Twisted reduction in large N QCD with adjoint Wilson fermions

M. Okawa with A. Gonzalez-Arroyo

Twisted space-time reduced model of large N QCD with adjoint Wilson fermions is constructed applying the symmetric twist boundary conditions with flux k for various number of flavors.

For two flavors, the string tension calculated at N=289 approaches zero as we decrease quark mass.

On the contrary, for one flavor, the string tension seems to remain finite as the quark mass decreases to zero.

A preliminary result for the 1/2 flavor theory is also presented （sign of Pfaffian is not included in the observables yet).

● Twisted reduced model of large N QCD with adjoint Wilson fermions

We consider gauge group $SU(N),\ N = L^2$

$$S = b N \sum_{\mu\neq\nu=1}^{d} Tr\left(I - Z_{\mu\nu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger\right) - \sum_{j=1}^{N_f} \bar{\Psi}_j D_W \Psi_j$$

$$Z_{\mu\nu} = \exp\left(k \frac{2\pi i}{L}\right), \quad Z_{\nu\mu} = Z_{\mu\nu}^*, \qquad \mu > \nu$$

$$D_W = 1 - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu) U_\mu^{adj} + (1+\gamma_\mu) U_\mu^{\dagger adj} \right], \qquad U_\mu^{adj} \Psi = U_\mu \Psi U_\mu^\dagger$$

k , L : co-prime

$k \neq 0$ corresponds to twisted boundary condition

A. Gonzalez-Arroyo, M.O.

$k = 0$ corresponds to periodic boundary condition

Kovtun, Unsal, Yaffe   Hietanen, Narayanan   Bringoltz, Koren, Sharpe

larger finite N effects compared with $k \neq 0$

Simulations have been done with $SU(N)$, $N = L^2$

$$N = 289 \quad (L = 17), \quad k = 5$$

Our model is related to the ordinary SU(N) lattice theory on $V = L^4$ space-time volume up to $O(1/N^2)$ corrections

$$N = 289 \quad \leftrightarrow \quad V = 17^4$$

We can, then, calculate Wilson loop W(R,R) up to $R = 8$.

In this talk, string tensions and the lowest eigenvalues of $Q^2 = (D_W \gamma_5)^2$ are calculated both for $N_f = 2$ and $N_f = 1$.

Large N QCD with $N_f = 2$ adjoint fermions

SU(N) LGT with two adjoint fermions is thought to be conformal or nearly conformal for any value of N.

Indeed, the first two coefficients of β function expressed in term of 't Hooft coupling $\lambda = g^2 N$ is independent of N.

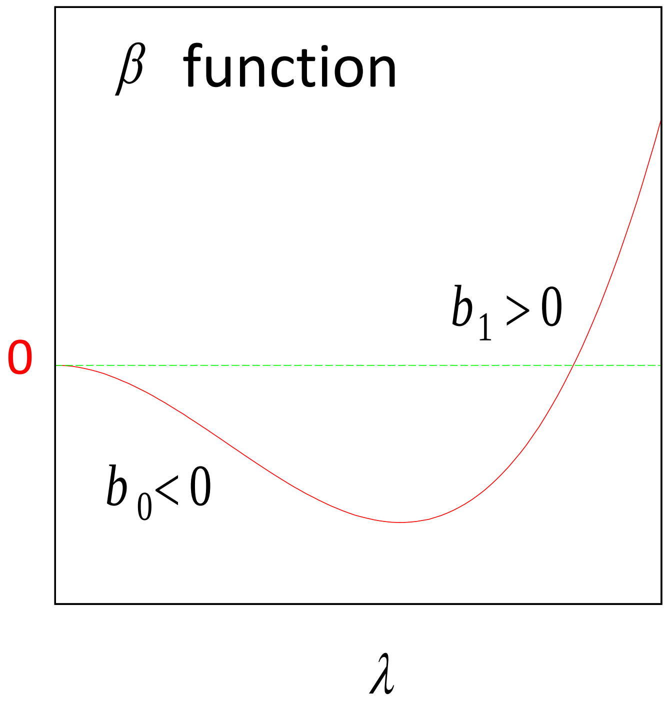
$$b_0 = \frac{4N_f - 11}{24\pi^2}, \quad b_1 = \frac{16N_f - 17}{192\pi^4}$$

● asymptotic free

$$b_0 < 0 \rightarrow N_f < \frac{11}{4} = 2.75$$

● infrared fixed point

$$b_1 > 0 \rightarrow N_f > \frac{17}{16} = 1.08$$



In the last lattice conference at Cairns, we have reported the preliminary results of string tension for $N_f = 2$.

The HMC updation had been done for the following stopping condition during molecular dynamic steps

Let $r = s - Q^2 x$ be the residue with s the source.

Then we require $|r|^2 / |s|^2 < 10^{-7}$.

We could only achieve rather low global acceptance ratio R . For example, at $b = 0.35, \kappa = 0.14$

$R = 0.500$ with $\Delta\tau = 1/800$.

This is due to the weak stopping condition! In fact, we have

$R = 0.894$ with $\Delta\tau = 1/800$ and $|r|^2 / |s|^2 < 10^{-10}$

Run parameters of the present talk

$$N = 289 \quad (L = \ 17), \quad k = 5, \quad b = 0.35 \ \& \ 0.36$$

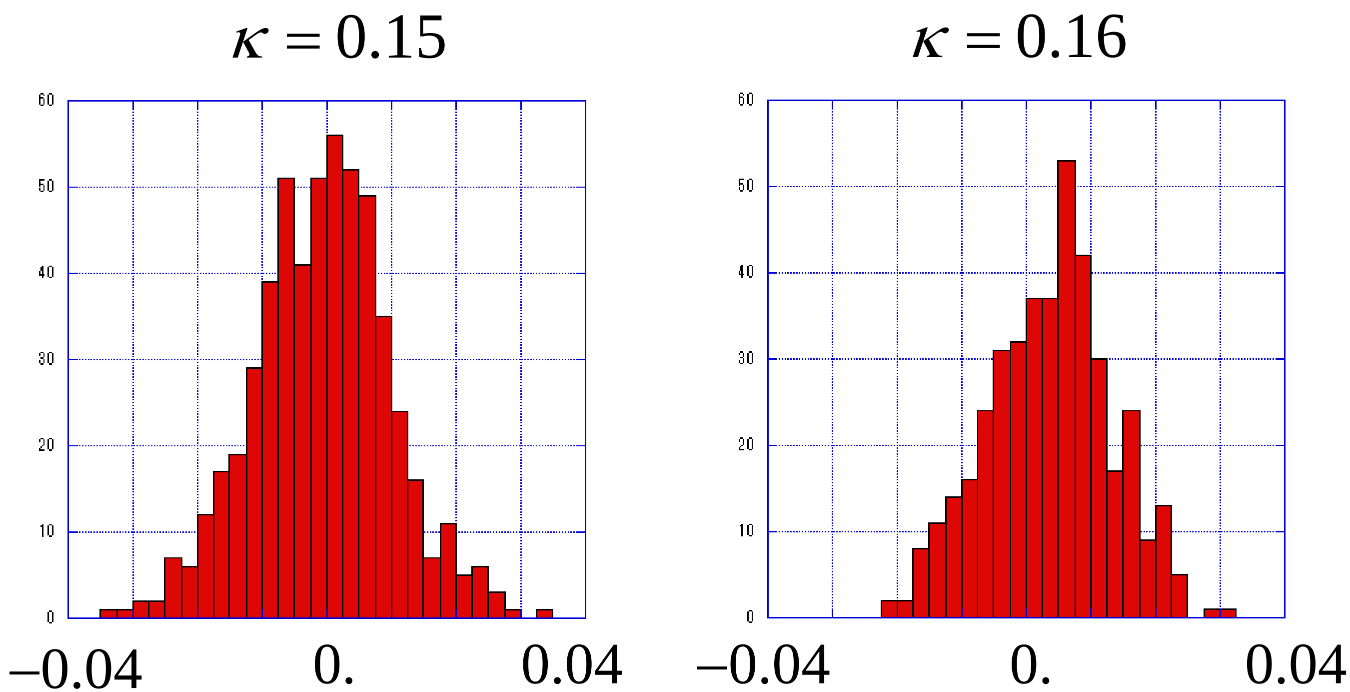
$N_f = 2$	$N_f = 1$
$\kappa = 0.10, 0.11, 0.12, 0.13$ $0.14, 0.15, 0.16$	$\kappa = 0.10, 0.11, 0.12, 0.13$ $0.14, 0.15, 0.155$
$\lvert r \rvert^2 / \lvert s \rvert^2 < 10^{-10}$	$\lvert r \rvert^2 / \lvert s \rvert^2 < 10^{-7}$

$\Delta\tau$ is chosen so that the global acceptance ratio is around 0.9.

For $N_f = 2$, the run at $\kappa = 0.16$ is suffered from finite size effect.

For $N_f = 1$, we can not make simulation for $\kappa > 0.155$ since CG iteration does not converge.

