RESEARCH WITH PLYMOUTH UNIVERSITY

Lattice 2013

Space-time symmetries and the Yang-Mills gradient flow

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Introduction

- Lattice breaks translational invariance.
- Energy momentum tensor requires proper definition through approximate Ward identities.
- Local Ward identities on the lattice are plagued by a plethora of contact terms.
- These contanct terms are artifacts of the probes rather than the regularization, and can be avoided by using observables defined through the gradient flow.
- I will discuss local translation Ward identities and global dilatation Ward identities.

Translations

- Local transformation

$$\delta_{x\rho} P = \frac{\delta P}{\delta A^A_\mu(x)} F^A_{\rho\mu}(x)$$

- Global transformation on local fields

$$\int d^4x \; \delta_{x\rho}\phi_R(x_0) = \partial_\rho \phi_R(x_0)$$

- Energy-momentum tensor

$$\delta_{x\rho} S = -\partial_\mu T_{\mu\rho}(x)$$

- Translation Ward identity (TWI) $\langle \delta P \rangle = \langle P \, \delta S \rangle$

$$\langle \delta_{x\rho}\phi_R(x_1)\phi_R(x_2)\rangle = -\langle \phi_R(x_1)\phi_R(x_2)\partial_\mu T_{\mu\rho}(x)\rangle$$

Translations

$\rightarrow$ Caracciolo, Curci, Menotti, Pelissetto, Annals Phys. 197 (1990) 119.

- Local transformation

$$\delta_{x\rho} P = \frac{\delta P}{\delta A^A_\mu(x)} F^A_{\rho\mu}(x) \qquad \longrightarrow \qquad \delta_{x\rho} P = \frac{1}{a^3} F^A_{\rho\mu}(x) \partial^A_{U_\mu(x)} P$$

- Global transformation on local fields

$$\int d^4x \; \delta_{x\rho} \phi_R(x_0) = \frac{1}{Z_\delta} \partial_\rho \phi_R(x_0) + \sum_{d_\Theta \leq d_\phi + 1} c_\Theta \, a^{d_\Theta - d_\phi - 1} \Theta_{R,\rho}(x_0) + O(a^{-2})$$

- Energy-momentum tensor

$$\delta_{x\rho} S = -\partial_\mu T_{\mu\rho}(x) - R_\rho(x) = \frac{-\partial_\mu T_{R,\mu\rho}(x) + X_\rho(x)}{Z_\delta}$$

- Translation Ward identity (TWI) $\langle \delta P \rangle = \langle P \, \delta S \rangle$

$$\langle [Z_\delta \delta_{x\rho} - X_\rho(x)] \, \phi_R(x_1) \phi_R(x_2) \rangle = -\langle \phi_R(x_1) \phi_R(x_2) \partial_\mu T_{R,\mu\rho}(x) \rangle$$

Translation Ward Identity (TWI)

$$T_{R,\mu\rho} = \sum_i c_i \left\{ T^{(i)}_{\mu\rho} - \langle T^{(i)}_{\mu\rho} \rangle \right\}$$

dim-4 operator mixing

valid only up to $O(a^2)$

$$\langle \partial_\mu \phi_R(x_1)\phi_R(x_2)\rangle = \langle \int_{V_1} d^4x\ \delta_{R,x\rho}\phi_R(x_1)\phi_R(x_2)\rangle = -\langle \phi_R(x_1)\phi_R(x_2)\int_{\partial V_1} dS_\mu\, T_{R,\mu\rho}(x)\rangle$$

$\delta_{R,x\rho} = Z_\delta \delta_{x\rho} - X_\rho(x)$
$X\phi_R$ and $Z_\delta \delta\phi_R$ have the same contact terms

avoids contact terms in $\phi_R T_R$

NB: Since we need to use the integrated TWI, we must use *at least* two local probes as $\langle \partial_\rho \phi_R(x) \rangle = 0$.

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artifacts of the probe

NB: Since we need to use the integrated TWI, we must use *at least* two local probes as $\langle \partial_\rho \phi_R(x) \rangle = 0$.

Probes at positive flow time

→ Lüscher's plenary talk on Tuesday.

