

Chiral phase transition of $N_f=2+1$ QCD with the HISQ action

Heng-Tong Ding

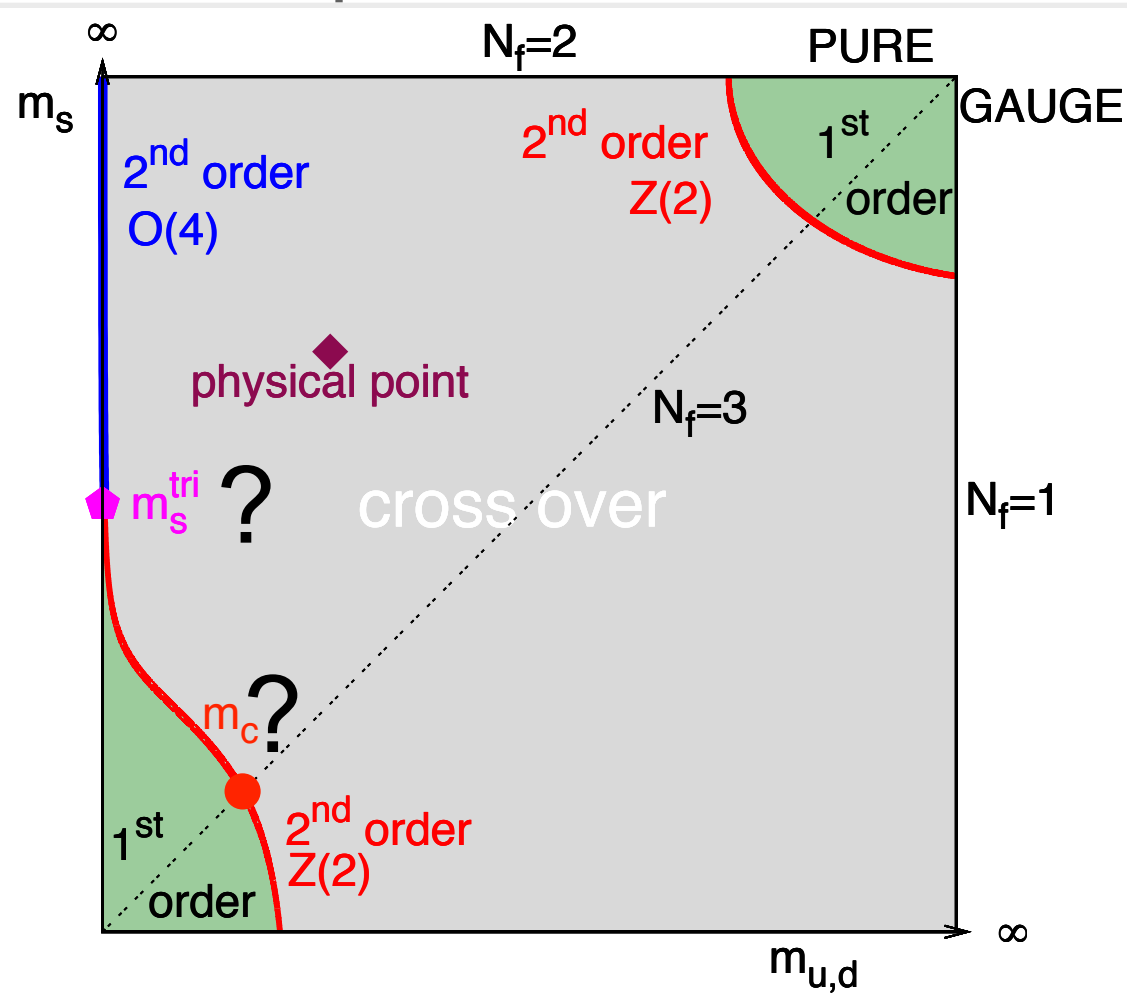
Brookhaven National Lab & Columbia University

in collaboration with A. Bazavov, F. Karsch, Y. Maezawa, S. Mukherjee and P. Petreczky

Lattice conference at Mainz
July 29, 2013

QCD phase diagram at $\mu=0$

columbia plot:



- $N_f=2+1$ theory: at $m=0$ or ∞ has a first order phase transition Pisarski, Wilczek PRD '84, Alexandrou et al., PRD'99...
- Intermediate quark mass region an analytic cross over is expected
- At physical quark masses, a cross over is confirmed Bernard et al., PRD '05, Cheng et al., PRD '06, Aoki et al., Nature '06...
- Critical lines of second order transition
 - $N_f=2$: $O(4)$ universality class Ejiri et al., PRD '09, ...
 - $N_f=3$: Ising universality class Karsch, Laermann, Schmidt PLB '04,...

★ How large is the chiral phase transition T_c ?

★ How large is the influence of scaling regimes to the physical world ?

$O(N)$ spin models and $N_f=2$ QCD

QCD at low energies can be described effectively by $O(N)$ symmetric spin models

- $SU(2)_L \times SU(2)_R$ is isomorphic to $O(4)$
- $O(4)$ fields: $\sigma = \bar{q}q$, $\pi = \bar{q}\gamma_5 \tau^i q$, and $\eta = \bar{q}\gamma_5 q$, $\delta = \bar{q}\tau^i q$
- external field H corresponds to quark mass m
- order parameter “magnetization” $\Sigma = \langle \sigma \rangle$

This description is valid both below and in the vicinity of the chiral phase transition region

chiral phase transition and universal scaling

Behavior of the free energy close to critical lines

$$f(m,T) = h^{1+1/\delta} f_s(z) + f_{\text{reg}}(m,T), \quad z = t/h^{1/\beta\delta}$$

h : external field, t : reduced temperature, β, δ : universal critical exponents

$f_s(z)$: universal scaling function, O(N) etc.

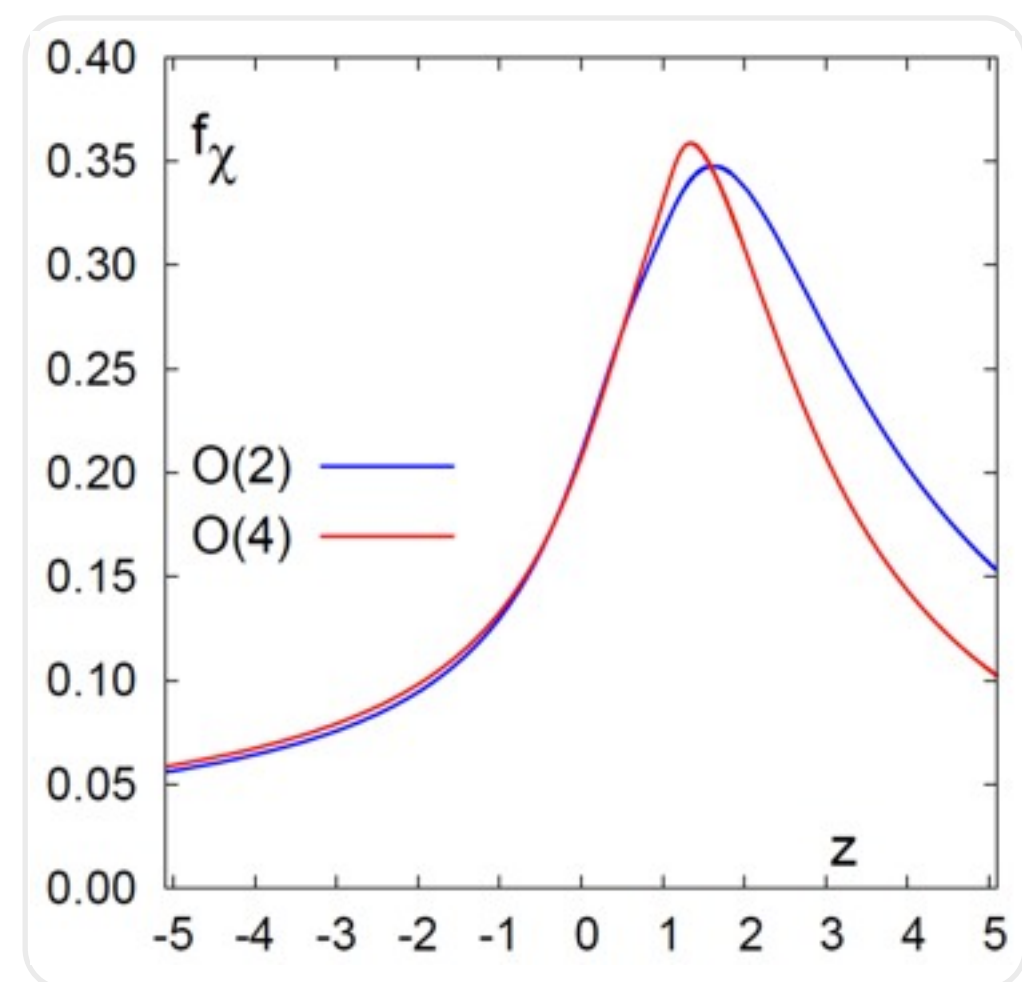
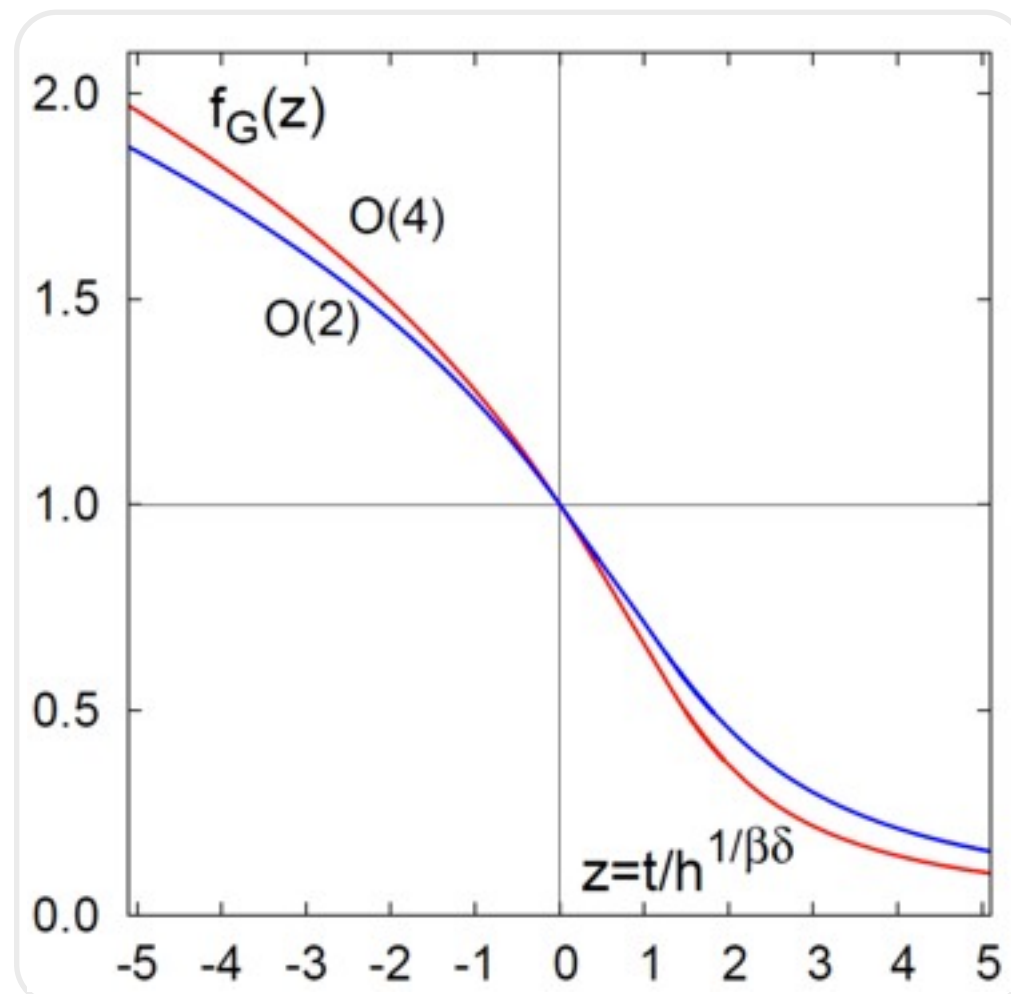
Magnetic Equation of State (MEoS):