Histogram of $\frac{1}{N}\mathrm{Re}\,Tr(U_\mu)^L$ at $N_f = 2,\ b = 0.35$



Since the effective lattice size of our system is L^4,

$\frac{1}{N}\left\langle \mathrm{Re}\,Tr(U_\mu)^L \right\rangle \neq 0$ is a finite size (temperature) effect.

We always check that $\frac{1}{N}\left\langle \mathrm{Re}\,Tr(U_\mu)^\ell \right\rangle = 0, \quad (\ell = 1, L-1)$

● Large Wilson loop at N=289 and k=5

In the reduced model, the Wilson loop $W(R,T)$ is defined by

$$W(R,T) = Z_{\mu\nu}^{RT} \left\langle Tr(U_\mu^R U_\nu^T U_\mu^{\dagger R} U_\nu^{\dagger T}) \right\rangle$$

We use smearing method to obtain good statistics

$$U_\mu^{smeared} = \Pr oj_{SU(N)} \left[U_\mu + c \sum_{\nu \neq \mu} (z_{\nu\mu} U_\nu U_\mu U_\nu^\dagger + z_{\mu\nu} U_\nu^\dagger U_\mu U_\nu) \right]$$

with $z_{\mu\nu}$ the twist tensor and $\Pr oj_{SU(N)}$ stands for the operator that projects onto the SU(N) matrices.

We choose c=0.1 and made 20 smearing.

String tension

String tension is extracted from the Creutz ratio

$$\chi(R,R) = -\log \frac{W(R+0.5,R+0.5)W(R-0.5,R-0.5)}{W(R+0.5,R-0.5)W(R-0.5,R+0.5)}$$

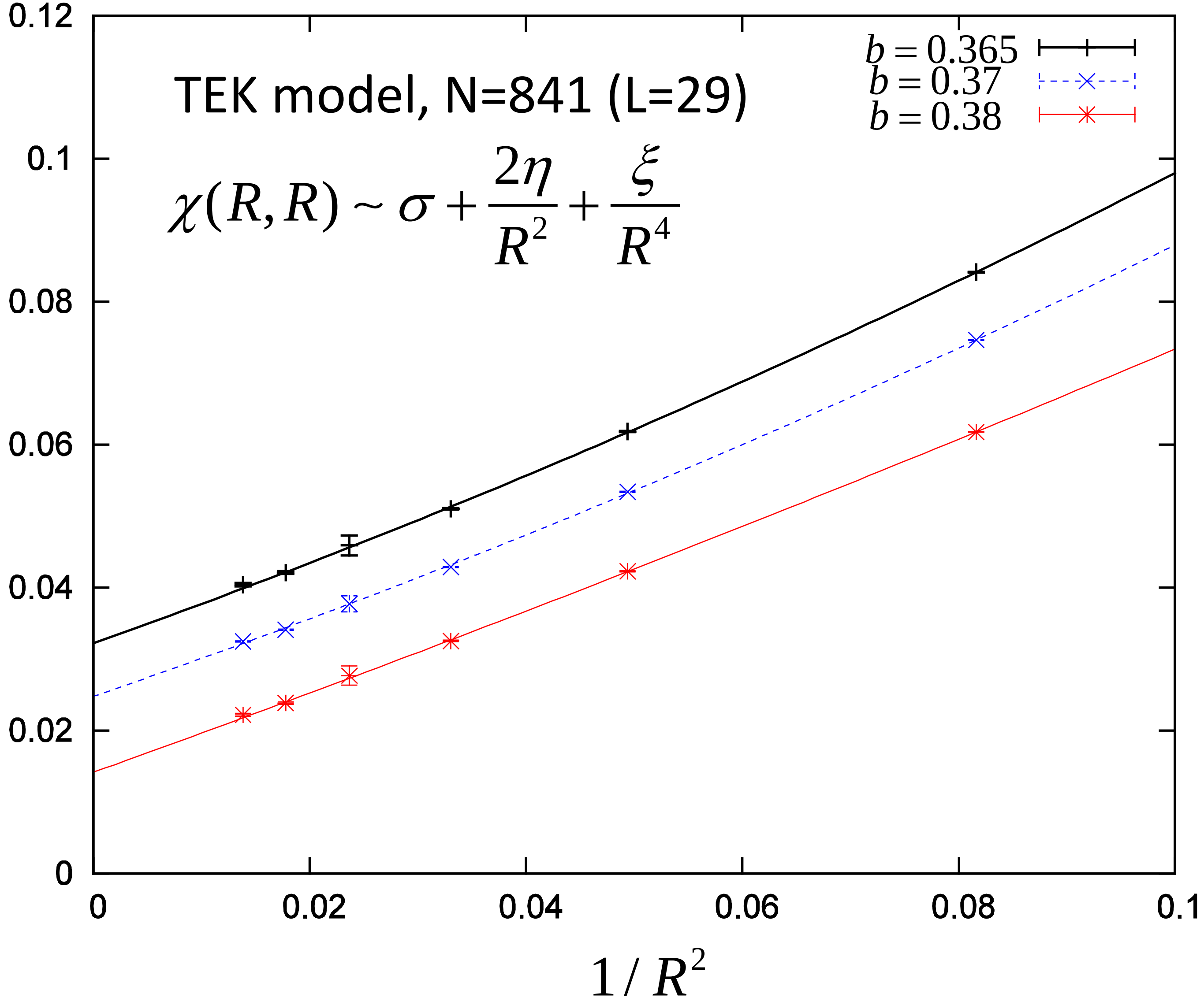
$$\sim \sigma + \frac{2\eta}{R^2} + \frac{\xi}{R^4}$$

with half integer R.

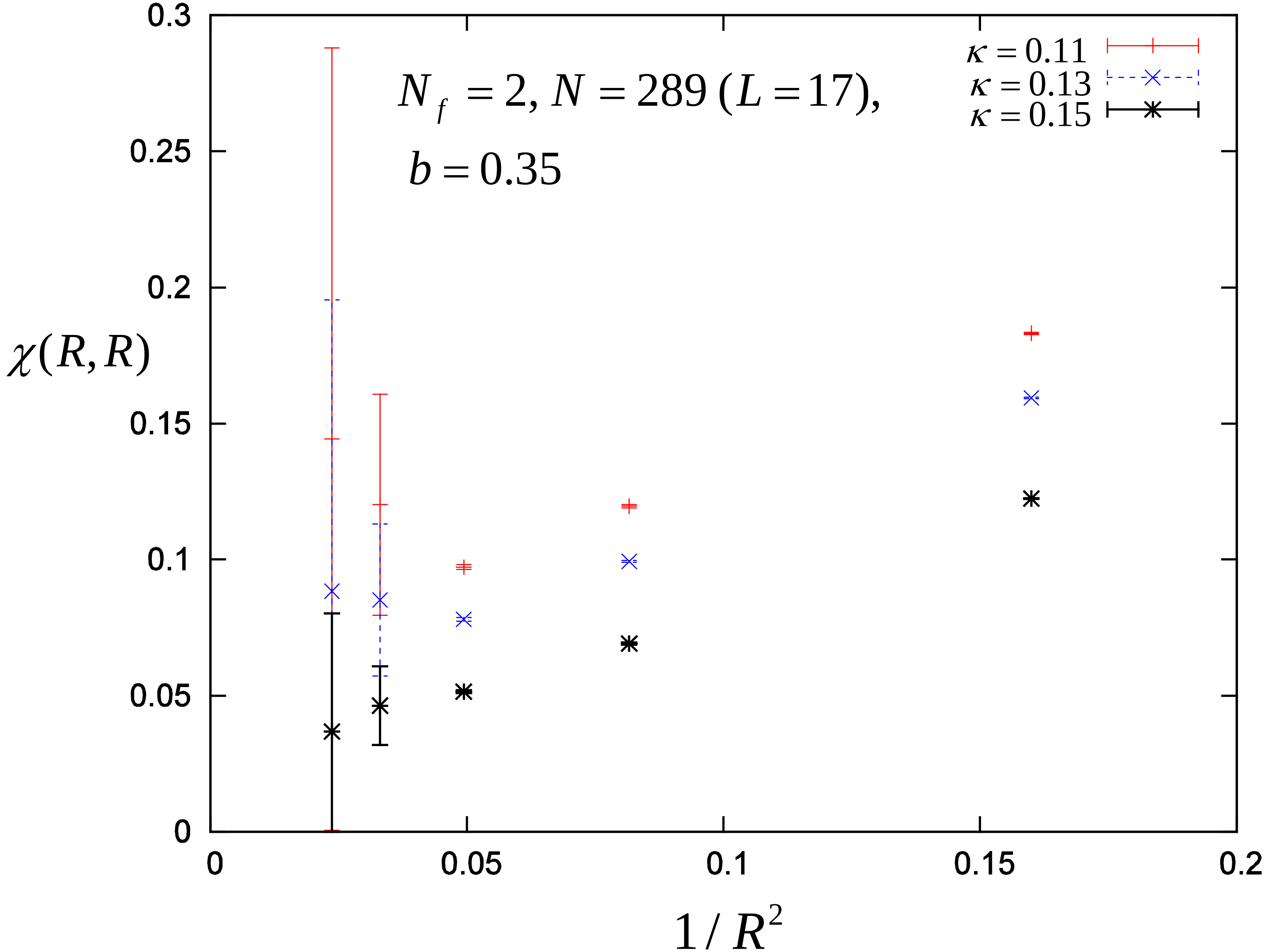
This method works quite well for the twisted Eguchi-Kawai model for pure gauge theory. In fact, we can determine σ, η, ξ rather precisely.