- Gradient flow.

$$\frac{d}{dt}V_\mu(t,x) = -g_0^2 \partial_{U_\mu(x)} S_W[V] V_\mu(t,x)$$
$$V_\mu(0,x) = U_\mu(x)$$

- The gauge field gets smoothed over a range of order $\sqrt{8t}$.

- Non-renormalization properties.

  Composite local operators at positive flow time stay finite when the cutoff is removed, and therefore they do not require renormalization. E.g.

$$E(t,x) = \frac{1}{4} G^A_{\mu\nu} G^A_{\mu\nu}(t,x)$$

TWI with probes at positive flow time

- Renormalization of infinitesimal translations.

  When acting on probes at positive flow time, the operator $\delta_{x\rho}$ renormalizes multiplicatively.

  $$\delta_{R,x\rho}\phi(t,x_1) = Z_\delta \delta_{x\rho}\phi(t,x_1)$$

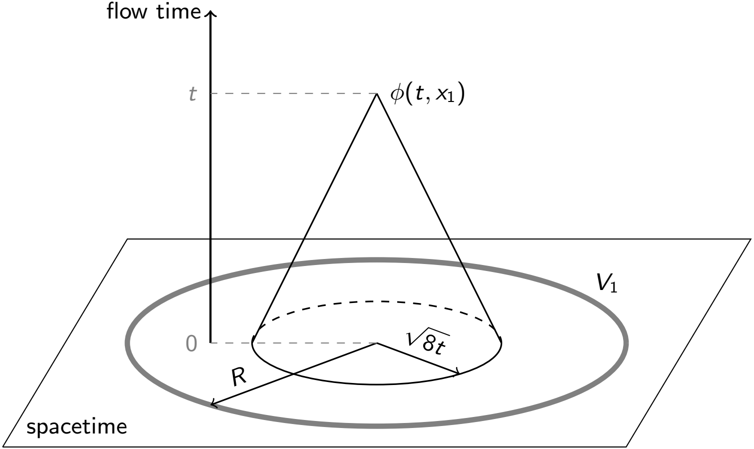
- The translation Ward identity is regular. The continuum limit is well defined for any value of t, x and x_1.

  $$Z_\delta \langle \delta_{x\rho}\phi(t,x_1)\rangle = \langle \phi(t,x_1)\, T_{R,\mu\rho}(x)\rangle$$

- This equation can be used to determine c_i/Z_δ.

  $$\langle \delta_{x\rho}\phi(t,x_1)\rangle = \sum_i \frac{c_i}{Z_\delta}\langle \phi(t,x_1)\, T^{(i)}_{\mu\rho}(x)\rangle$$

Normalization of translations



$$Z_\delta \langle \int_{V_1} d^4x \; \delta_{x\rho} \phi(t, x_1) \phi(t, x_2) \rangle = \langle \partial_\rho \phi(t, x_1) \phi(t, x_2) \rangle + O(e^{-\frac{R^2}{4t}})$$

Dilatations

- Ward identity for a global dilatation.

$$\left(2t\frac{d}{dt} + d_\phi\right)\langle\phi(t,0)\rangle = \langle\phi(t,0)\int d^4x\ T_{R,\mu\mu}(x)\rangle_c$$

- Renormalization of the trace of the energy-momentum tensor.

$$T_{R,\mu\mu} = \hat{c}\,\{\mathrm{tr}\,F_{\mu\nu}F_{\mu\nu} - \langle\mathrm{tr}\,F_{\mu\nu}F_{\mu\nu}\rangle\}$$

$$\left(2t\frac{d}{dt} + d_\phi\right)\langle\phi(t,0)\rangle = \hat{c}\ \langle\phi(t,0)\int d^4x\ \mathrm{tr}\,F_{\mu\nu}F_{\mu\nu}(x)\rangle_c$$

Conclusions

- Probes at positive flow time can be used to write local Ward identities that are free of contact terms and spurious singularities.
- The differential operator that generates local translations renormalizes multiplicatively when applied to probes at positive flow time.
- We have seen an example of how the gradient flow can be used to suppress certain lattice artifacts.
- A strategy to renormalize the energy-momentum tensor is outlined. Is it practical?
- Can we use the Wilson flow to explore infrared conformality?