Spectral densities from the lattice

David Dudal [1], Orlando Oliveira [2], Paulo Silva [2]

[1] Department of Physics and Astronomy, Ghent University, Belgium
[2] Centro de Física Computacional, Universidade de Coimbra, Portugal

July 29, 2013

Outline

1. Introduction
2. Gluon spectral densities
   - How-to
   - First results
3. Finite temperature
   - Lattice setup
   - Positivity violation
   - Spectral density

Landau gauge @ T=0

$$D^{ab}_{\mu\nu}(\hat{q}) = \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) D(q^2) \, ,$$

Lattice computation of the gluon propagator:

- Large volume: access to the deep IR region, infinite volume limit
  - SU(2): La = 27 fm, a = 0.22 fm

    A. Cucchieri, T. Mendes, PoS (LAT 2007) 297
  - SU(3): La = 17 fm, a = 0.18 fm

    I. L. Bogolubsky et al., Phys. Lett. B676, 69 (2009)
- Small lattice spacing:
  - large *a* also changes the propagator

O. Oliveira, P. J. S., Phys. Rev. D86, 114513 (2012)

Positivity violation

Spectral representation

$$D(p^2) = \int_0^{+\infty} d\mu \frac{\rho(\mu)}{p^2 + \mu^2}$$

On the lattice: study the temporal correlator

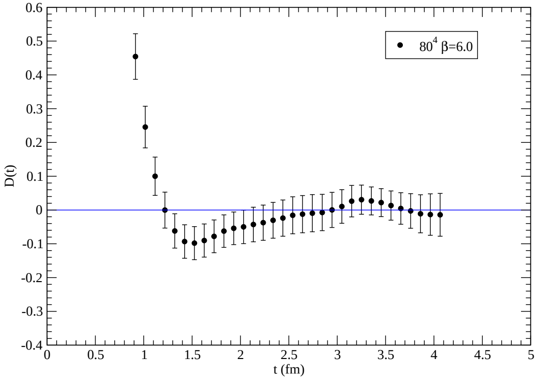
$$C(t) = \int_{-\infty}^{\infty} \frac{dp}{2\pi} D(p^2) \exp(-ipt) = \int_0^{\infty} d\omega \rho(\omega^2) e^{-\omega t}$$

$C(t) < 0$

- negative spectral density
- positivity violation
- gluon confinement

$C(t) > 0$ says nothing about $\rho(\mu)$

Positivity violation for the gluon propagator



Already observed in lattice simulations

C. Aubin, M. C. Ogilvie, Phys. Rev D70, 074514 (2004)

A. Cucchieri, T. Mendes, A. R. Taurines, Phys. Rev. D71, 051902 (2005)

Spectral density

- Euclidean momentum-space propagator of a (scalar) physical degree of freedom

$$\mathcal{G}(p^2) \equiv \langle \mathcal{O}(p)\mathcal{O}(-p) \rangle$$

- Källén-Lehmann spectral representation

$$\mathcal{G}(p^2) = \int_0^\infty \mathrm{d}\mu \frac{\rho(\mu)}{p^2 + \mu} , \qquad \text{with } \rho(\mu) \geq 0 \text{ for } \mu \geq 0 .$$

- spectral density contains information on the masses of physical states described by the operator $\mathcal{O}$

$$\rho(\mu) = \sum_\ell \delta(\mu - m_\ell^2) \, |\langle 0|\mathcal{O}|\ell_0\rangle|^2 \; ,$$

Spectral density

- $\mathcal{G} = \mathcal{L}^2\hat{\rho} = \mathcal{L}\mathcal{L}^*\hat{\rho}$ where $(\mathcal{L}f)(t) \equiv \int_0^\infty dse^{-st}f(s)$ is a Laplace transform
- inversion of Laplace transform: ill-posed problem
- Way out: Tikhonov regularization
  - ill-posed problem $y = \mathcal{K}x$
  - minimize $||\mathcal{K}x - y|| + \lambda||x||^2$
    - $\lambda > 0$ is a regularization parameter
  - x^λ is the unique solution of the normal equation

$$\mathcal{K}^*\mathcal{K}x^\lambda + \lambda x^\lambda = \mathcal{K}^*y$$

  the operator $\mathcal{K}^*\mathcal{K} + \lambda$ is strictly positive, hence invertible
- Morozov discrepancy principle: choose $\overline{\lambda}$ s.t. $||\mathcal{K}x^{\overline{\lambda}} - y^\delta|| = \delta$
  - δ: “noise of input data”
  - A unique solution $x^{\overline{\lambda},\delta}$ exists