$$M = -\partial f_s(t,h)/\partial h = h^{1/\delta} f_G(z)$$

$$h = \frac{1}{h_0} \frac{m_l}{m_s}$$

$$t = \frac{1}{t_0} \frac{T-T_c}{T_c}$$

$$f_\chi(z) = h_0^{1/\delta} (m_l/m_s)^{1-1/\delta} \partial M / \partial h$$

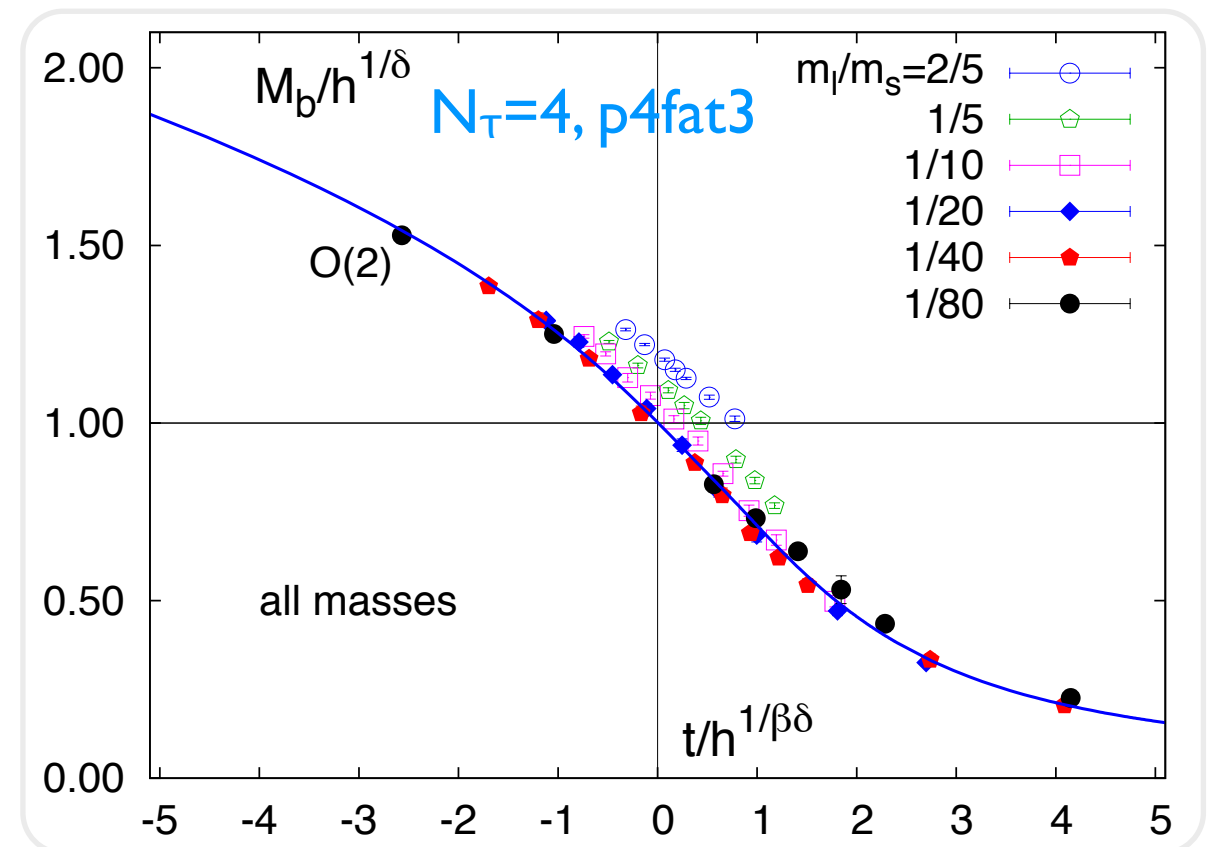
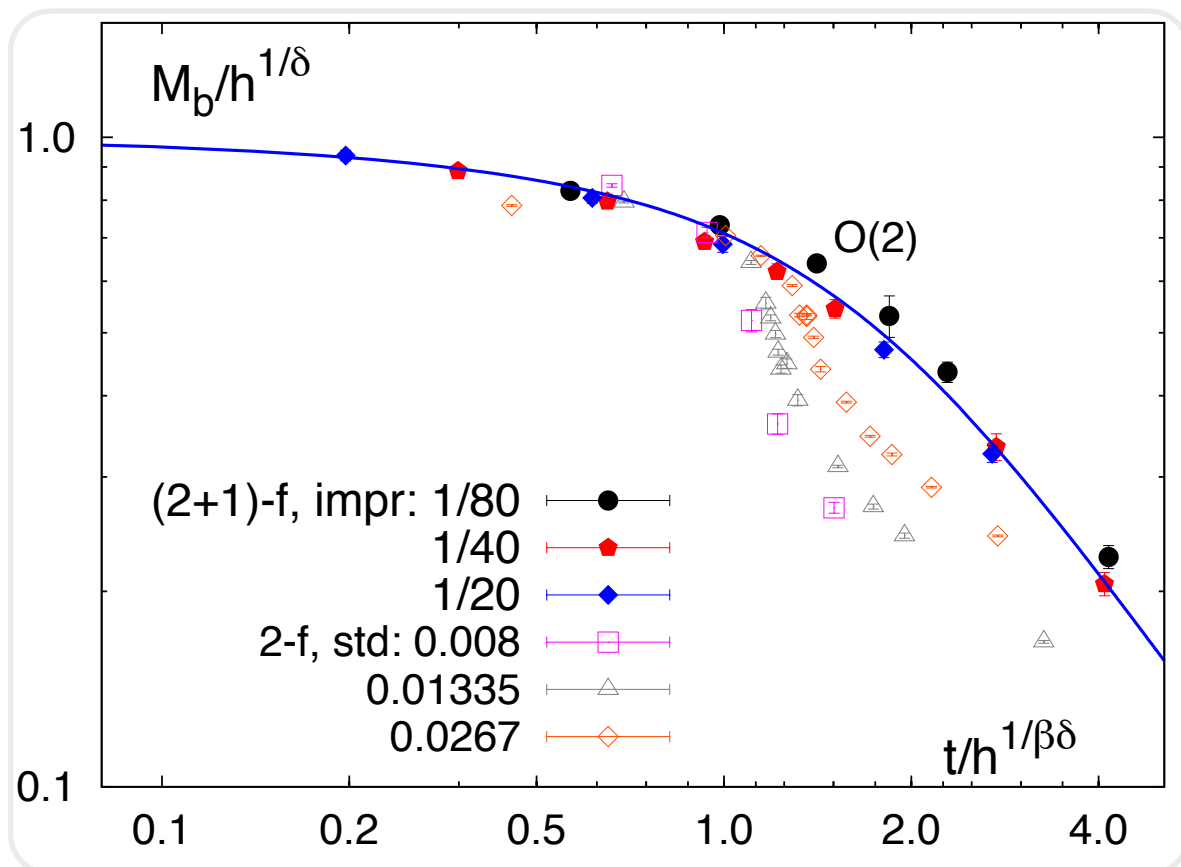


Recent $O(N)$ universal scaling studies

$$f(m,T) = h^{1+1/\delta} f_s(z) + f_{\text{reg}}(m,T),$$

$$z = t/h^{1/\beta\delta}$$

$$M_b = m_s \langle \bar{\psi} \psi \rangle / T^4$$

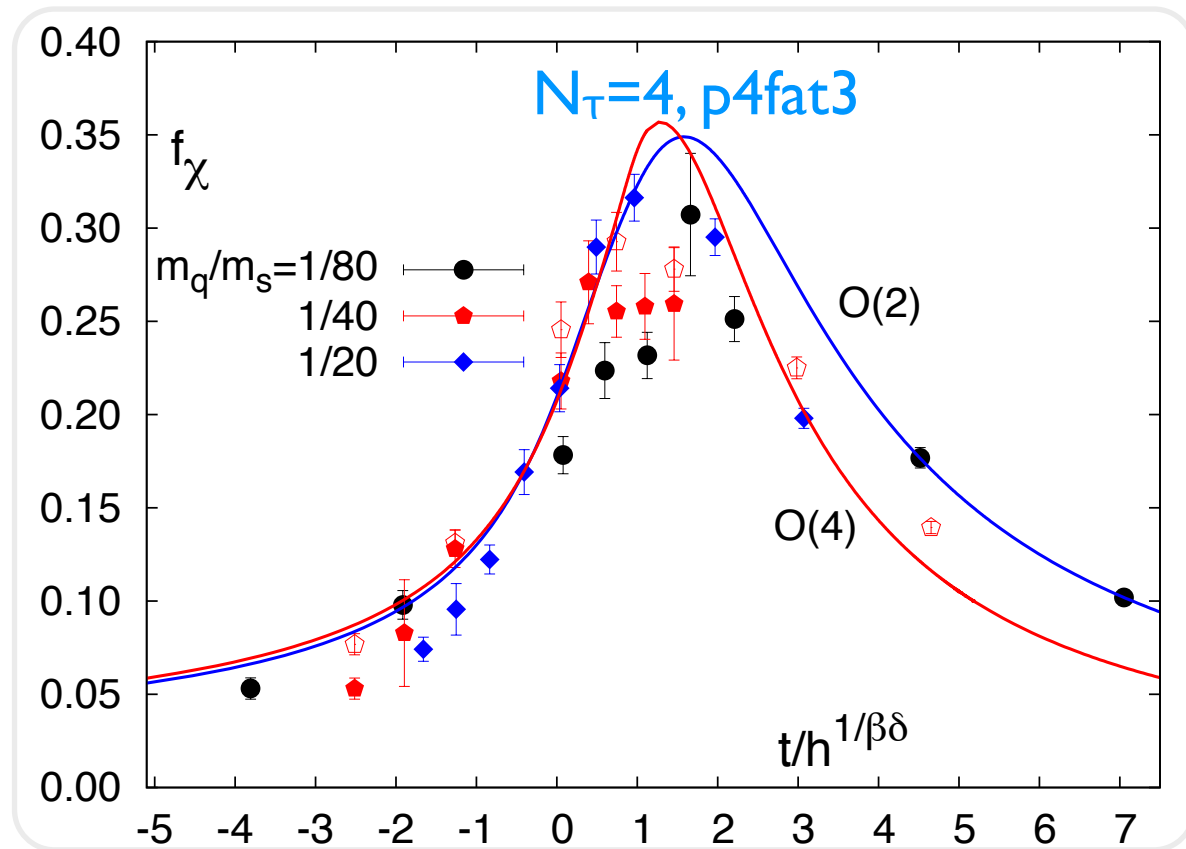


BNL-Bielefeld PRD '09

- the scaling window depends on discretization schemes: standard v.s. improved staggered fermions
- scaling violations seen at $m_l/m_s > 1/10$ using p4 action on $N_t=4$ lattices