η depends only slightly on b .

η is dimensionless, which could be a universal quantity as is expected from string theory.



TEK model, N=841 (L=29)
$\chi(R,R) \sim \sigma + \frac{2\eta}{R^2} + \frac{\xi}{R^4}$
$b = 0.365$
$b = 0.37$
$b = 0.38$
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.02
0.04
0.06
0.08
0.1
$1/R^2$



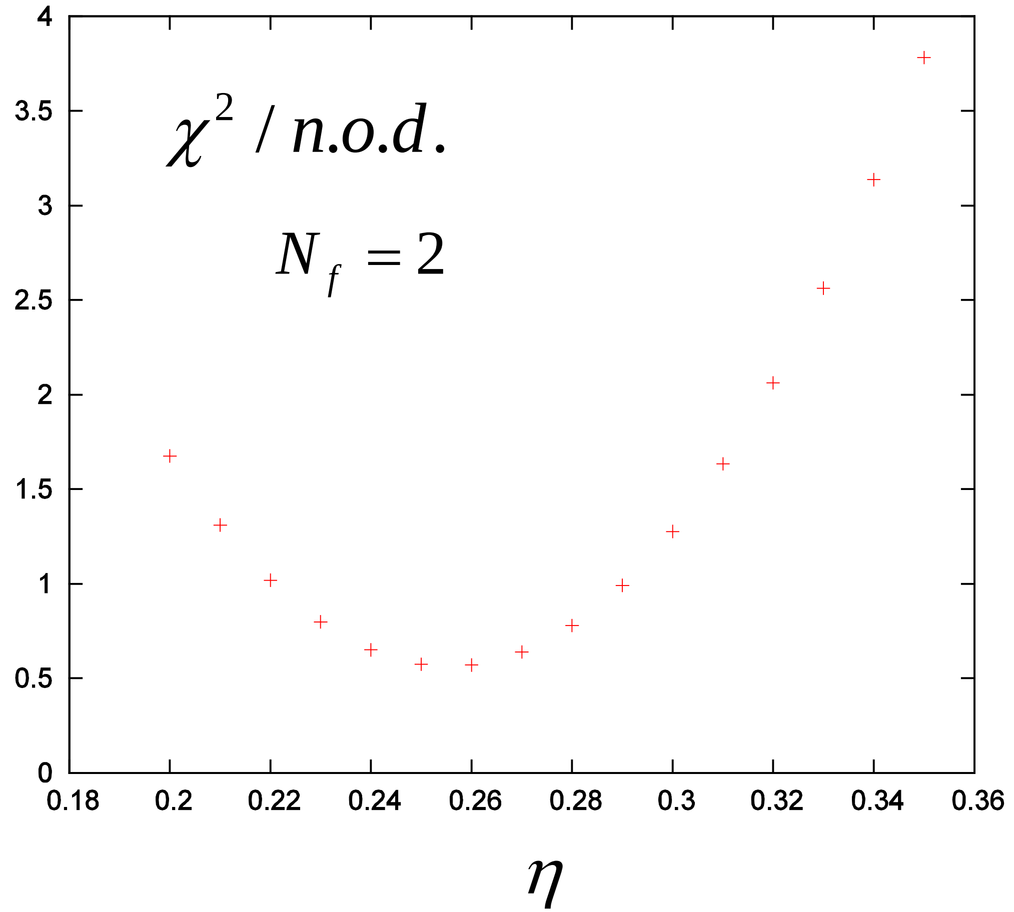
$N_f = 2, N = 289 \, (L = 17),$
$b = 0.35$
$\kappa = 0.11$
$\kappa = 0.13$
$\kappa = 0.15$
$\chi(R,R)$
$1/R^2$
0
0.05
0.1
0.15
0.2
0.25
0.3

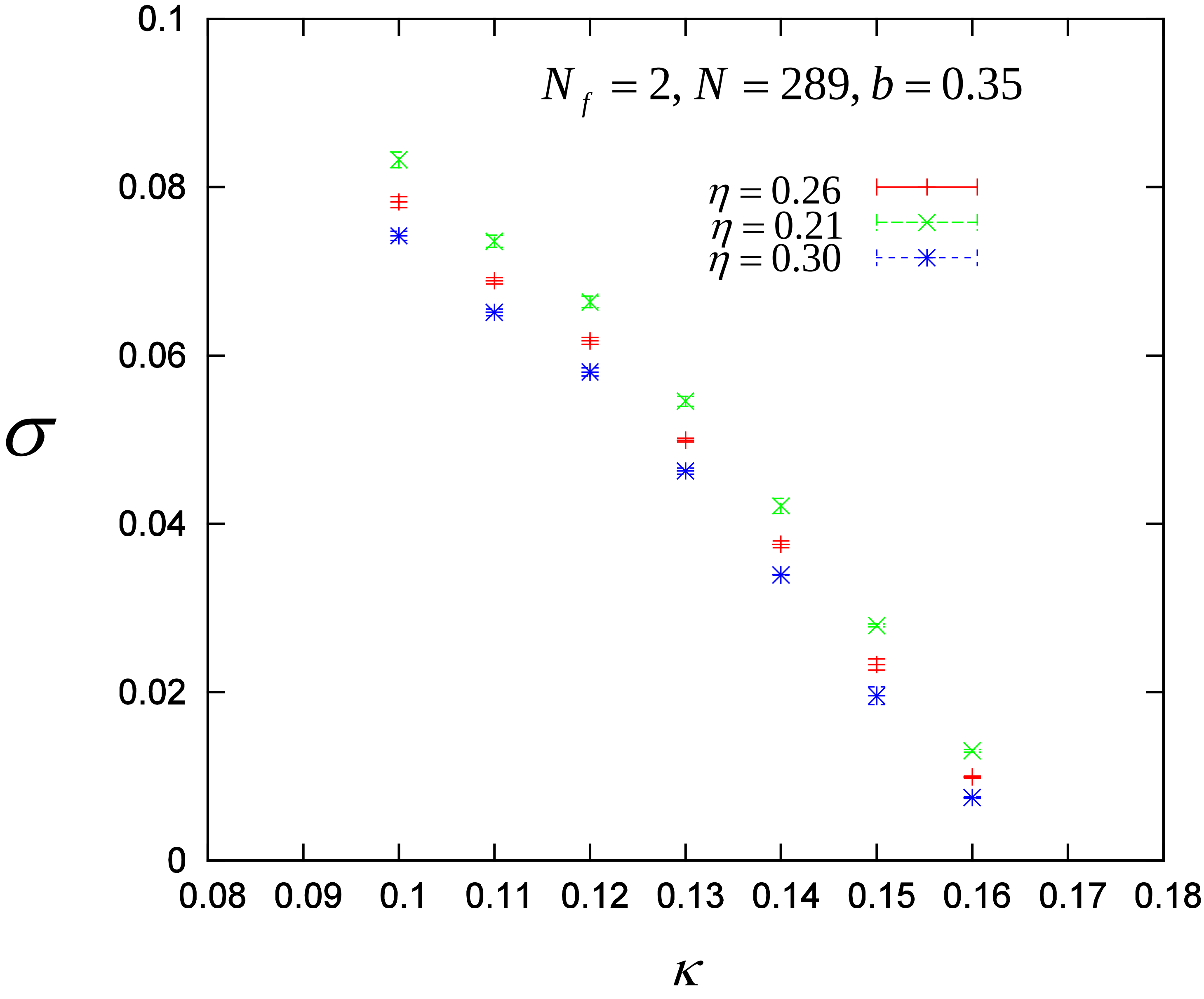
For the theory with adjoint fermions, three parameter fit

$$\chi(R,R) = \sigma + \frac{2\eta}{R^2} + \frac{\xi}{R^4}$$

is unstable, due to small $L = 17$ compared with $L = 29$ of TEK model, and due to less statistics.

We fix the value of η to the universal constant depending only on N_f .