Outline

CENTRO DE FÍSICA COMPUTACIONAL

Getting gluon spectral density

$$\mathcal{L}^2 \rho = D$$

$$\mathcal{L}^4 \rho + \lambda \rho = \mathcal{L}^2 \mathcal{D}$$

$$\int_0^\infty \mathrm{d}t \, \rho(t) \frac{\ln \frac{z}{t}}{z - t} + \lambda \rho(z) = \int_0^\infty \mathrm{d}t \, \frac{\mathcal{D}(t)}{t + z}$$

- consider 1-loop perturbative behaviour after $p_{max}^{(latt)}$
- integrals computed using Gauss-Legendre quadrature
- discretization leads to a linear system
- IR and UV cut-offs
- lattice data (80^4, $\beta = 6.0$) interpolated using splines

CENTRO de FÍSICA COMPUTACIONAL

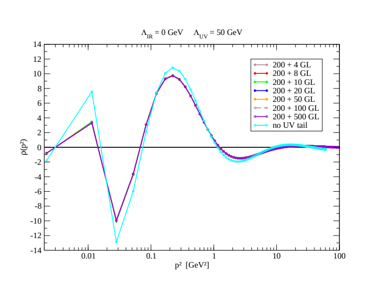
Outline

CENTRO DE FÍSICA COMPUTACIONAL

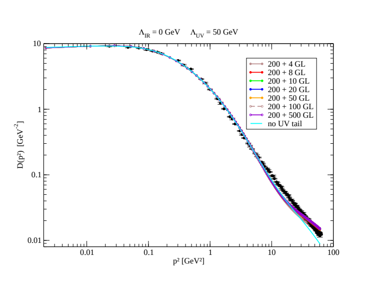
Results (preliminary)

Changing number of GL points

Spectral density



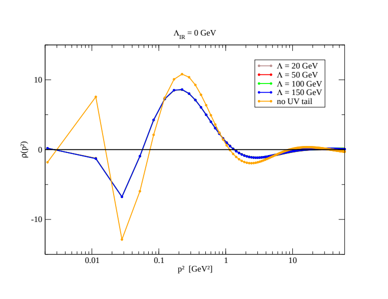
Reconstructed propagator



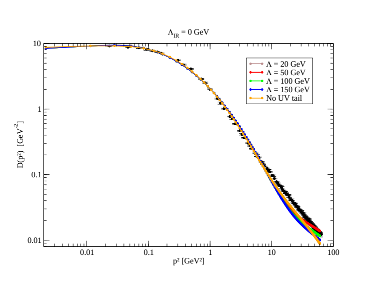
Results (preliminary)

Changing UV cutoff

Spectral density



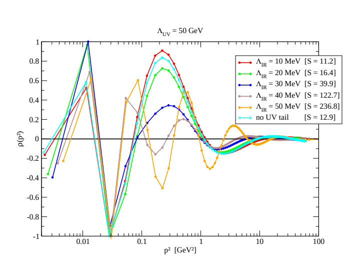
Reconstructed propagator



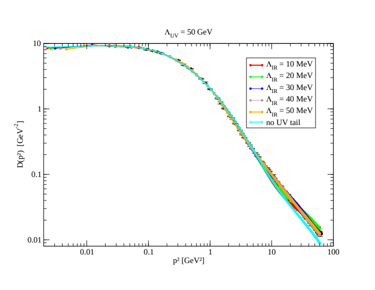
Results (preliminary)

Changing IR cutoff

Spectral density



Reconstructed propagator



Outline

CENTRO DE FÍSICA COMPUTACIONAL

Getting hotter

- Gluon propagator at finite T splitted into two components
  - transverse D_T
  - longitudinal D_L

$$D^{ab}_{\mu\nu}(\hat{q}) = \delta^{ab}\left(P^T_{\mu\nu}D_T(q_4^2,\vec{q}) + P^L_{\mu\nu}D_L(q_4^2,\vec{q})\right)$$

- Finite temperature on the lattice: $L_t << L_s$

$$T = \frac{1}{aL_t}$$

- Simulations: use of Chroma and PFFT libraries
- keep a constant (spatial) physical volume $\sim (6.5fm)^3$
- all data renormalized at $\mu = 4GeV$

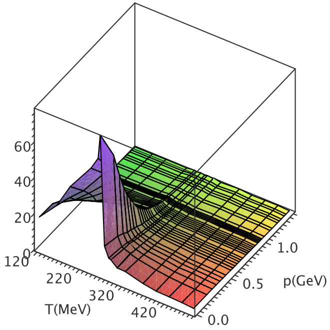
O. Oliveira, PJS, Acta Phys.Polon.Supp. 5 (2012) 1039, PoS(LATTICE2012)216, PoS(Confinement X)045

