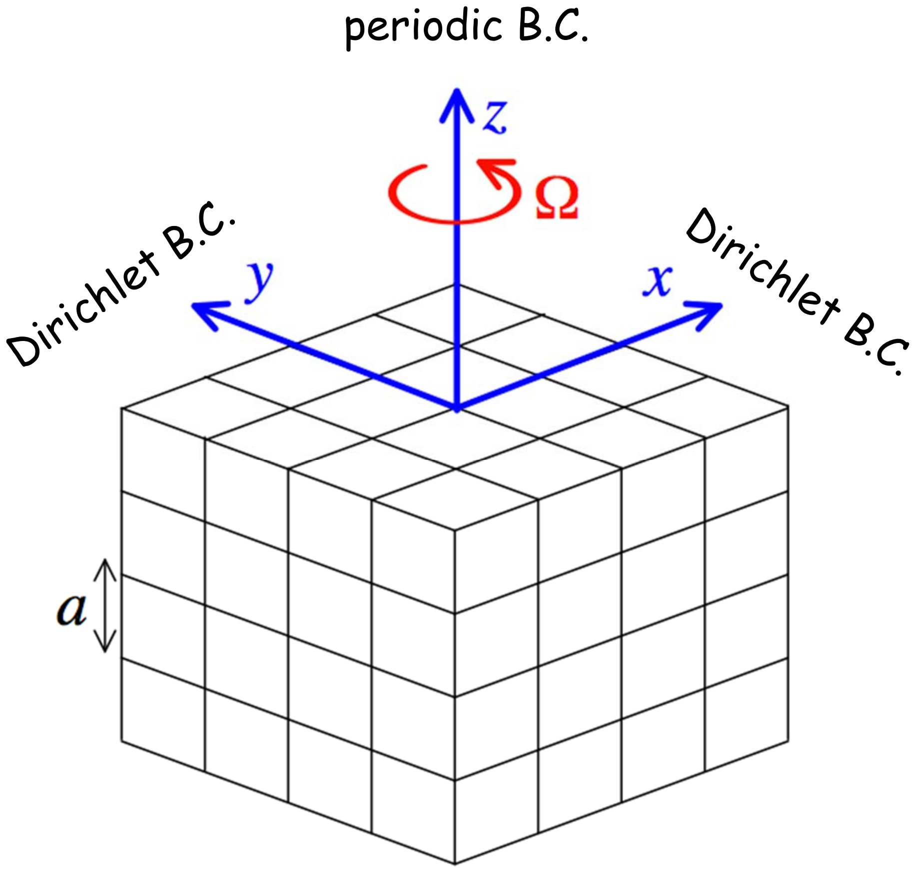
$$+ (1-\gamma^4)\exp\left(ia\Omega\frac{\sigma^{12}}{2}\right)T_{\tau+}$$

$$+ (1+\gamma^4)\exp\left(-ia\Omega\frac{\sigma^{12}}{2}\right)T_{\tau-}\Big\}\Big]\psi(x_2)$$

spin-rotation coupling

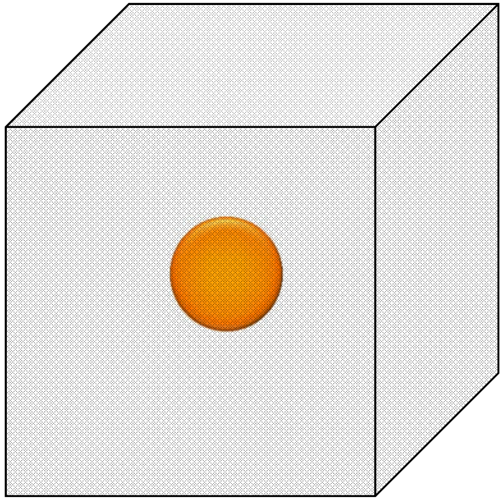
$$T_{\mu+} \equiv U_\mu(x_1)\delta_{x_1+\hat{\mu},x_2} \qquad T_{\mu-} \equiv U_\mu^\dagger(x_2)\delta_{x_1-\hat{\mu},x_2}$$