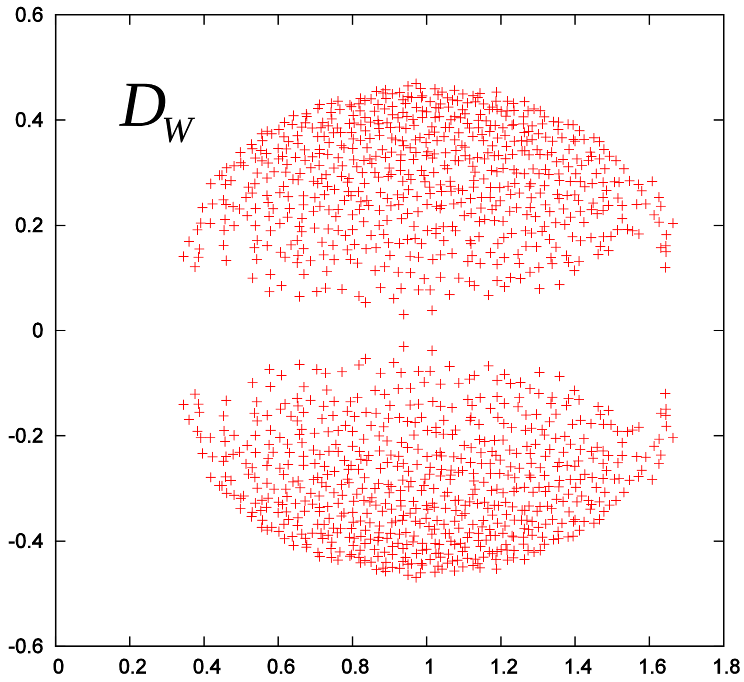


$N_f = 2, N = 289, b = 0.35$
$\eta = 0.26$
$\eta = 0.21$
$\eta = 0.30$
σ
κ
0.1
0.08
0.06
0.04
0.02
0
0.08
0.09
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18

Lowest eigenvalue λ of positive hermitian Wilson Dirac operator

$Q^2 = (D_W \gamma_5)^2$ is related to the quark mass as

$$D_W = 1 - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu) U_\mu^{adj} + (1+\gamma_\mu) U_\mu^{\dagger adj} \right] = 2\kappa m_q + 2\kappa \partial_\mu \gamma^\mu + \cdots$$



$$\lambda = 4\kappa^2 m_q^2$$

$$\therefore m_q = \sqrt{\lambda} / (2\kappa)$$

In the usual QCD, m_q is expected to depend on κ as

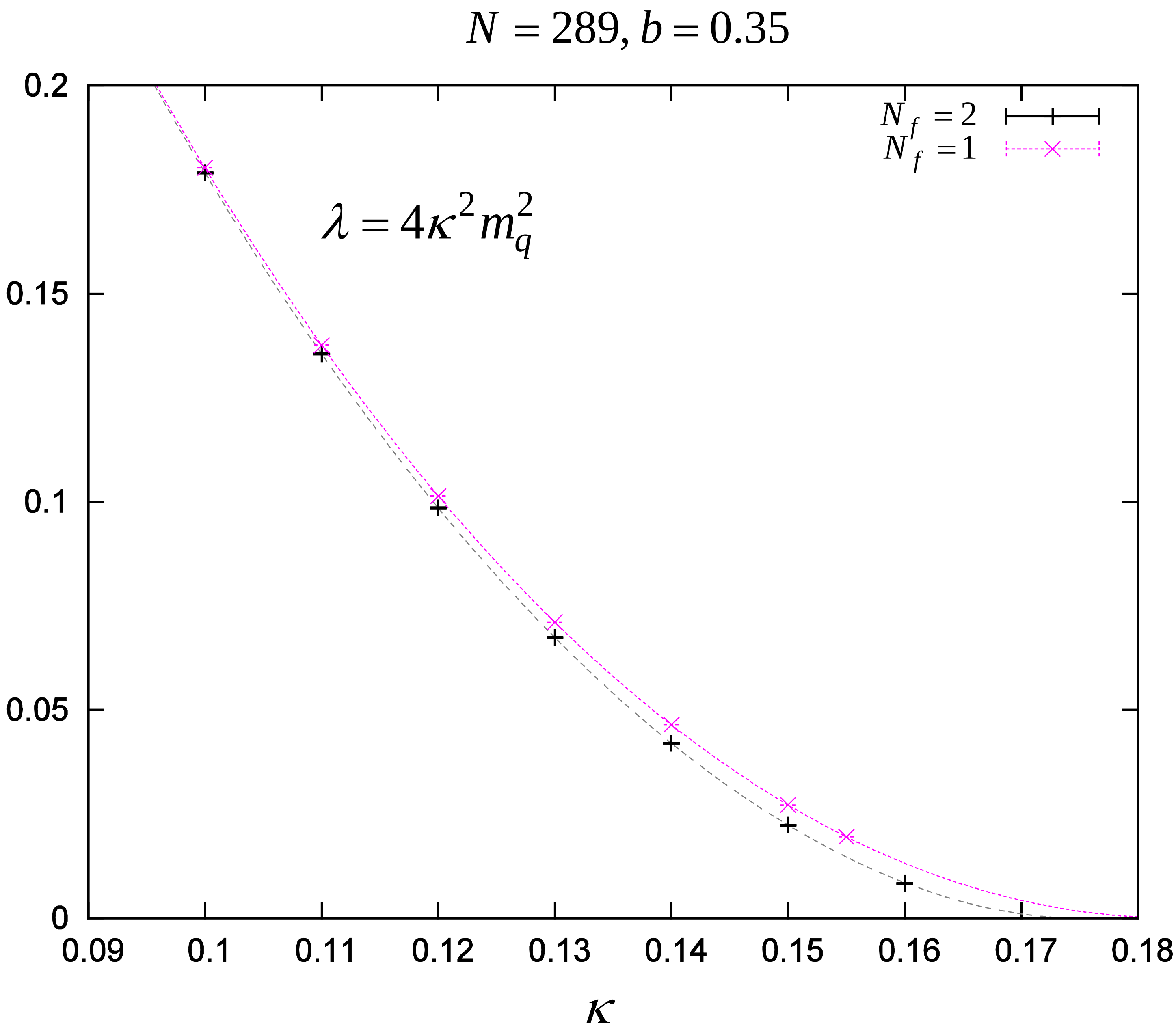
$$m_q = (1/\kappa - 1/\kappa_c) / 2$$

However, for $N_f = 2$ adjoint theory, this is not the case.
In fact, by fitting m_q with the following function,

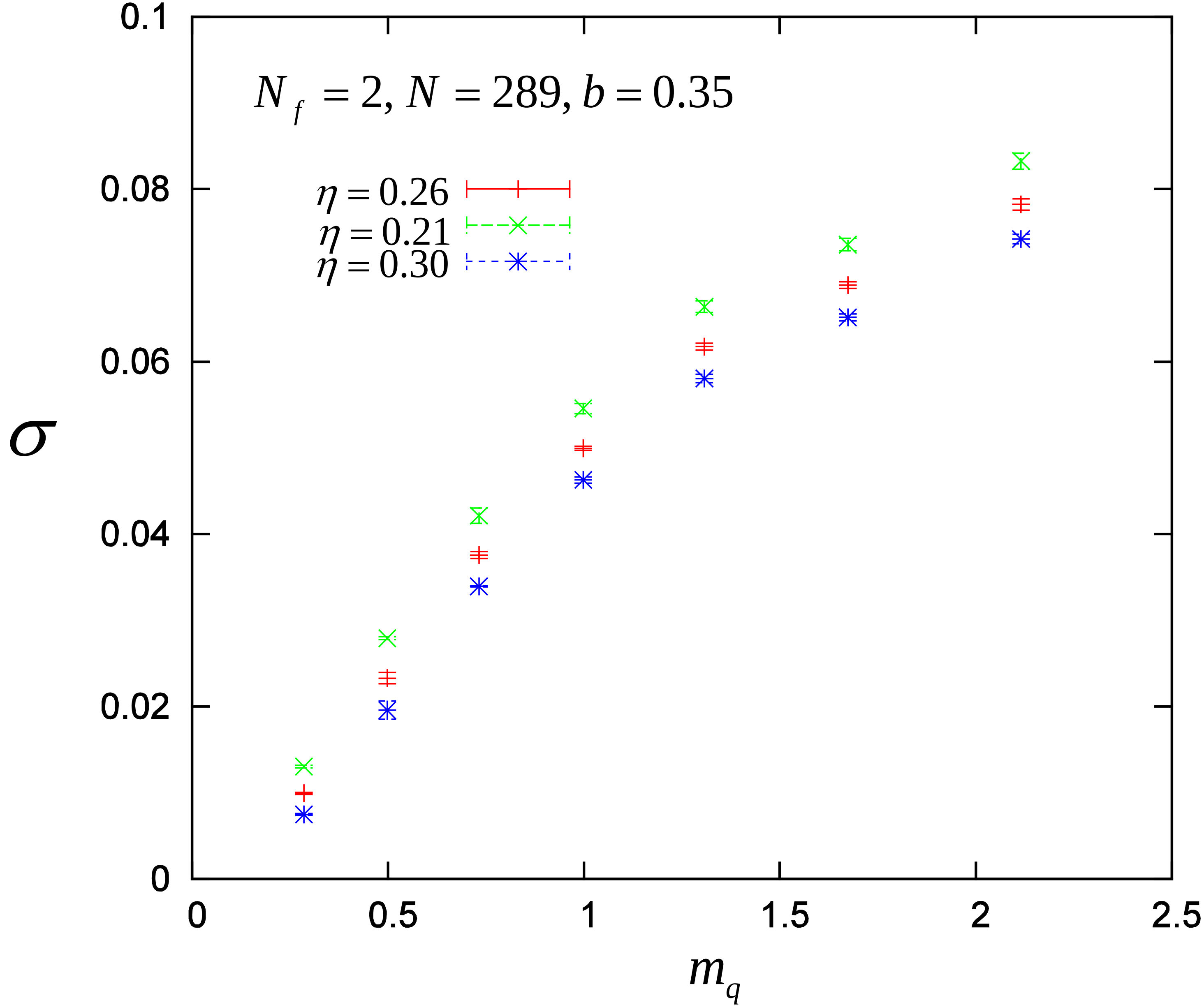
$$m_q = \sqrt{\lambda}/(2\kappa) = A\left(1/\kappa - 1/\kappa_c\right)^{\delta}\left(1 + B\left(1/\kappa - 1/\kappa_c\right)\right)$$

We have $\delta = 0.914(1)$.