Lattice setup finite T

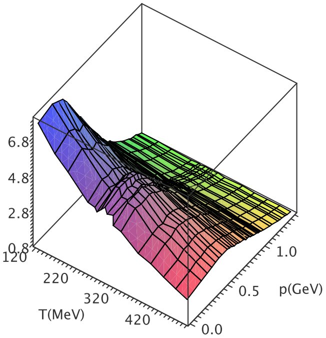
Temp. (MeV)	β	L_s	L_t	a [fm]	1/a (GeV)
121	6.0000	64	16	0.1016	1.943
162	6.0000	64	12	0.1016	1.943
194	6.0000	64	10	0.1016	1.943
243	6.0000	64	8	0.1016	1.943
260	6.0347	68	8	0.09502	2.0767
265	5.8876	52	6	0.1243	1.5881
275	6.0684	72	8	0.08974	2.1989
285	5.9266	56	6	0.1154	1.7103
290	6.1009	76	8	0.08502	2.3211
305	5.9640	60	6	0.1077	1.8324
305	6.1326	80	8	0.08077	2.4432
324	6.0000	64	6	0.1016	1.943
366	6.0684	72	6	0.08974	2.1989
397	5.8876	52	4	0.1243	1.5881
428	5.9266	56	4	0.1154	1.7103
458	5.9640	60	4	0.1077	1.8324
486	6.0000	64	4	0.1016	1.943

Surface plots

Longitudinal component



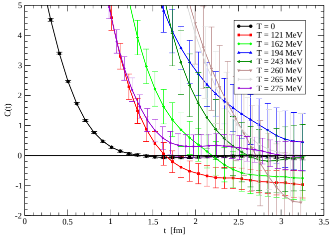
Transverse component



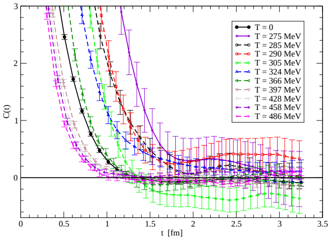
Outline

Positivity violation finite T - longitudinal component

Below T_C

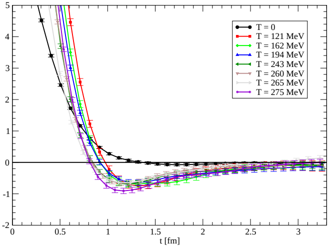


Above T_c

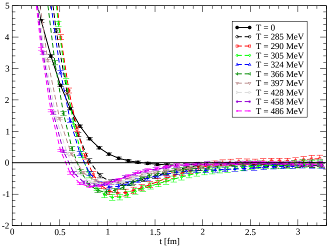


Positivity violation finite T - transverse component

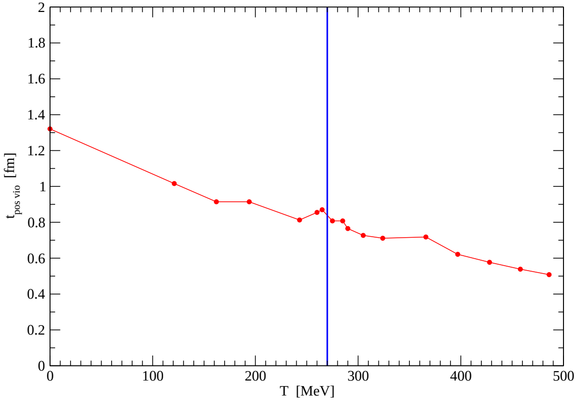
Below T_C



Above T_c



Positivity violation scale – transverse component

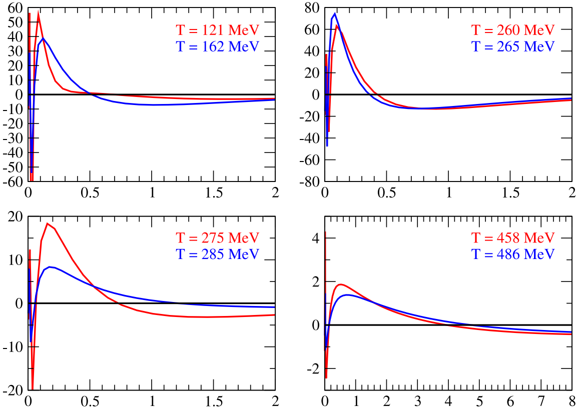


CENTRO DE FÍSICA COMPUTACIONAL

Outline

CENTRO DE FÍSICA COMPUTACIONAL

Longitudinal propagator spectral densities



Conclusions and outlook

- Gluon unphysical for all T up to 500 MeV
- Access to the spectral density
  - Preliminary results
- Positivity violation scale increases with temperature
  - Gluons behave as quasi-particles for high T?

FCT Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

Supported by FCT via project CERN/FP/123612/2011. D.D. acknowledges financial support from the Research-Foundation Flanders (FWO Vlaanderen). P.S. supported by FCT grant SFRH/BPD/40998/2007.

CENTRO DE FÍSICA COMPUTACIONAL