Simulation



Angular momentum

rest frame

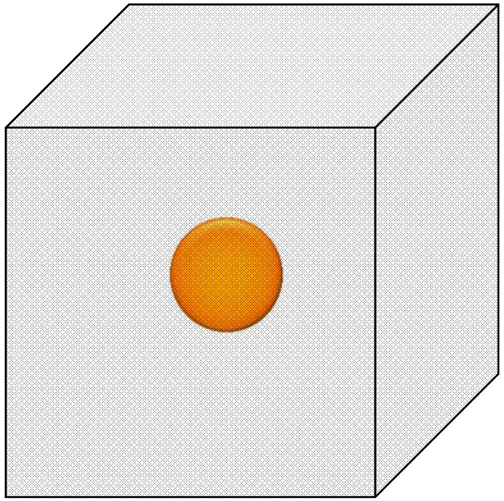


$$\mathcal{L} = \frac{1}{2} m r^2 \dot{\theta}^2_{\text{rest}}$$

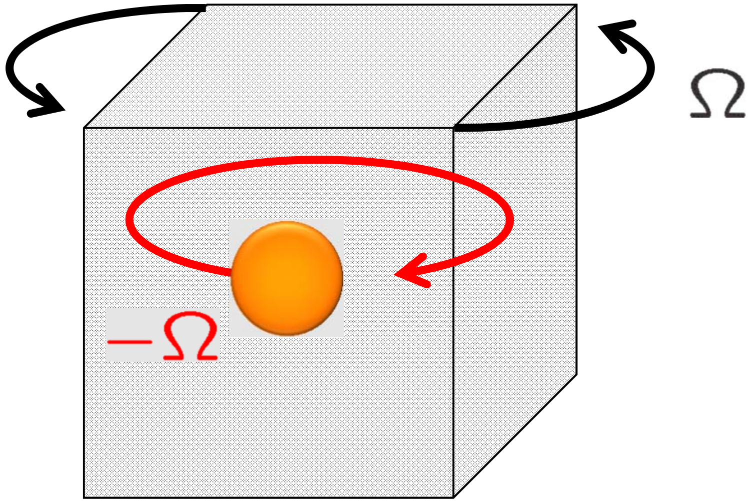
$$J = 0$$

Angular momentum

rest frame



rotating frame



$$\mathcal{L} = \frac{1}{2} m r^2 \dot{\theta}^2_{\text{rest}}$$

$$J = 0$$

$$\mathcal{L} = \frac{1}{2} m r^2 (\dot{\theta} + \Omega)^2$$

$$J = -m r^2 \Omega$$

Angular momentum density

gluon :

$$J_G = \left\langle \frac{1}{g_{\rm YM}^2} \mathrm{tr}[2yF_{xy}F_{y\tau} - 2xF_{yx}F_{x\tau} + 2yF_{xz}F_{z\tau} - 2xF_{yz}F_{z\tau}] \right\rangle$$

fermion orbit :

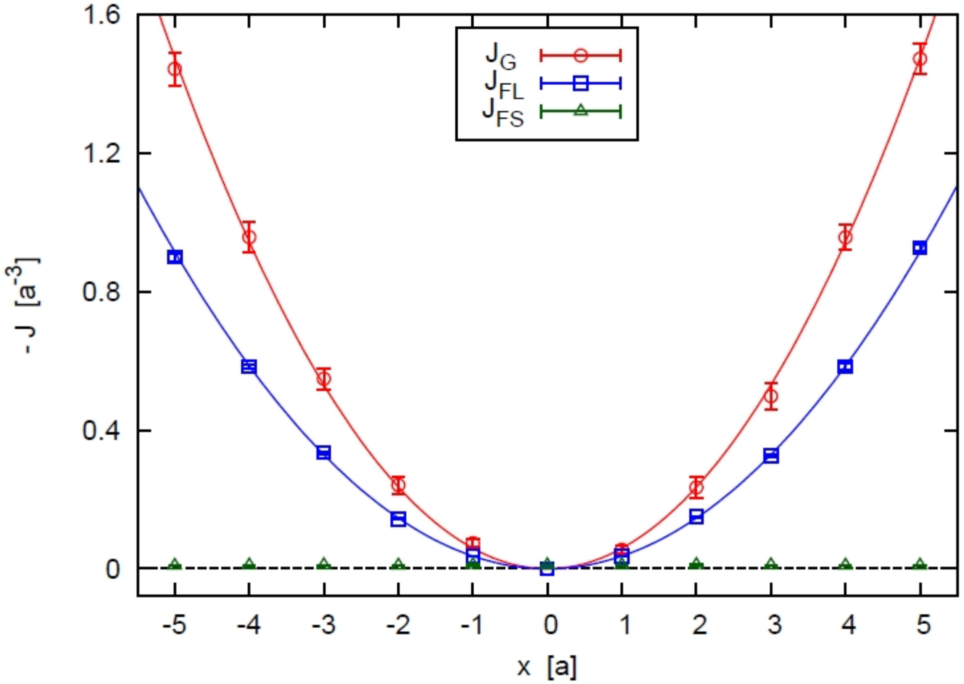
$$J_{FL} = \langle \bar{\psi}\gamma^4 (xD_y - yD_x)\psi \rangle$$