On the contrary, for $N_f = 1$ case, we have $\delta = 1.001(1)$.



$N = 289, b = 0.35$
$N_f = 2$
$N_f = 1$
$\lambda = 4\kappa^2 m_q^2$
0.2
0.15
0.1
0.05
0
0.09
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
κ



$N_f = 2, N = 289, b = 0.35$
$\eta = 0.26$
$\eta = 0.21$
$\eta = 0.30$
σ
m_q
0.1
0.08
0.06
0.04
0.02
0
0.5
1
1.5
2
2.5

If the theory is governed by an infrared fixed point with the relevant mass term $m_q \bar{\psi}\psi$, all physical quantity having mass dimension should vanish as $m_q \to 0$.

In particular, the string tension having mass square dimension should behave as

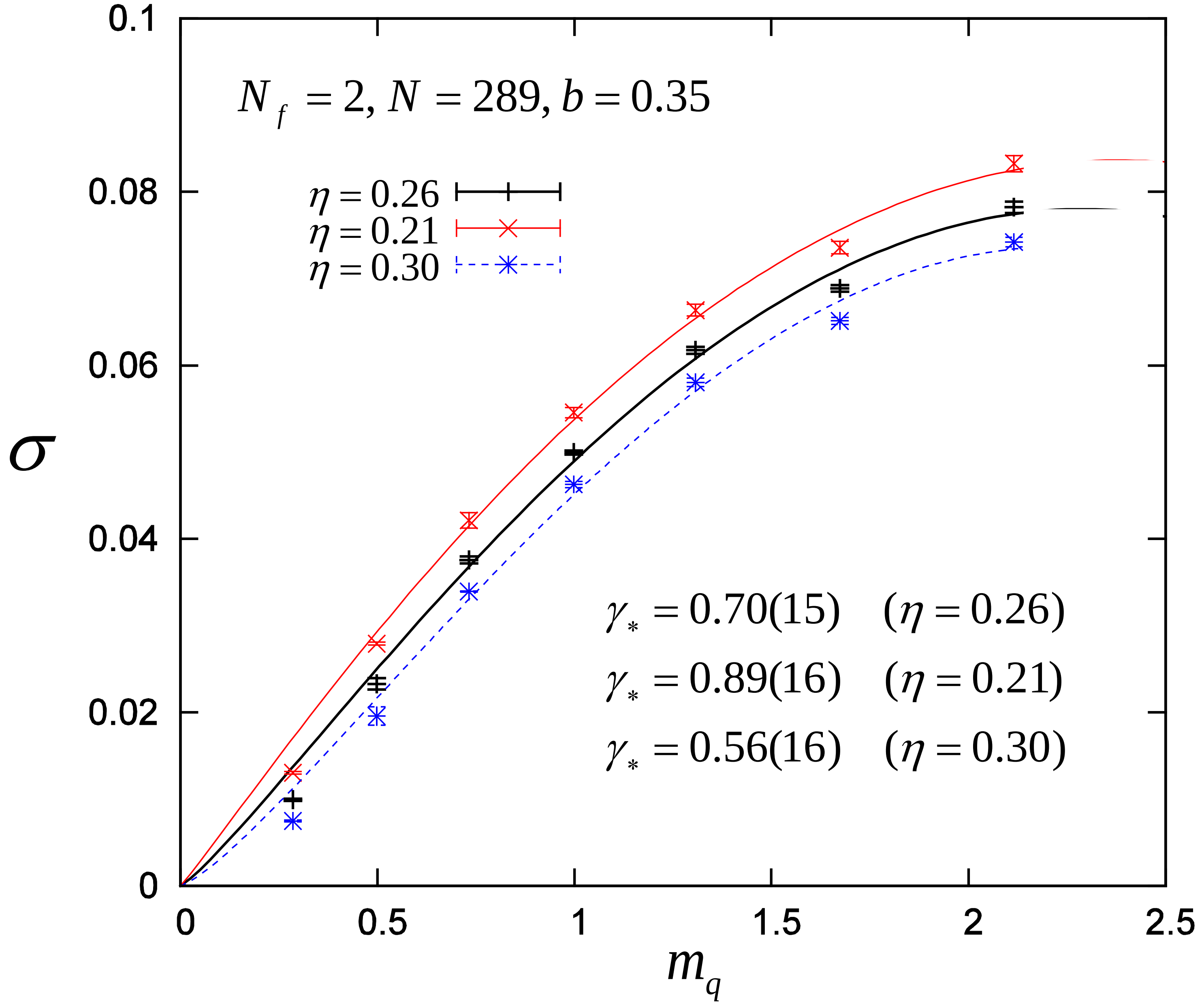
$$\sigma = A m_q^{2/(1+\gamma_*)}(1 + B m_q)$$

with γ_* the mass anomalous dimension at infrared fixed point and we have included possible $O(m_q)$ correction. We get

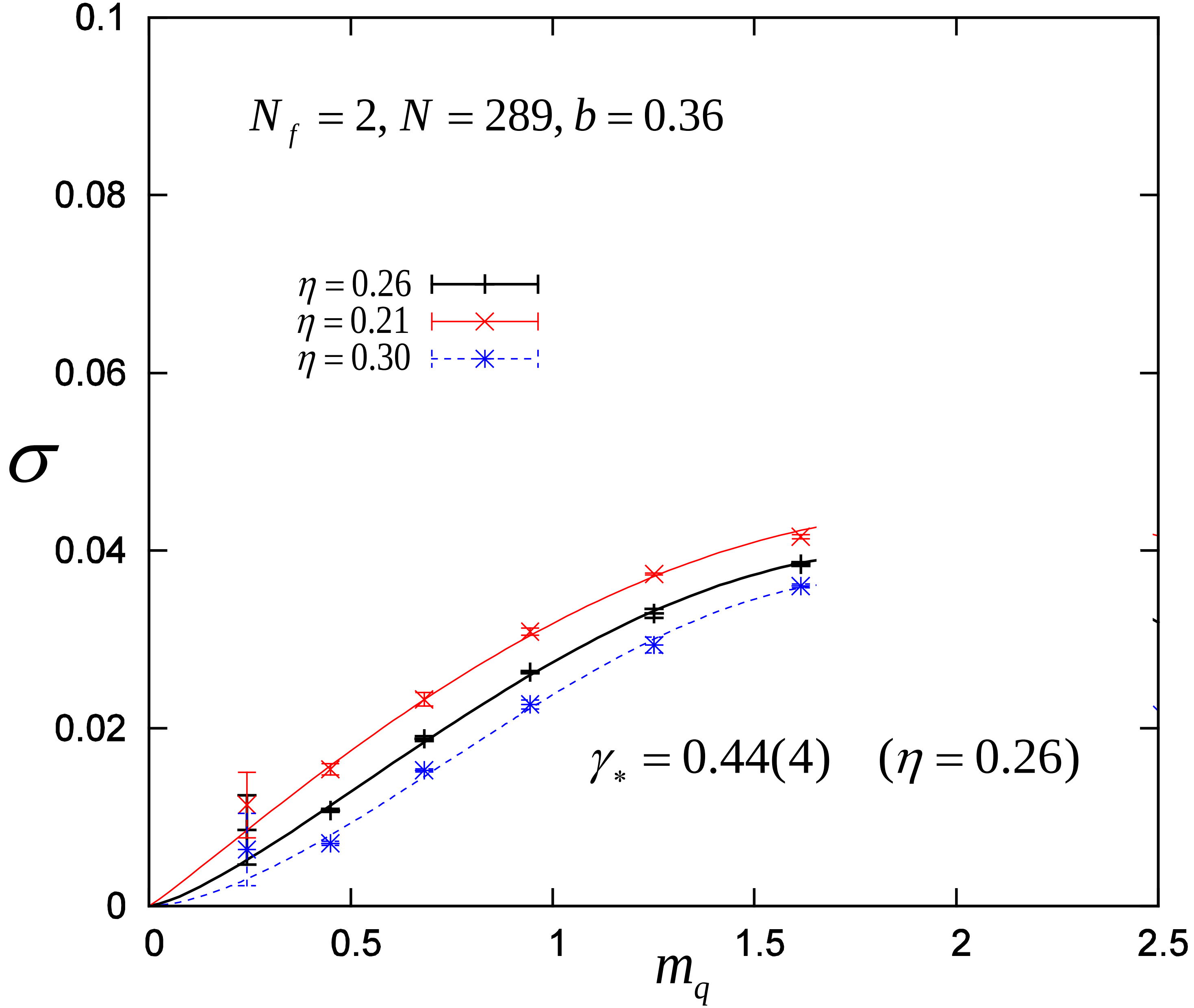
$$\gamma_* = 0.70(15) \quad (\eta = 0.26)$$

$$\gamma_* = 0.89(16) \quad (\eta = 0.21)$$

$$\gamma_* = 0.56(16) \quad (\eta = 0.30)$$



$N_f = 2, N = 289, b = 0.35$
$\eta = 0.26$
$\eta = 0.21$
$\eta = 0.30$
$\gamma_* = 0.70(15) \quad (\eta = 0.26)$
$\gamma_* = 0.89(16) \quad (\eta = 0.21)$
$\gamma_* = 0.56(16) \quad (\eta = 0.30)$
σ
m_q
0
0.02
0.04
0.06
0.08
0.1
0.5
1
1.5
2
2.5