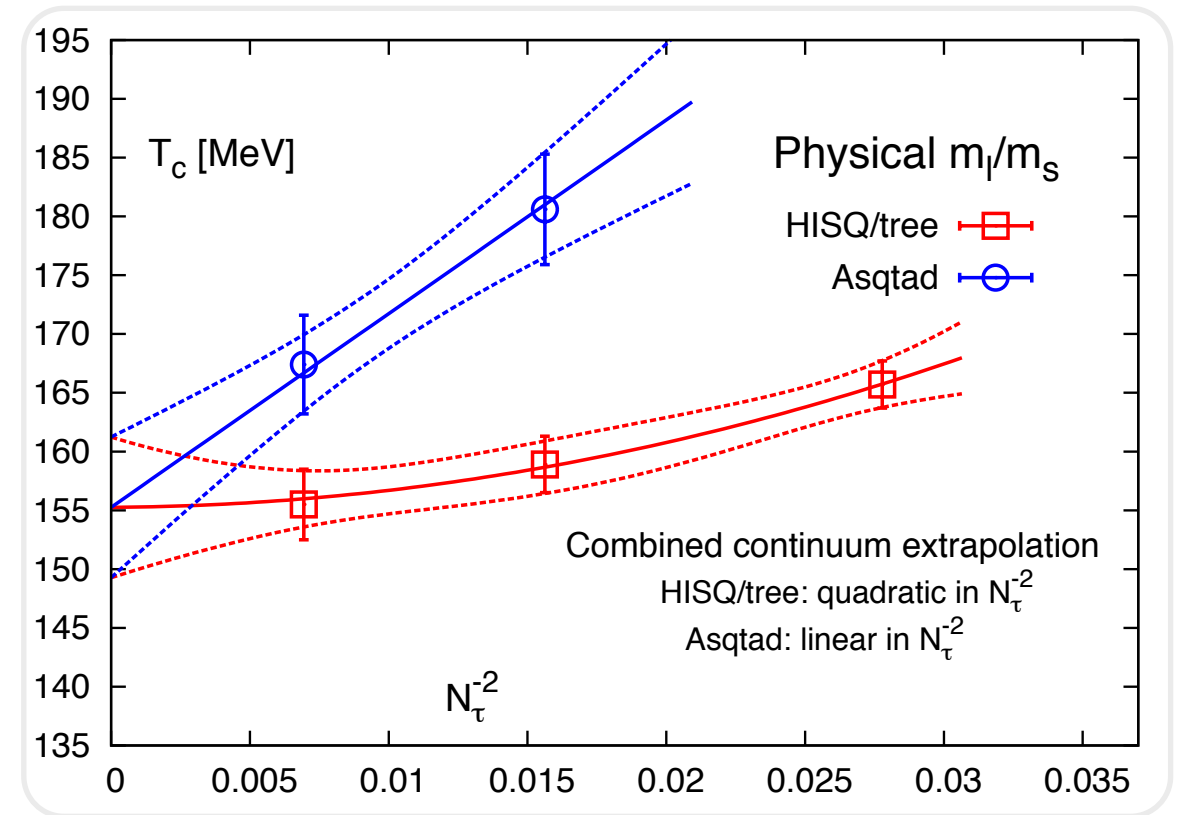
Recent $O(N)$ universal scaling studies

no fitting



BNL-Bielefeld PRD '09

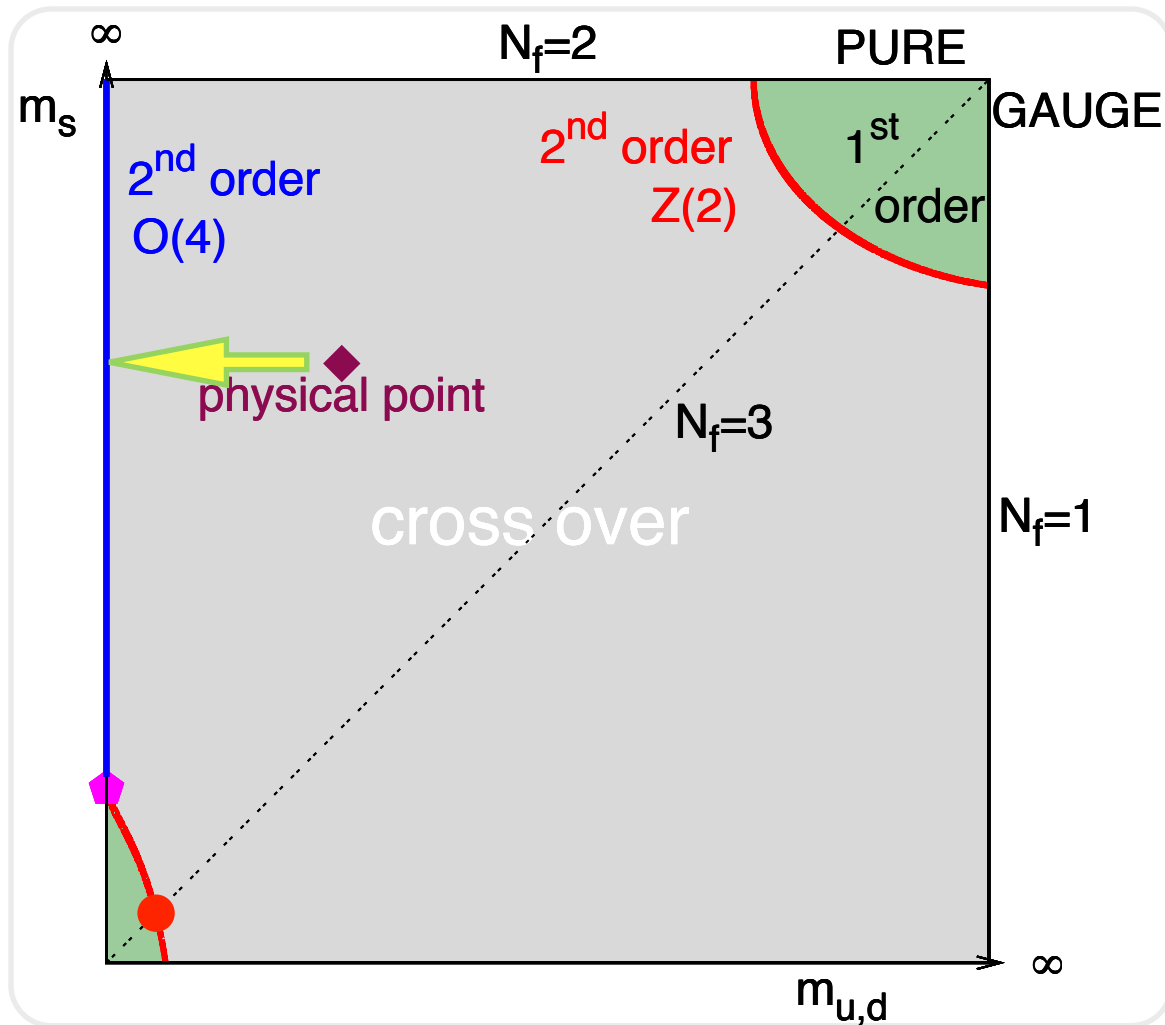
combined fitting to
condensates & susceptibilities



HotQCD, PRD '11

- Reasonably good prediction of chiral susceptibilities using parameters obtained from the scaling fit to chiral condensates
- Useful tool to determine the critical temperature, chiral curvature etc.