fermion spin :

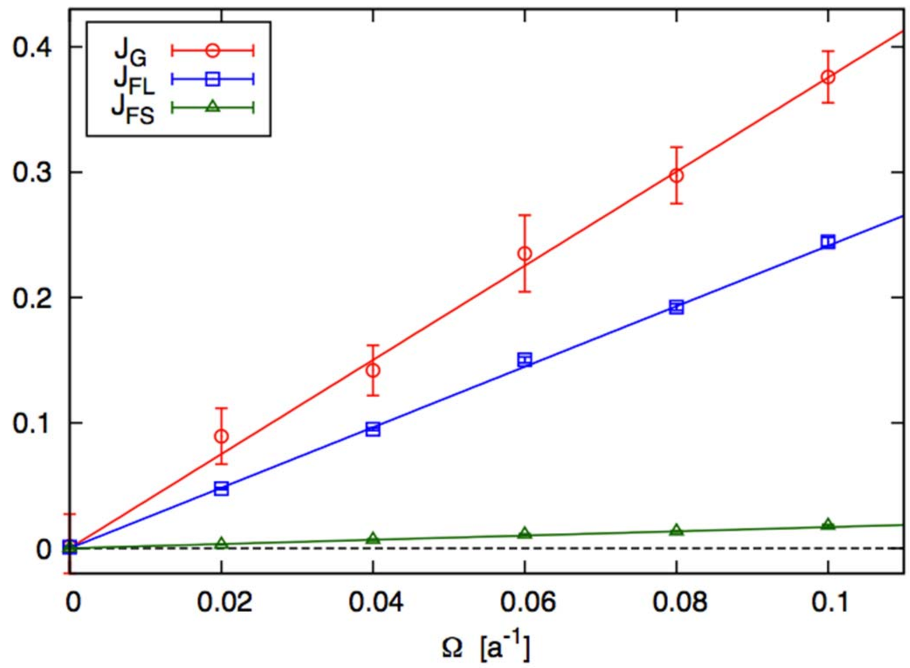
$$J_{FS} = \left\langle i\bar{\psi}\gamma^4 \frac{\sigma^{12}}{2}\psi \right\rangle$$

Angular momentum density

radius dependence



angular-velocity dependence



Angular momentum density

gluon : $$J_G = -(0.94 \pm 0.01)a^{-4} \times r^2\Omega$$

fermion orbit : $$J_{FL} = -(0.60 \pm 0.01)a^{-4} \times r^2\Omega$$

fermion spin : $$J_{FS} = -(0.17 \pm 0.01)a^{-2} \times \Omega$$

cf.) classical particle $$J = -I\Omega = -mr^2\Omega$$

Summary

- ✓ We formulated lattice QCD in rotating frames.
- ✓ We tested this framework by calculating angular momenta.
- ✓ There are many applications:

  rotating quark potential
  rotating hadron & QCD matter,
  quantum vortex nucleation,
  chiral vortical effect,
  etc.