0.1
0.08
0.06
0.04
0.02
0
0
0.5
1
1.5
2
2.5
σ
m_q
$N_f = 2, N = 289, b = 0.36$
$\eta = 0.26$
$\eta = 0.21$
$\eta = 0.30$
$\gamma_* = 0.44(4) \quad (\eta = 0.26)$

Motivation for $N_f = 1$ adjoint fermion

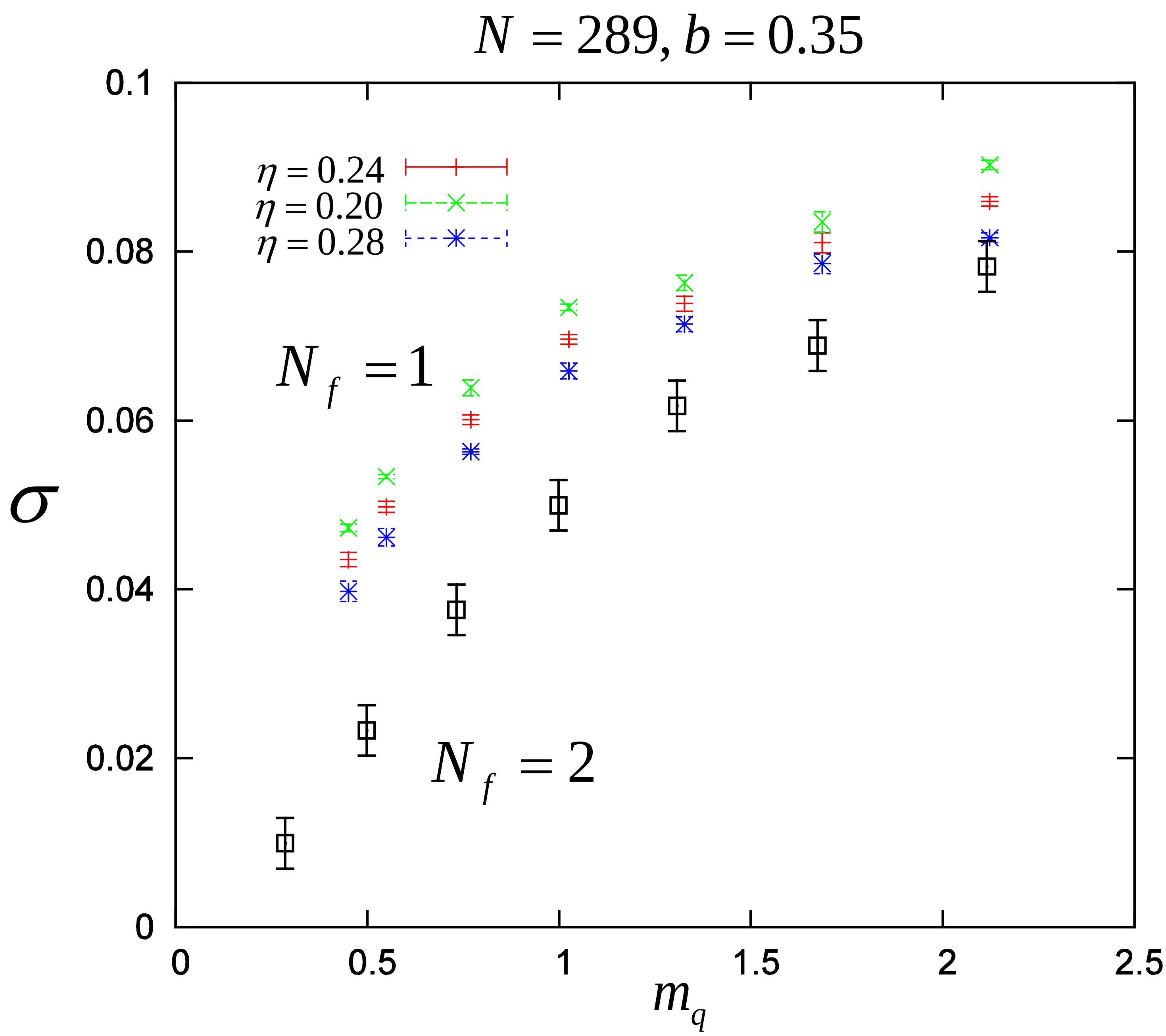
In the large N limit, $N_f = 1$ adjoint fermion is equivalent to $N_f = 2$ fundamental fermion in rank two anti-symmetric rep.

(Armoni, Shifman, Veneziano, Kovtun, Unsal, Yaffe)

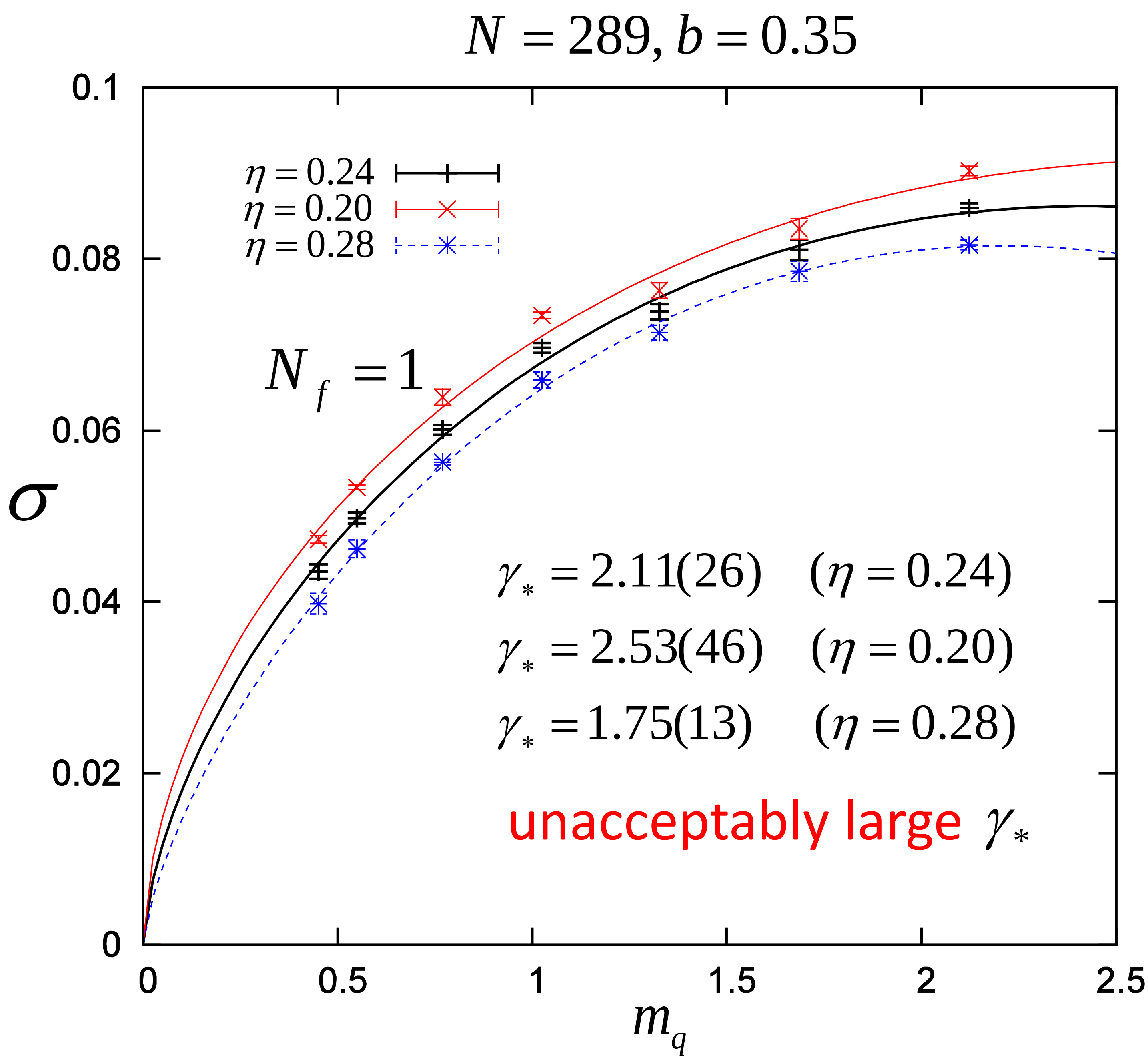
For N=3, the latter theory is just two flavor QCD and our model corresponds to Corrigan-Ramond large-N limit.