$N_f=2+1$ QCD



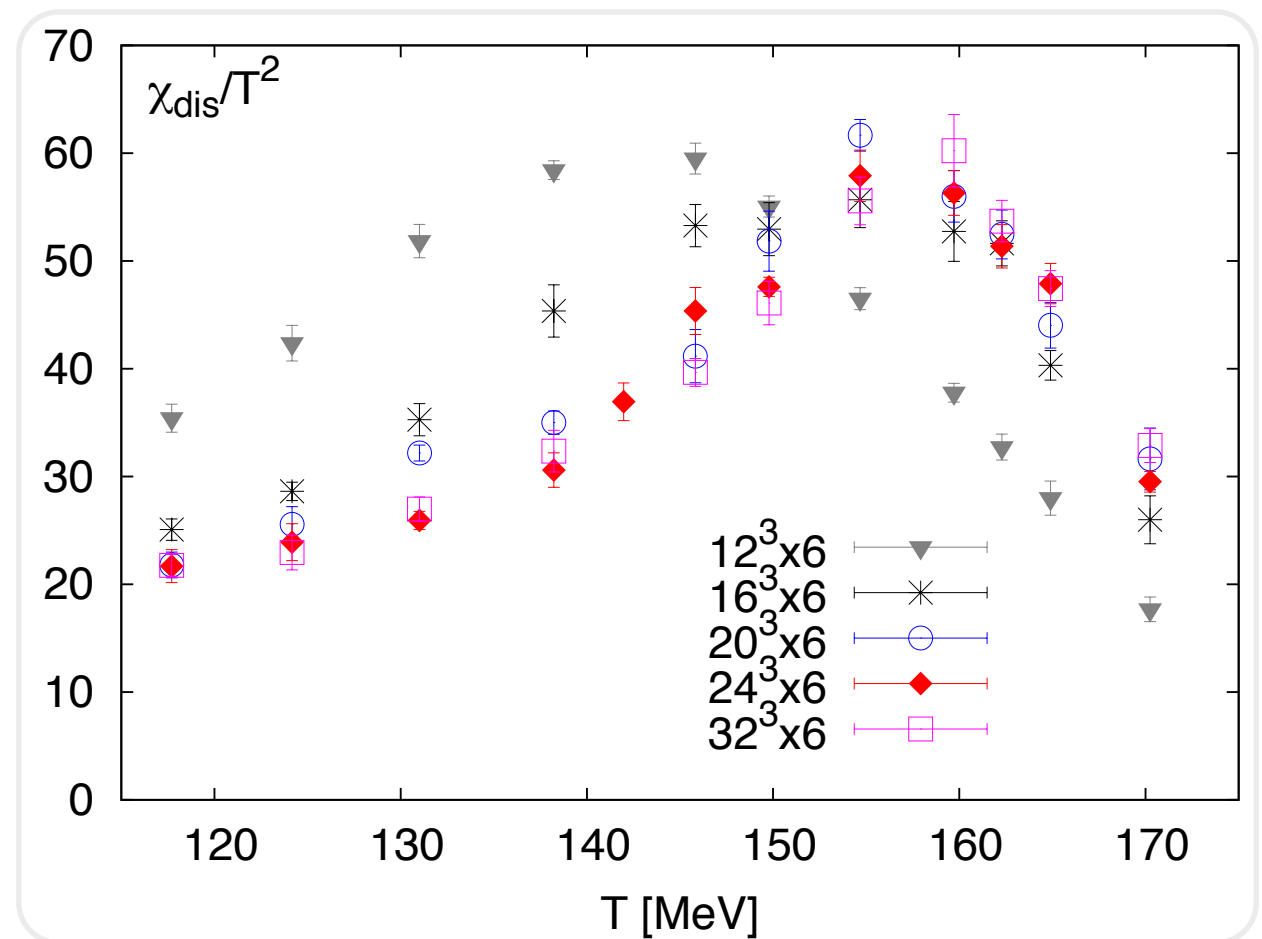
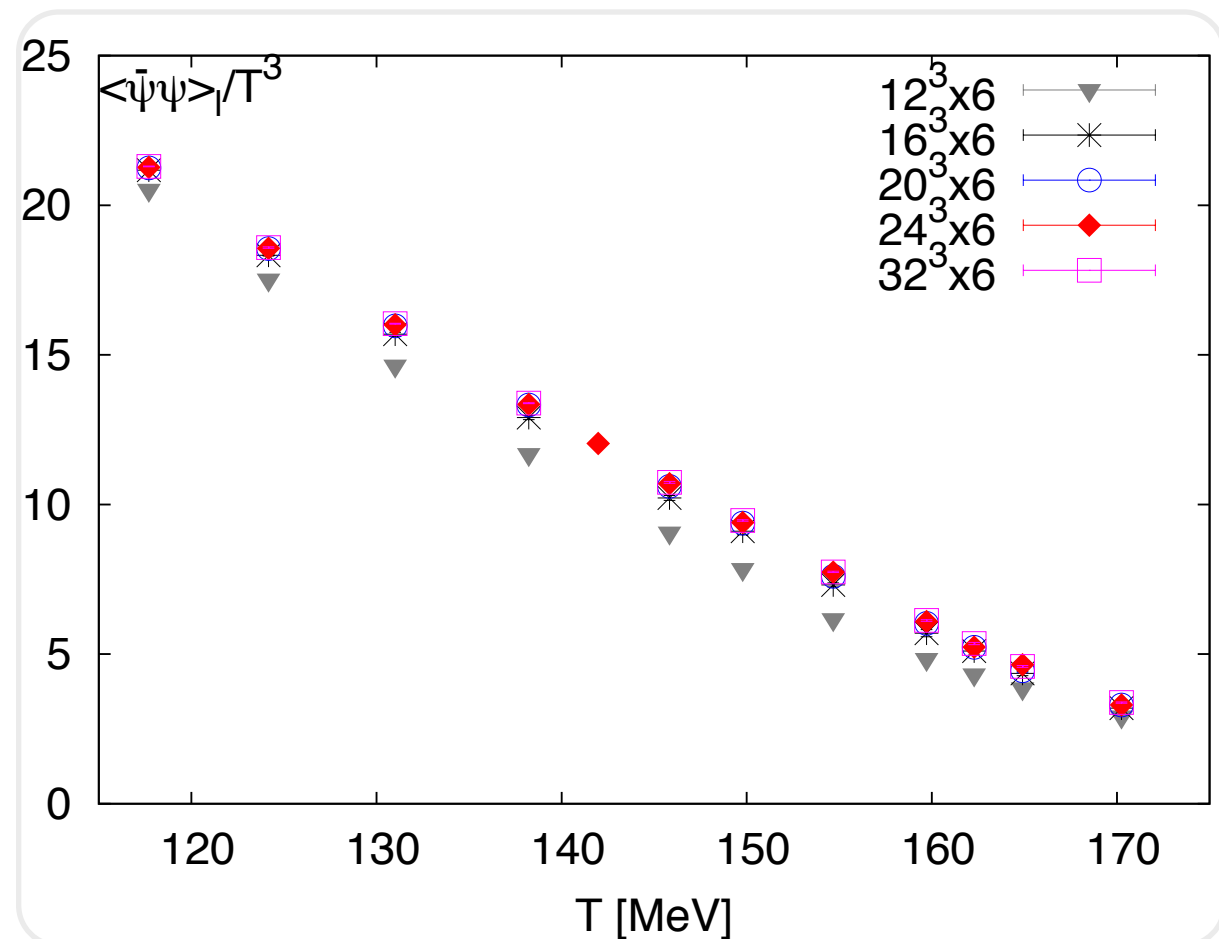
- Fix the strange quark mass to be physical
- Decrease the light quark mass approaching to the chiral limit
- Simulations on $N_t=6$ lattices using Highly Improved Staggered Quarks with 5 different quark masses corresponding to m_π from 160 MeV down to 80 MeV

• The chiral first order phase transition region shrinks with better improved staggered fermions $m_\pi^c \approx 290 \text{ MeV} \rightarrow m_\pi^c \approx 45 \text{ MeV}$

HTD, xQCD 2012, arXiv:1302.5740

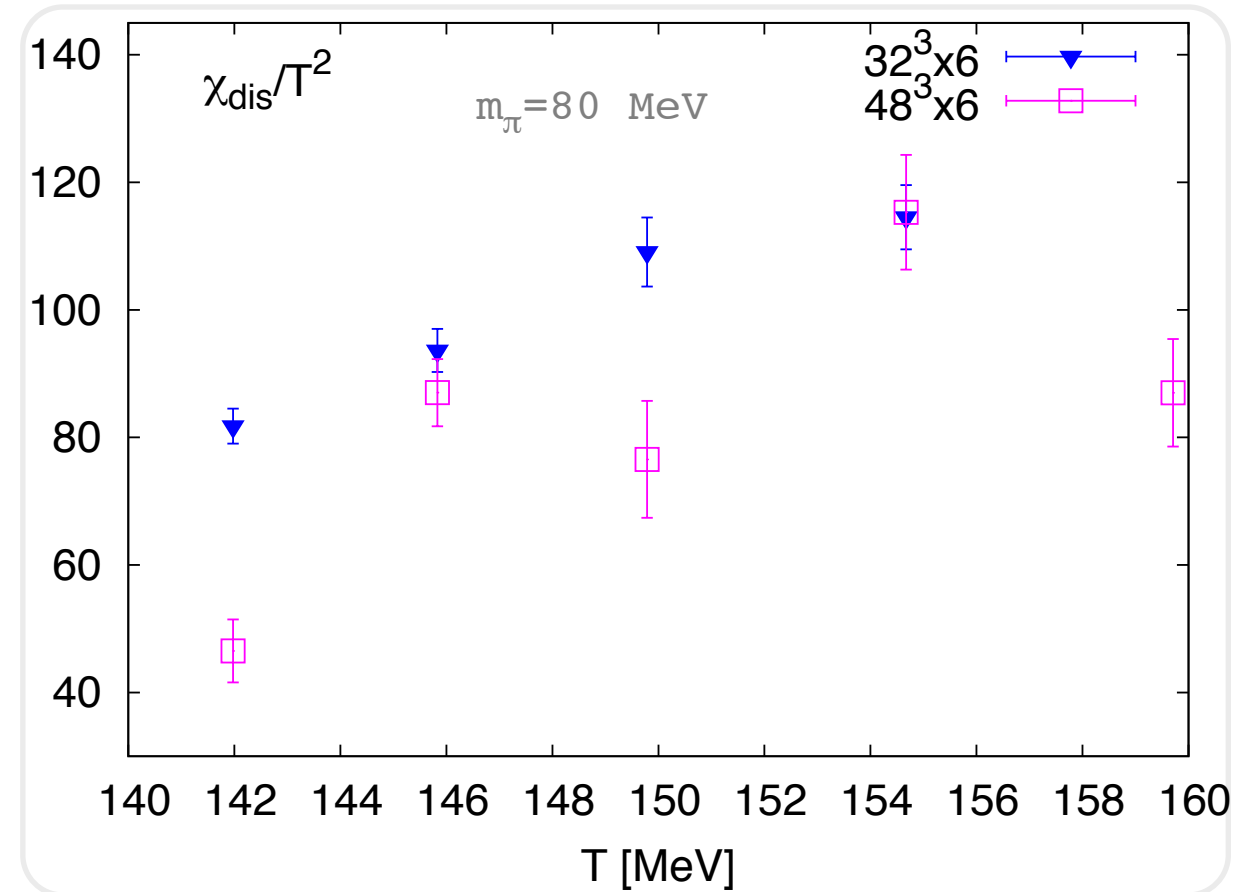
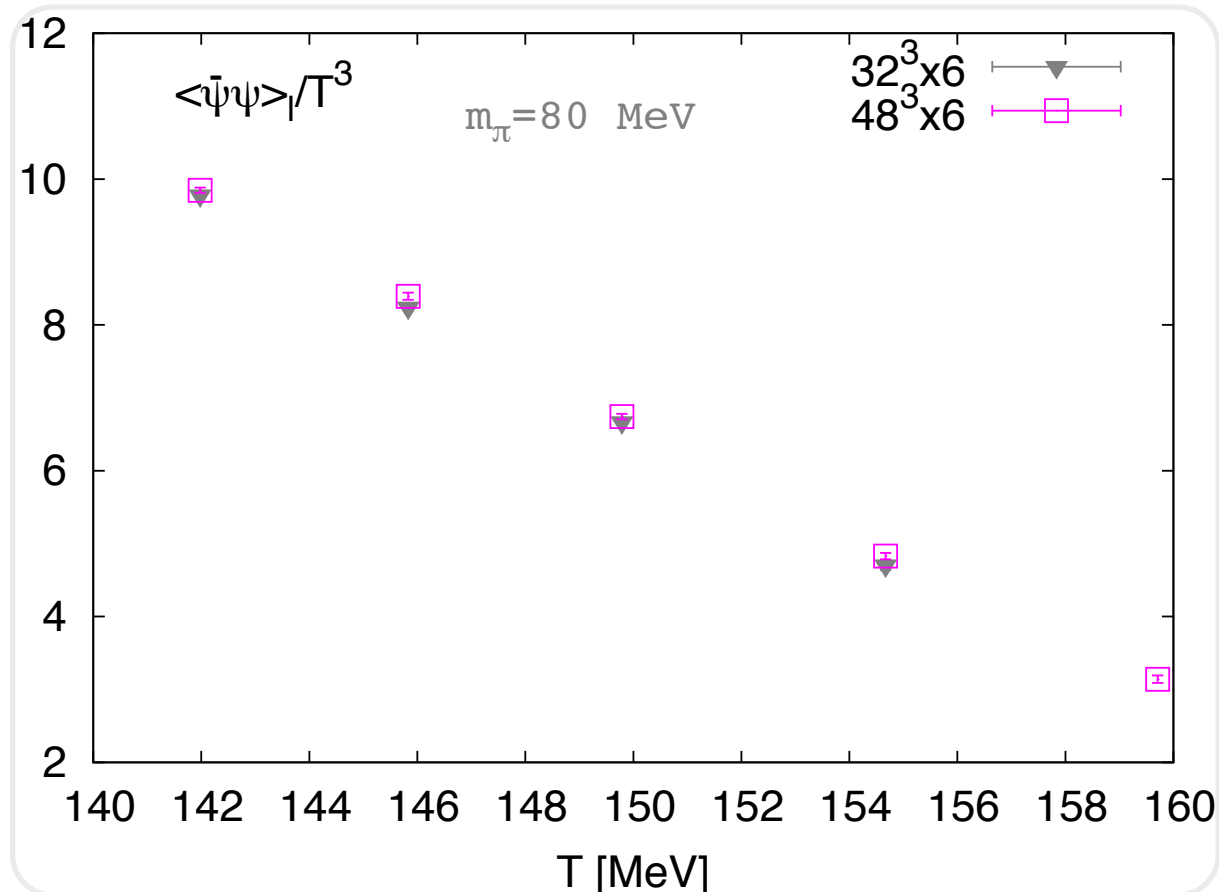
• 2nd order $O(4)$ scaling regime may have more influence on the physical world ?