We then expect the reduced model of $N_f = 1$ adjoint fermion as

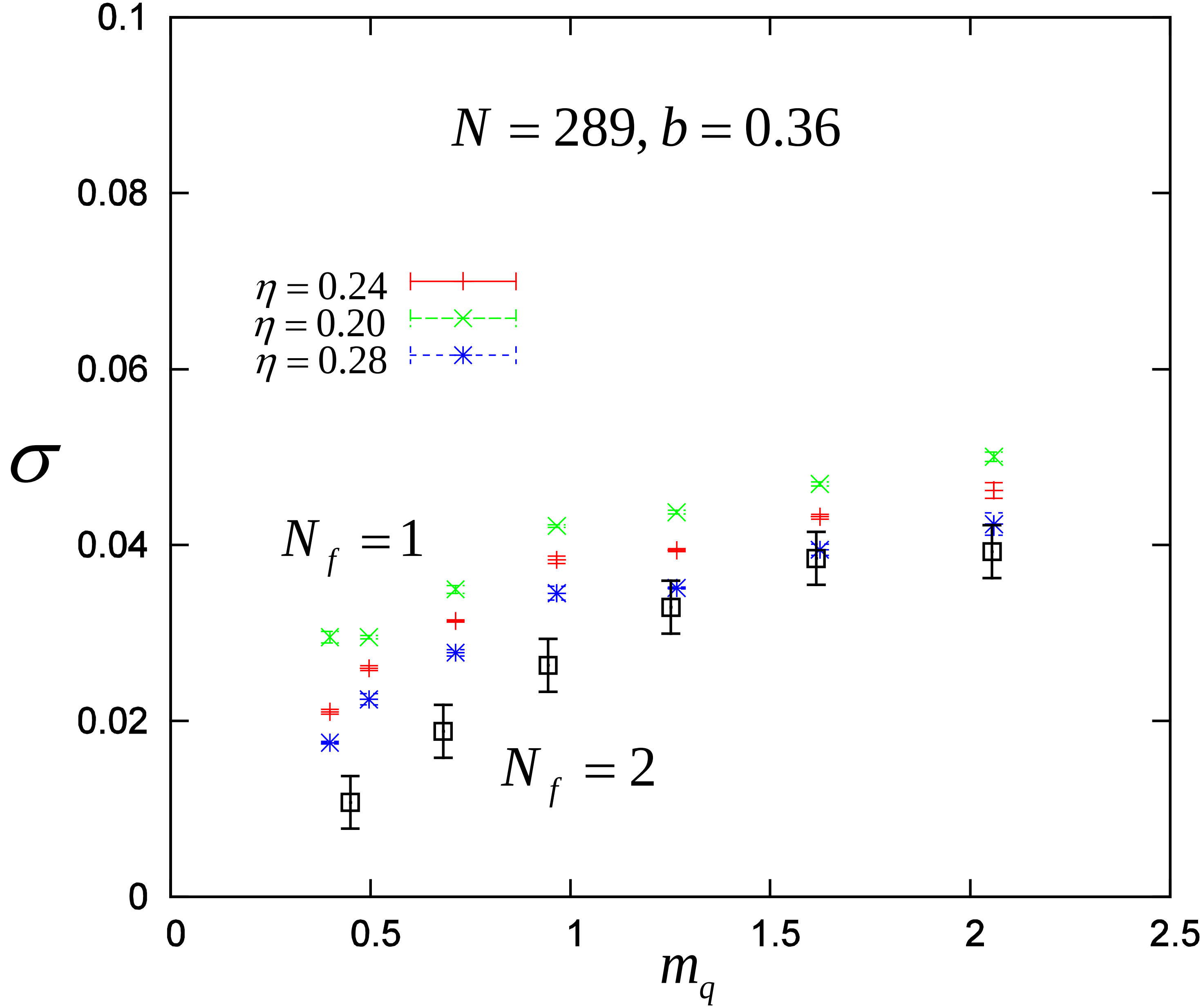
confining theory



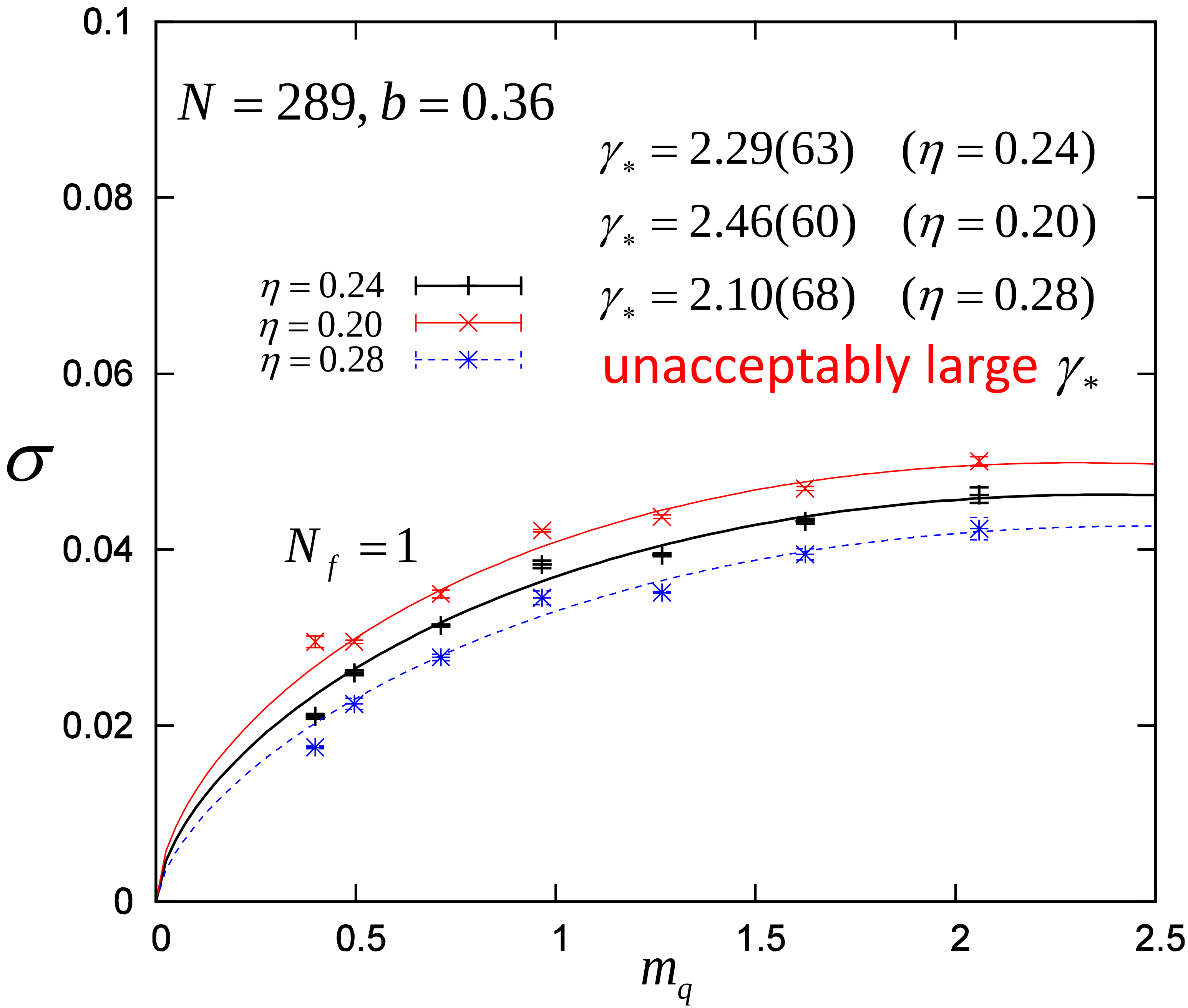
N = 289, b = 0.35
0.1
0.08
0.06
0.04
0.02
0
0
0.5
1
1.5
2
2.5
σ
m_q
η = 0.24
η = 0.20
η = 0.28
N_f = 1
N_f = 2



$N = 289, b = 0.35$
0.1
0.08
0.06
0.04
0.02
0
σ
$\eta = 0.24$
$\eta = 0.20$
$\eta = 0.28$
$N_f = 1$
$\gamma_* = 2.11(26) \quad (\eta = 0.24)$
$\gamma_* = 2.53(46) \quad (\eta = 0.20)$
$\gamma_* = 1.75(13) \quad (\eta = 0.28)$
unacceptably large γ_*
0
0.5
1
1.5
2
2.5
m_q



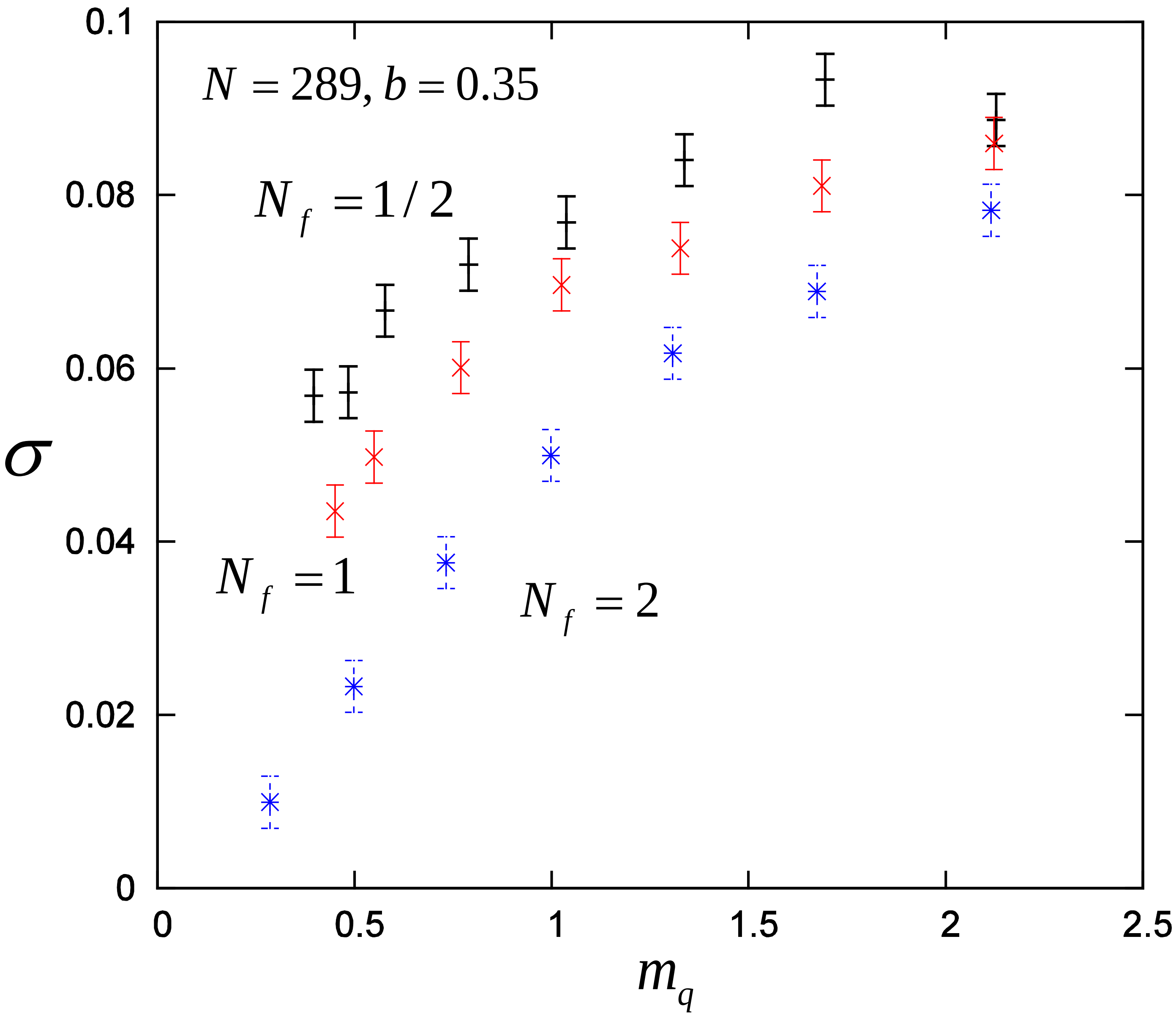
$N = 289, b = 0.36$
$\eta = 0.24$
$\eta = 0.20$
$\eta = 0.28$
$N_f = 1$
$N_f = 2$
σ
m_q
0.1
0.08
0.06
0.04
0.02
0
0
0.5
1
1.5
2
2.5



$N = 289, b = 0.36$
$\gamma_* = 2.29(63) \quad (\eta = 0.24)$
$\gamma_* = 2.46(60) \quad (\eta = 0.20)$
$\gamma_* = 2.10(68) \quad (\eta = 0.28)$
unacceptably large γ_*
$\eta = 0.24$
$\eta = 0.20$
$\eta = 0.28$
$N_f = 1$
σ
m_q
0.1
0.08
0.06
0.04
0.02
0
0.5
1
1.5
2
2.5

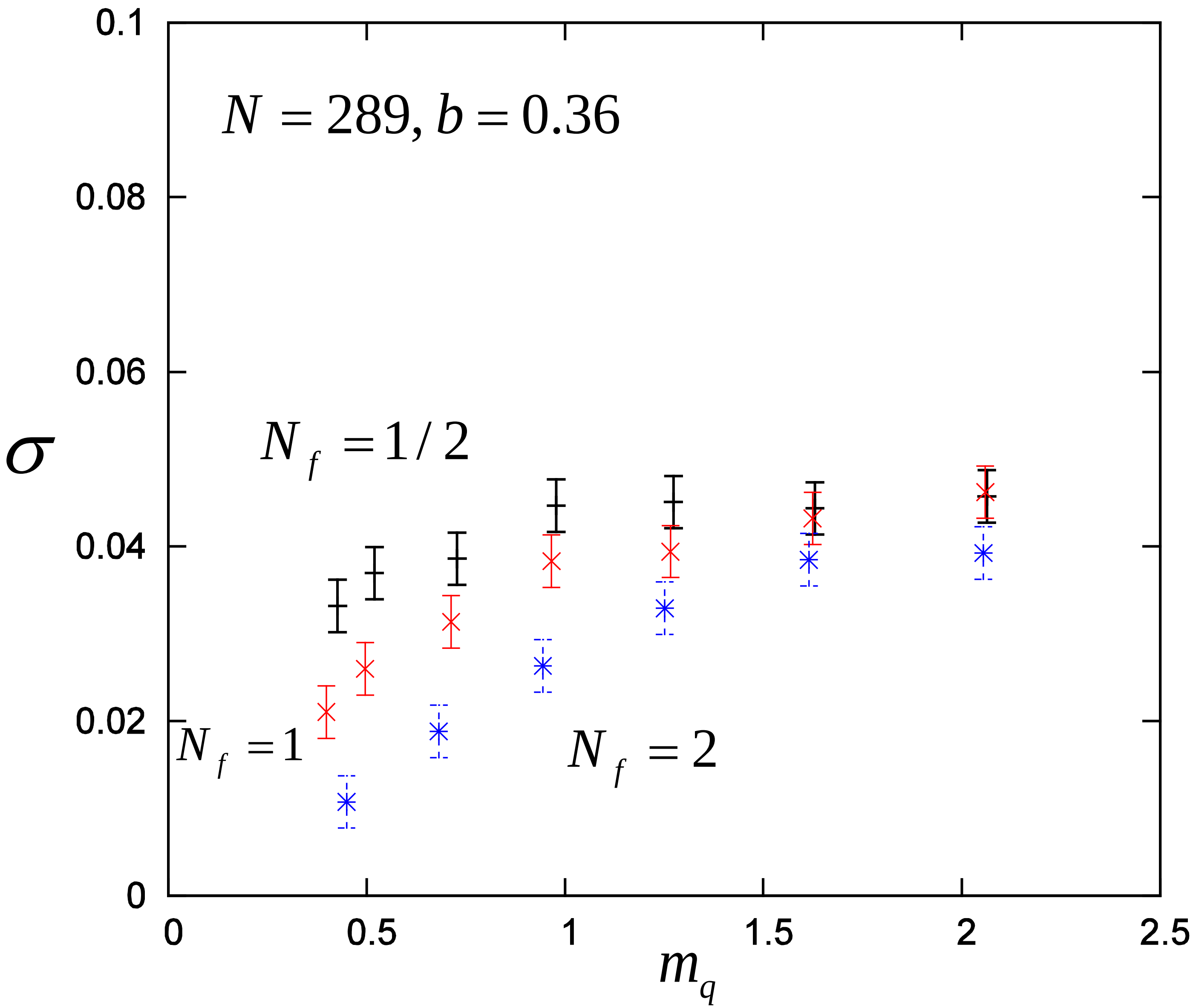
Preliminary result for the 1/2 flavor theory

(sign of Pfaffian is not included in the observables yet)



Preliminary result for the 1/2 flavor theory

(sign of Pfaffian is not included in the observables yet)



Conclusion

We have demonstrated that the twisted reduced model of large N QCD with adjoint Wilson fermions works quite well.

For two flavors, string tension is calculated at N=289, which clearly decreases as we decrease m_q and seems to vanish at $m_q = 0$.

Reliable estimation of mass anomalous dimension γ_* requires more data for small m_q.

Use larger N on 1^4 lattice.
Make simulation on 2^4 lattice (Narayanan Neuberger)

Use eigenvalue distribution of $Q^2 = (D_W \gamma_5)^2$ (next talk by Keegan)

For $N_f = 1,\ 1/2$, string tension remain finite as we decrease m_q , indicating that these theories are confining.