volume dependence at physical pion mass



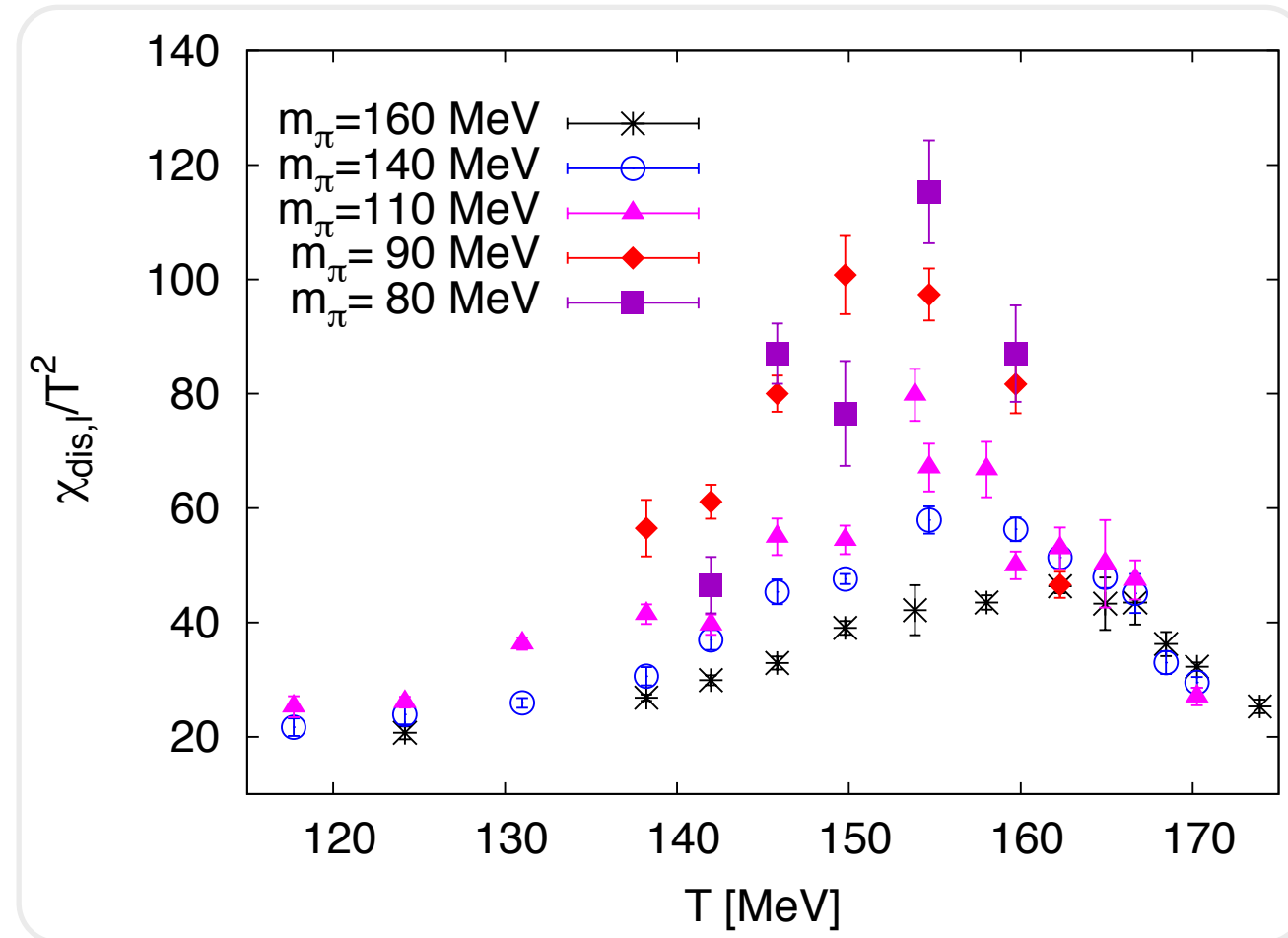
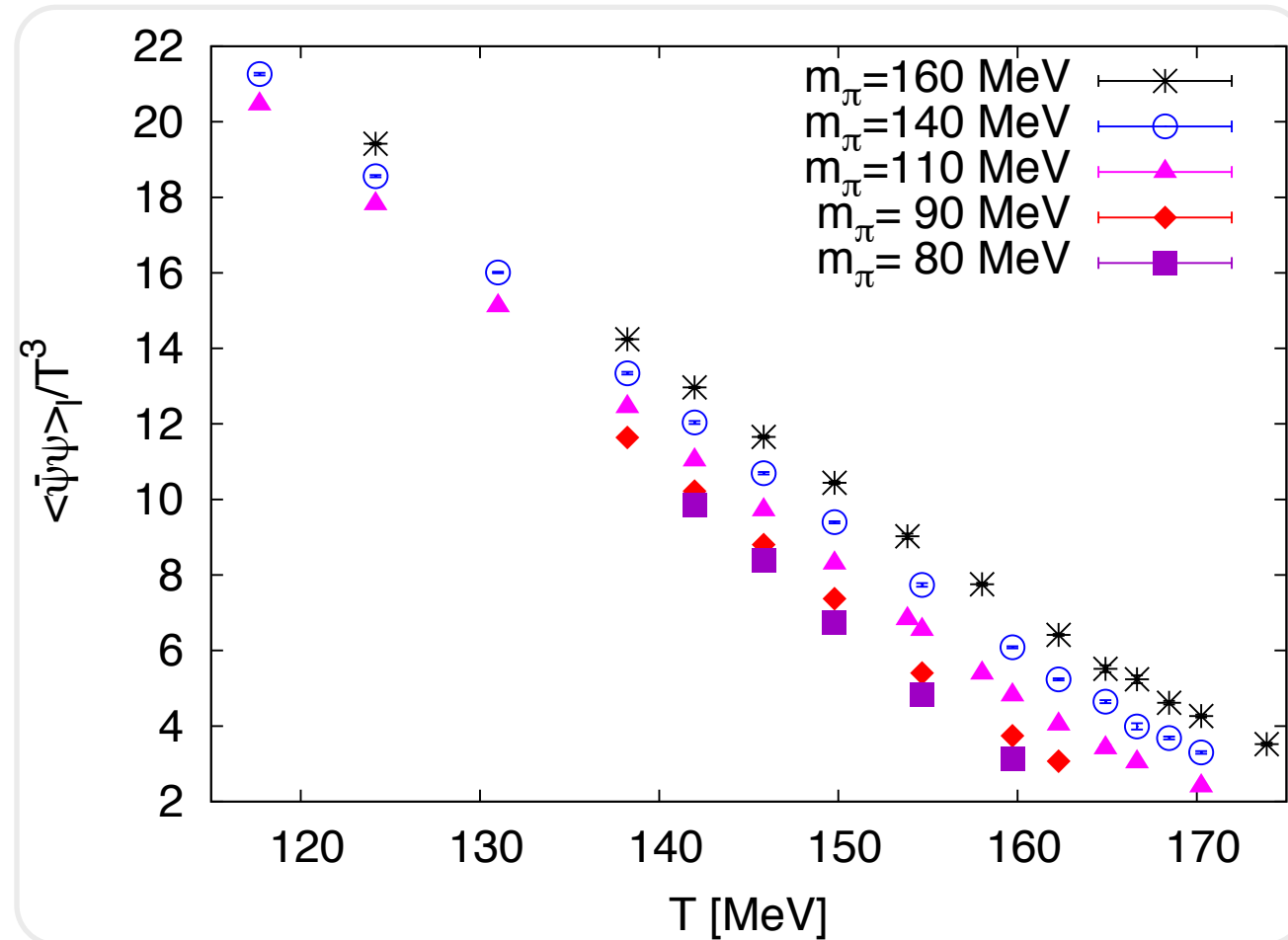
- volume effects are small in 3 largest volume
- $m_\pi L > 4$ is ensured in the following other datasets
 $48^3 \times 6$ with $m_\pi = 80$ MeV, $40^3 \times 6$ with $m_\pi = 90$ MeV,
 $32^3 \times 6$ with $m_\pi = 110$ MeV, $24^3 \times 6$ with $m_\pi = 160$ MeV

volume dependence at $m_\pi=80$ MeV



- Mild volume dependence is seen from chiral condensates
- No evidence of linear volume scaling as signatures of first order phase transition
- Volume scaling analysis needs to understand the volume effects

chiral condensates & susceptibilities



- chiral condensates decrease with increasing temperature and decreasing quark mass
- peak heights of chiral susceptibilities increase and peak locations shift to lower temperatures with decreasing quark mass

$O(N)$ scaling behavior

• For large negative values of z

$$z = t/h^{1/\beta\delta}$$

$$h = \frac{I}{h_0} \frac{m_I}{m_s}$$

$$t = \frac{I}{t_0} \frac{T - T_c}{T_c}$$

$$f_G(z) \simeq f_G^{-\infty}(z) = (-z)^\beta \left(1 + c_2 \beta (-z)^{-\beta\delta/2} \right) \quad \text{Engels et al., PLB 514(2001)299}$$

$$M = h^{1/\delta} f_G(z) \simeq h^{1/\delta} f_G^{-\infty}(z) = (-t)^\beta \left(1 + c_2 \beta (-t)^{-\beta\delta/2} \sqrt{h} \right)$$

contribution of Goldstone modes to the order parameter M
is enclosed in the scaling function in the low temperature

susceptibility of the order parameter $\sim 1/\sqrt{h}$

• For large positive values of z

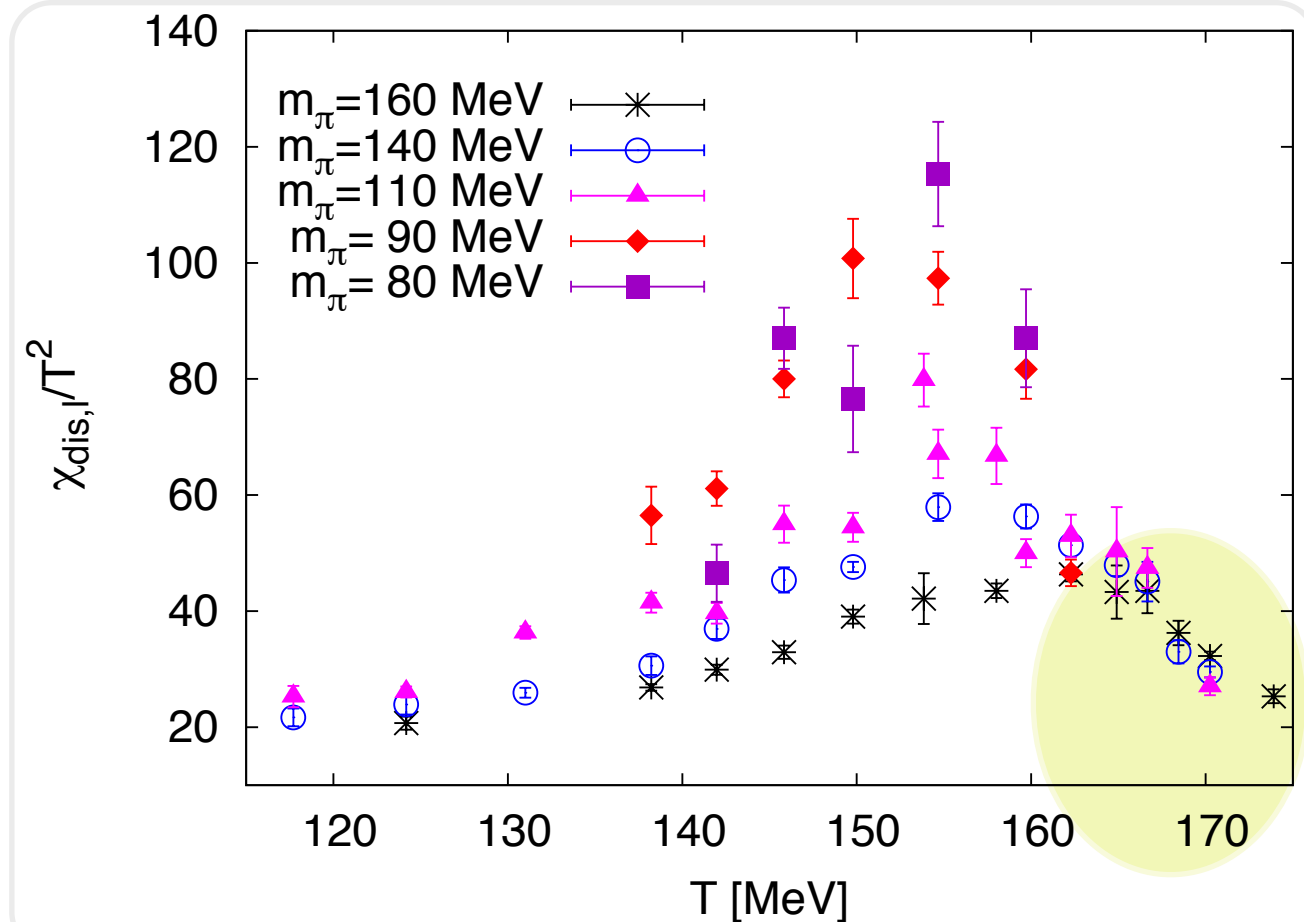
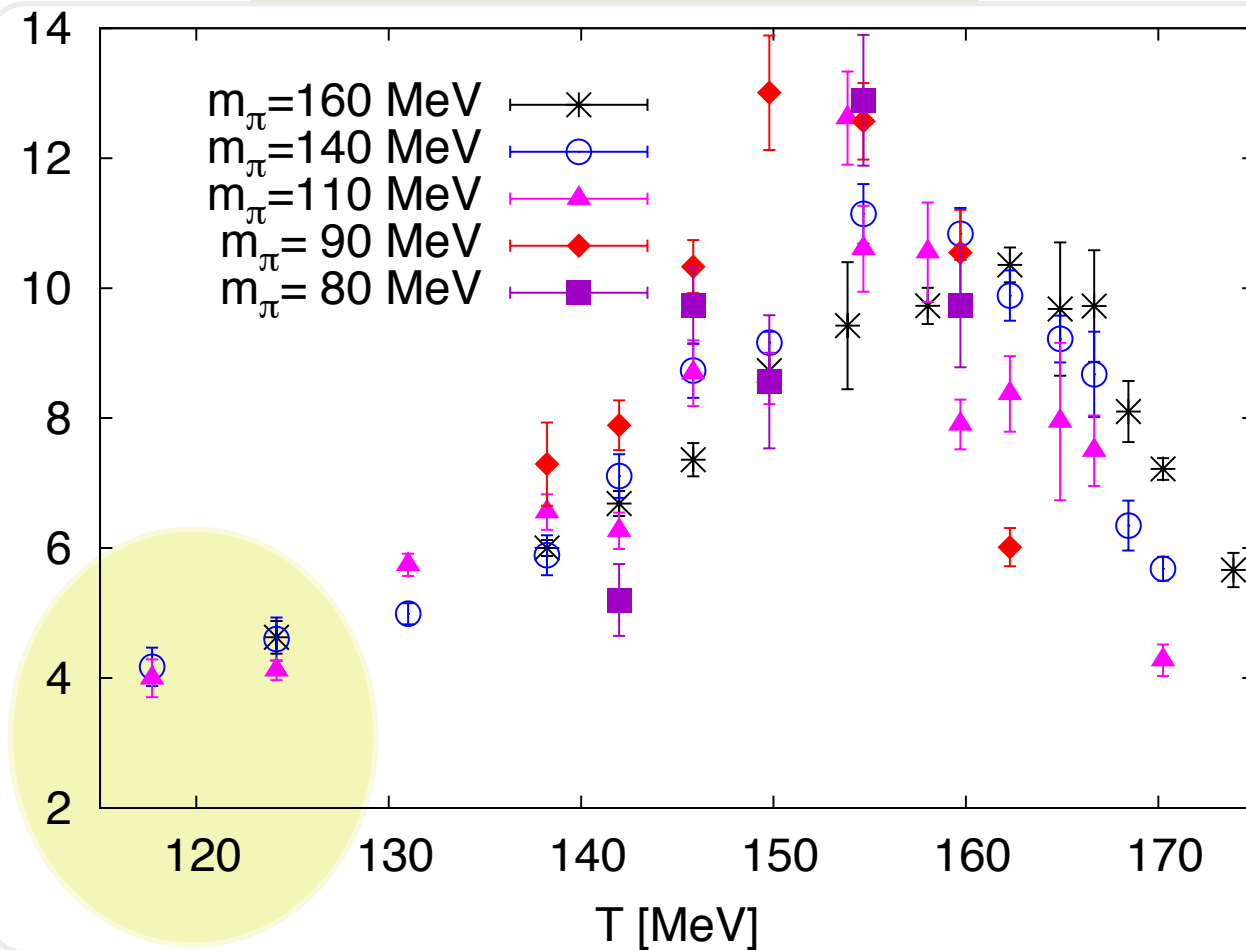
$$f_G(z) \sim R_\chi z^{-\beta(\delta-1)} \quad \text{Engels et al., NPB 675(2003)533}$$

$$M = h^{1/\delta} f_G(z) \sim R_\chi t^{-\beta(\delta-1)} h$$

susceptibility of the order parameter is independent of h

disconnected chiral sus. at low and high temperatures

$$\chi_{l,dis} (m_l/m_s)^{1/2}$$

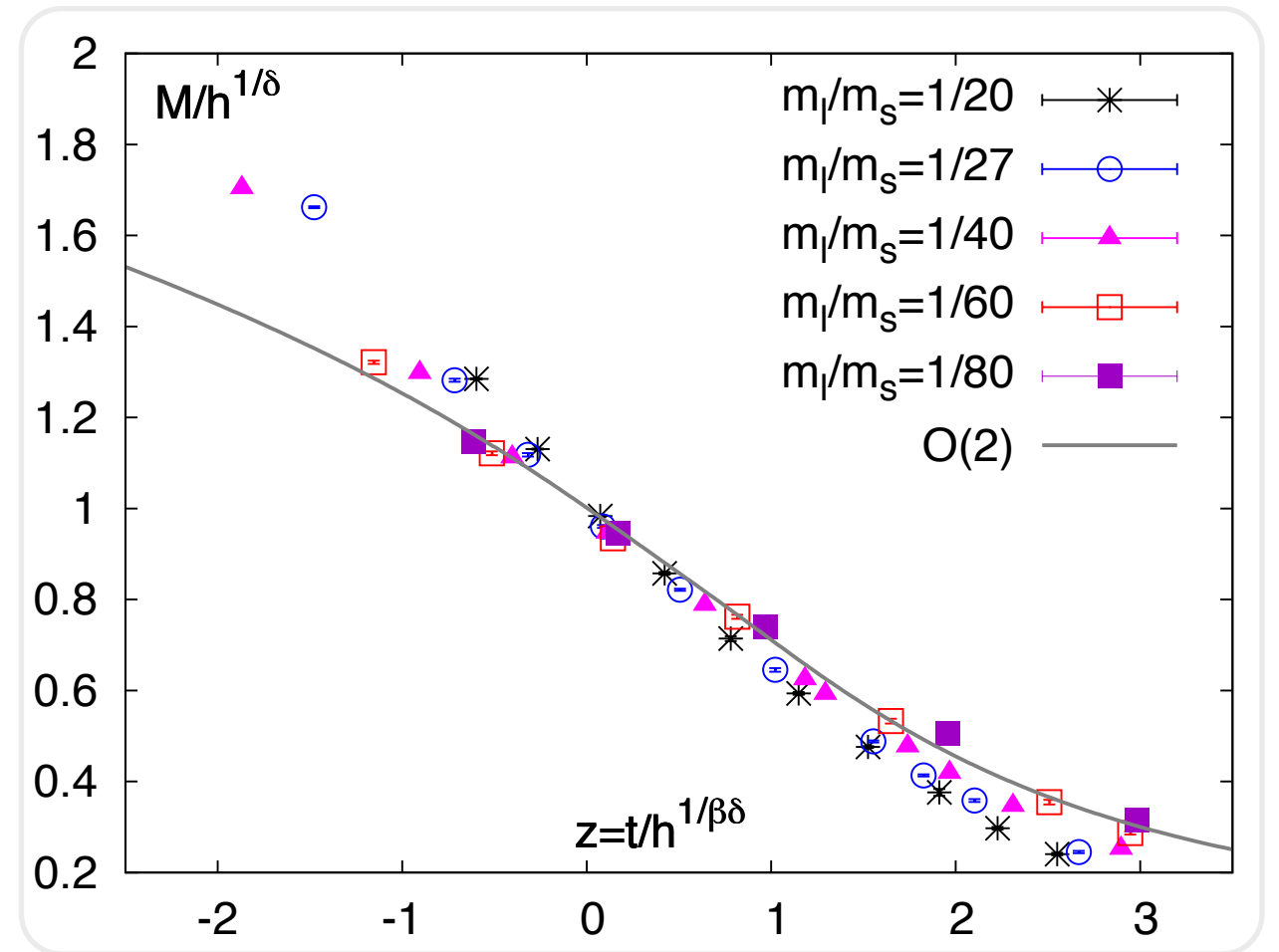
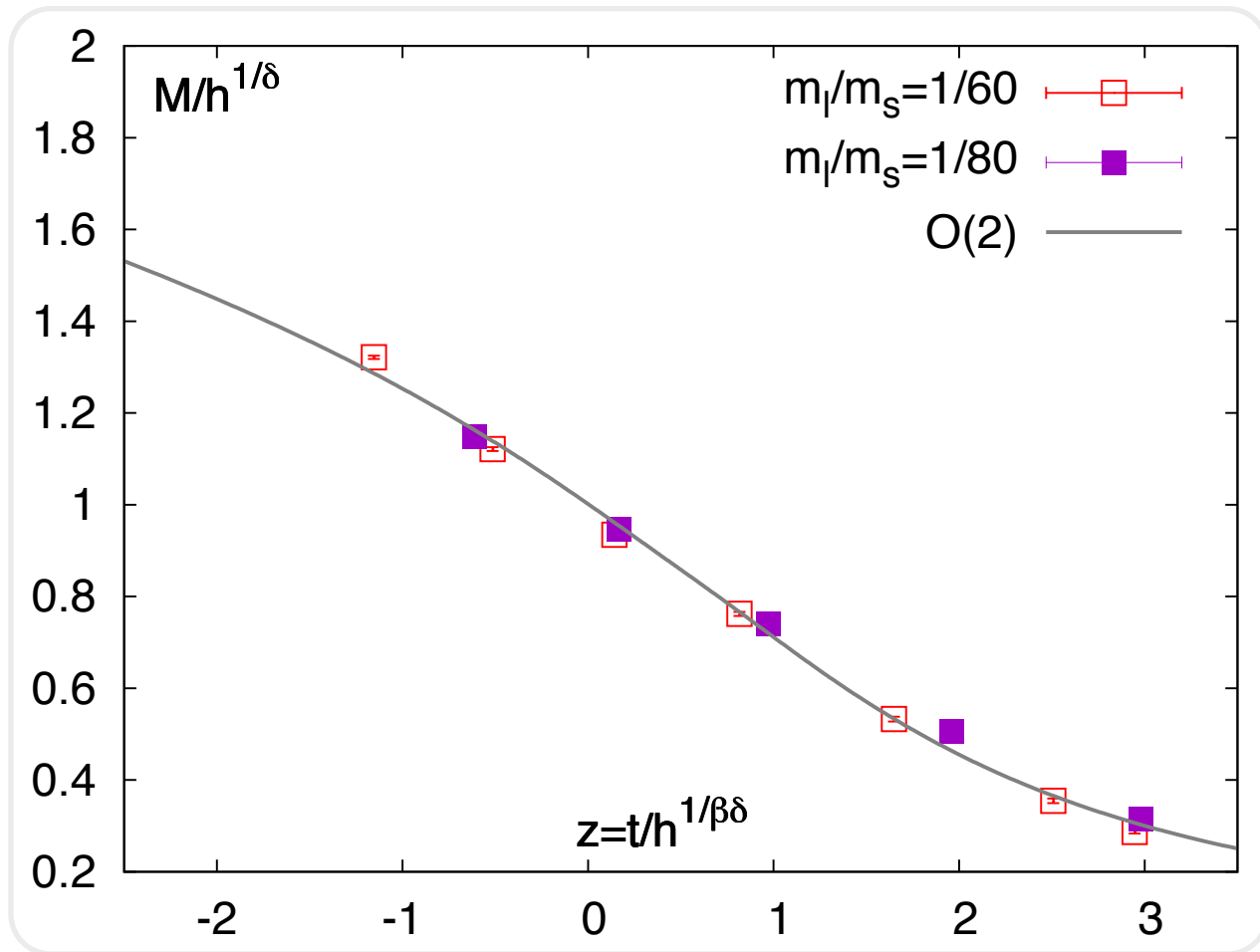


- At very low temperature, the disconnected susceptibilities scale as square root of quark mass
- At $T \sim 170$ MeV, the disconnected susceptibilities seem to be independent on quark mass: a likely indication of $U(1)_A$ symmetry breaking

Chris Schroeder's talk on $U(1)_A$ from DWF, 15:30 today

scaling and scaling violation of the chiral condensate

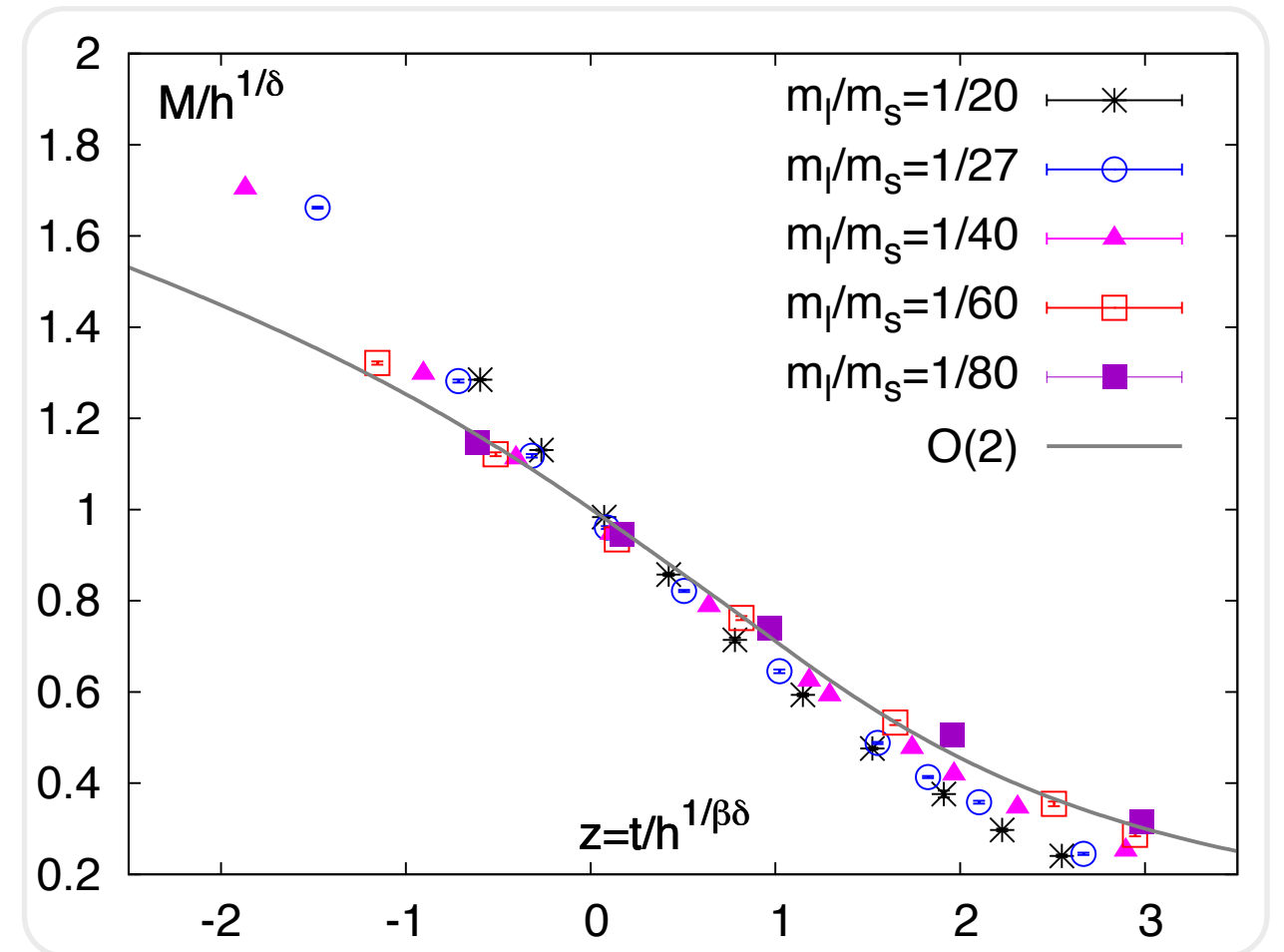
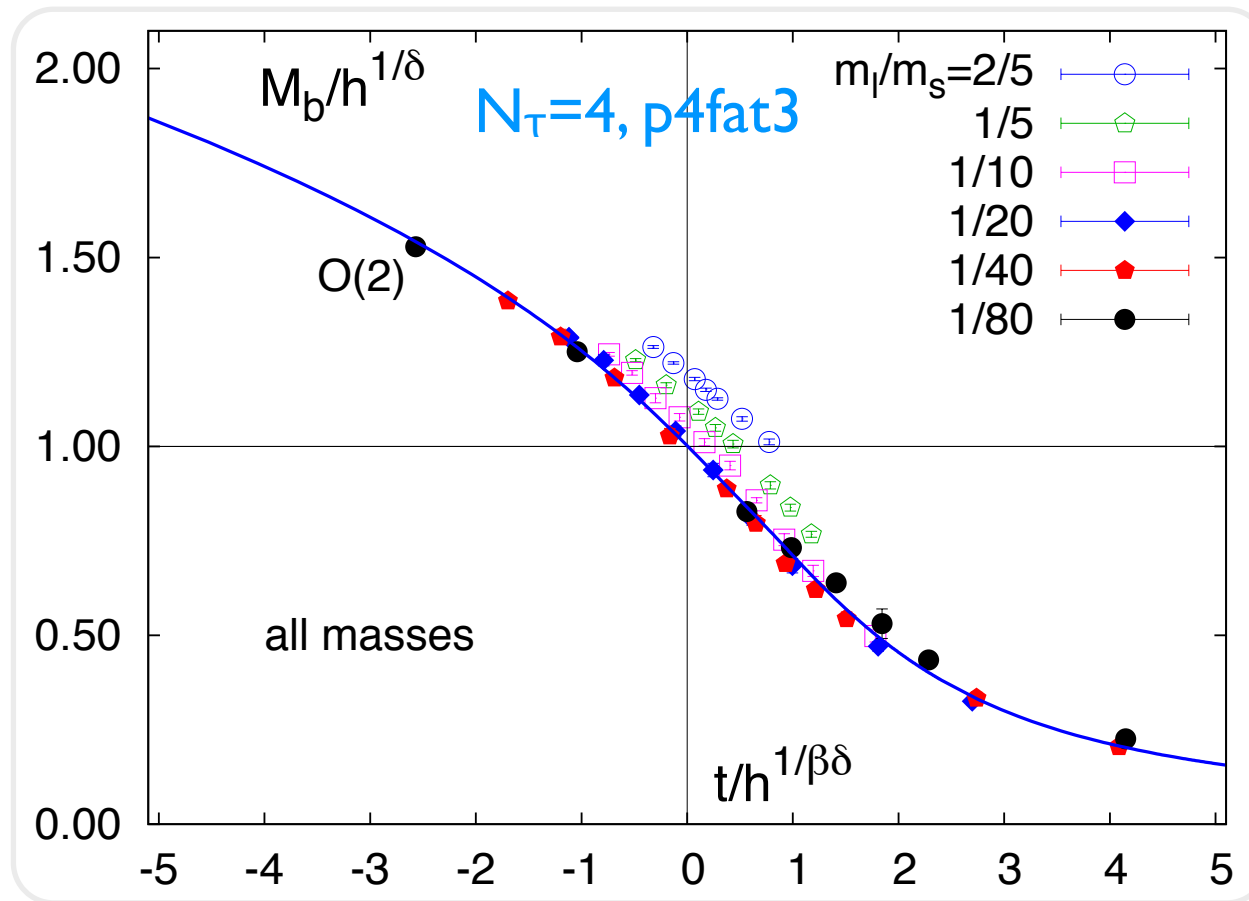
$$M = -\partial f_s(t,h)/\partial h = h^{1/\delta} f_G(z)$$



- The right plot is generated using the fitting parameters obtained from the fit to the two lightest quark mass shown in the left plot
- scaling violation of chiral condensates seen with $m_\pi \geq 110$ MeV ($m_l/m_s \geq 1/40$) using the HISQ action on $N_t=6$ lattices

scaling and scaling violation of the chiral condensate

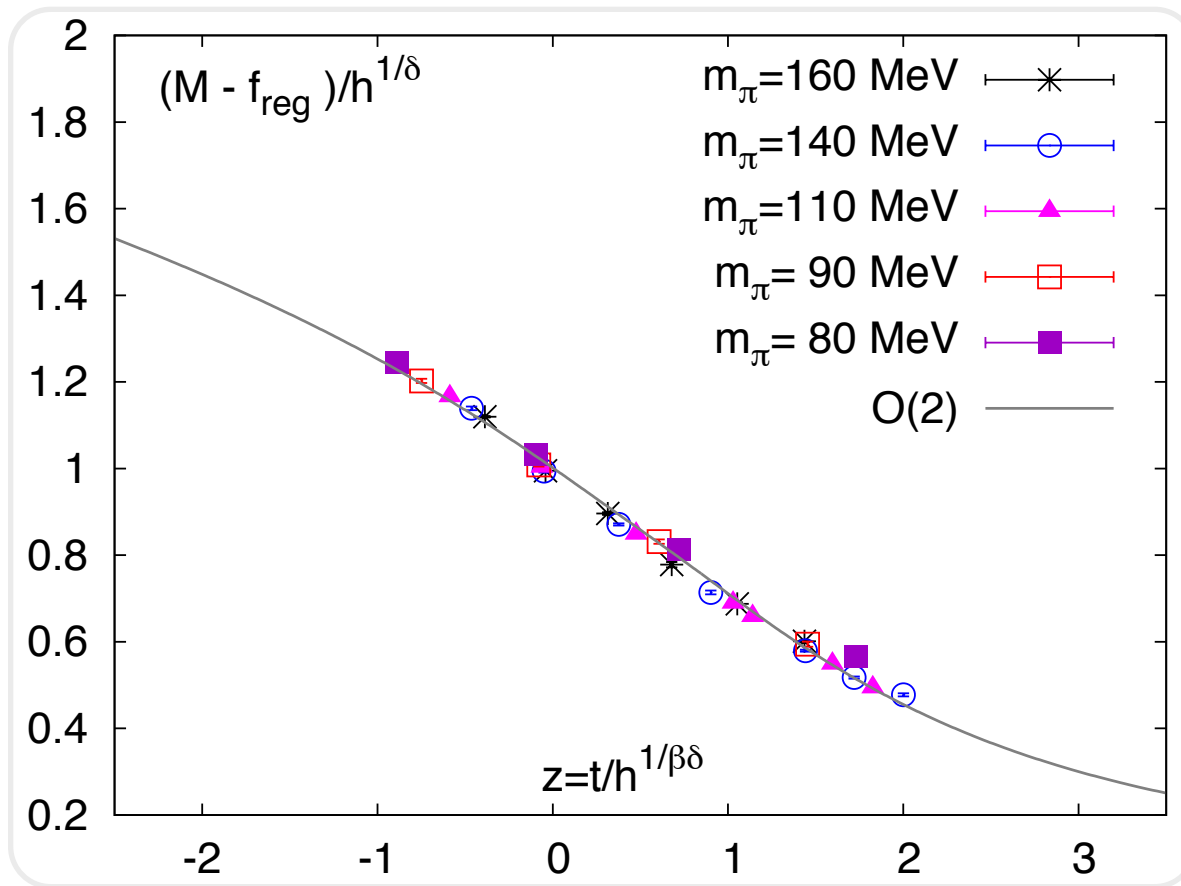
$$M = -\partial f_s(t,h)/\partial h = h^{1/\delta} f_G(z)$$



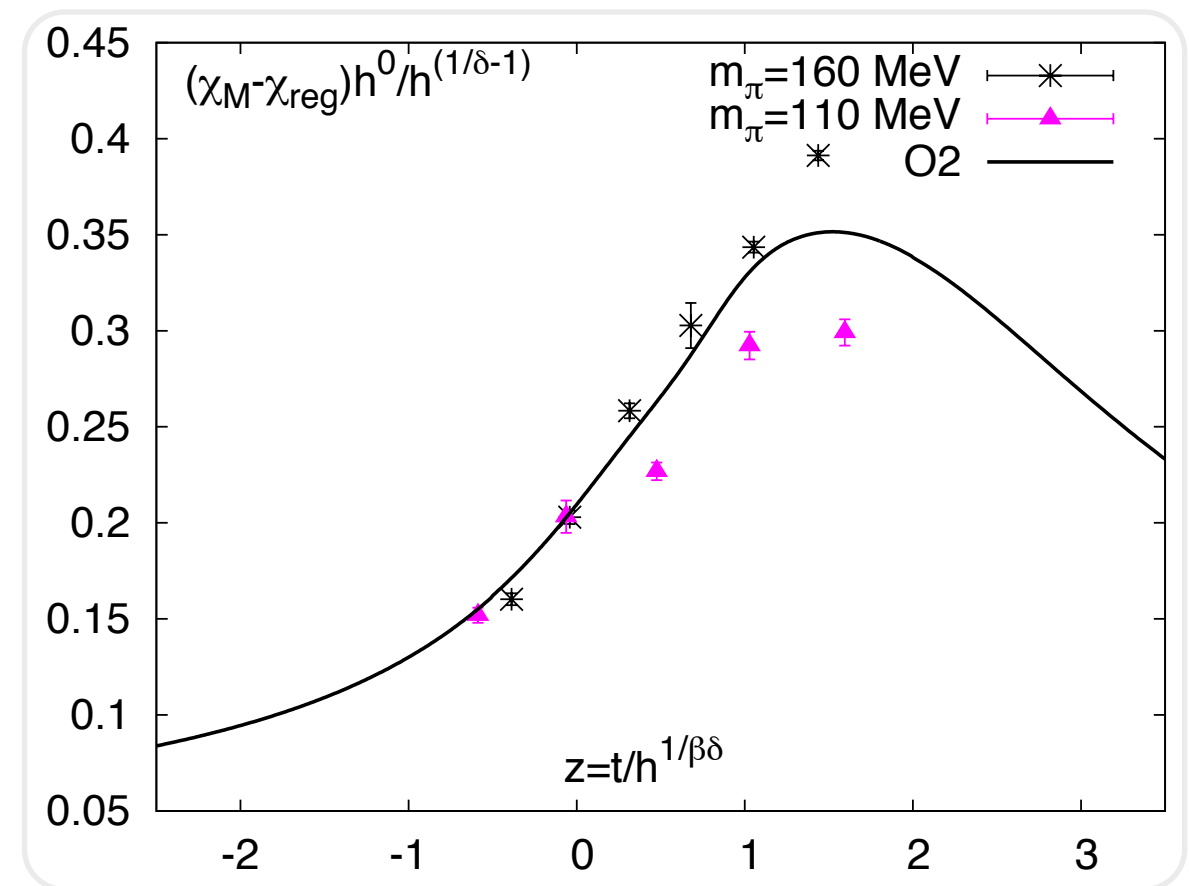
Navigation: $m_\pi=160$ MeV $\sim m_l/m_s=1/20$, $m_\pi=80$ MeV $\sim m_l/m_s=1/80$

- scaling violation of chiral condensates seen with $m_\pi \geq 110$ MeV ($m_l/m_s \geq 1/40$) using the HISQ action on $Nt=6$ lattices
- the scaling window shrinks compared to the results obtained using the p4 action on $Nt=4$ lattices

fit to chiral condensates and resulting sus.



$$f_G(z) = (M - f_{\text{reg}})/h^{1/\delta}$$



$$f_\chi(z) = (\chi_M - \chi_{\text{reg}})h^0/h^{1/\delta-1}$$

$$f_{\text{reg}} = (a_0 + a_1(T-T_c)/T_c) m_l/m_s, \quad \chi_{\text{reg}} = \partial f_{\text{reg}}/\partial(m_l/m_s)$$

- After including the regular terms, the chiral condensates can be described by the O(2) scaling function $f_G(z)$
- The susceptibilities can be reasonably reproduced using the fitting parameters obtained from the fit to the chiral condensate

Summary

- We study the chiral observables on $N_t=6$ lattices using the HISQ action with $m_\pi = 160, 140, 110, 90$ and 80 MeV
- No direct evidence of a first order phase transition in current pion mass window is found
- The scaling window shrinks in the HISQ results compared to that in the p4 results
- Regular terms need to be included to extract information on the singular structure