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Coulomb Gauge on the lattice

From Zero to Finite Temperature

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Content

- Gluon propagator
  - in SU(3) and SU(4) at $T = 0$
  - at $T > 0$ on anisotropic lattices
- Ghost propagator at $T > 0$
- Coulomb potential
  - Effect of Gribov copies
  - Results at $T > 0$
- Summary and Outlook

The gluon propagator

$$D(|\boldsymbol{p}|, p_0) = \frac{1}{9V} \langle A_i^a(|\boldsymbol{p}|, p_0) A_i^a(-|\boldsymbol{p}|, -p_0) \rangle$$

- Static propagator shows scaling violations
  - p_0 dependence in the non-instantaneous propagator after residual gauge fixing
- Scaling violations resolved
  - in lattice Hamiltonian limit $\xi \to \infty$ (anisotropic lattices)
  - by taking the static propagator at fixed p_0

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The gluon propagator

- Non-instantaneous bare propagator factorizes

$$D(|\boldsymbol{p}|, p_0) = f_\beta(|\boldsymbol{p}|) g_\beta(z), \qquad z = \frac{p_0}{|\boldsymbol{p}|}$$

- Identify $g_\beta(z)$ from fit of

$$g_\beta(z) = \frac{D(|\boldsymbol{p}|, p_0)}{D(|\boldsymbol{p}|, 0)}, \qquad g(0) \equiv 1$$

- Integrate out the z dependence

$$D(|\boldsymbol{p}|) = \frac{1}{N_t} \sum_{p_0} |\boldsymbol{p}| \frac{D(|\boldsymbol{p}|, p_0)}{g_\beta(z)}$$

The gluon propagator at T=0

- In SU(2) on isotropic lattices: Gribov form

$$D(|\boldsymbol{p}|) = \frac{1}{\sqrt{|\boldsymbol{p}|^2 + \frac{M^4}{|\boldsymbol{p}|^2}}}$$

- For SU(3) and SU(4) modify the mid-momentum region:

$$D(|\boldsymbol{p}|) = \frac{1}{\sqrt{|\boldsymbol{p}|^2 + \gamma M^2 + \frac{M^4}{|\boldsymbol{p}|^2}}}$$

- On anisotropic lattices:

$$D(|\boldsymbol{p}|) = \frac{1}{\sqrt{|\boldsymbol{p}|^2 + \gamma M^2 + \alpha \frac{M^3}{|\boldsymbol{p}|} + \frac{M^4}{|\boldsymbol{p}|^2}}}$$

The gluon propagator at T=0

- In SU(2) on isotropic lattices: Gribov form

$$\frac{D(|\boldsymbol{p}|)}{|\boldsymbol{p}|} = \frac{1}{\sqrt{|\boldsymbol{p}|^4 + M^4}}$$

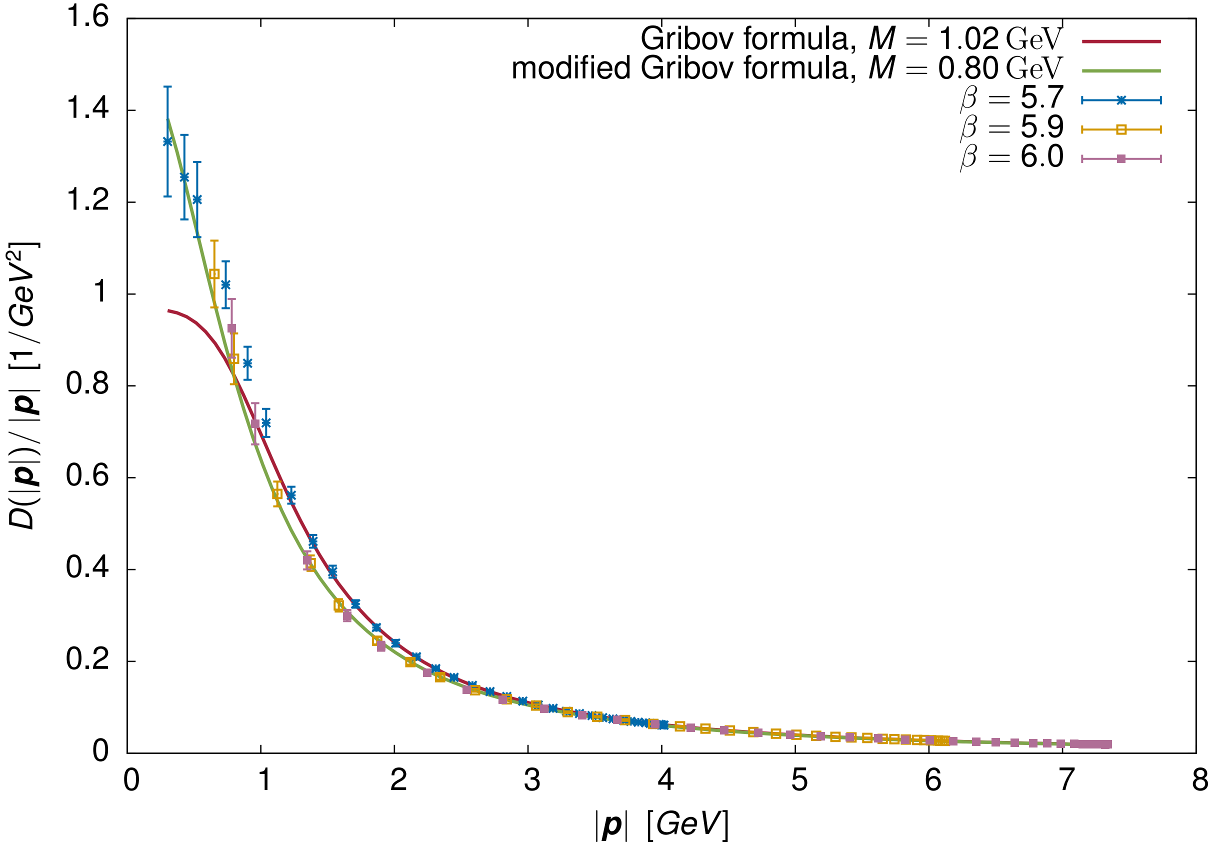
- For SU(3) and SU(4) modify the mid-momentum region:

$$\frac{D(|\boldsymbol{p}|)}{|\boldsymbol{p}|} = \frac{1}{\sqrt{|\boldsymbol{p}|^4 + \gamma M^2 |\boldsymbol{p}|^2 + M^4}}$$

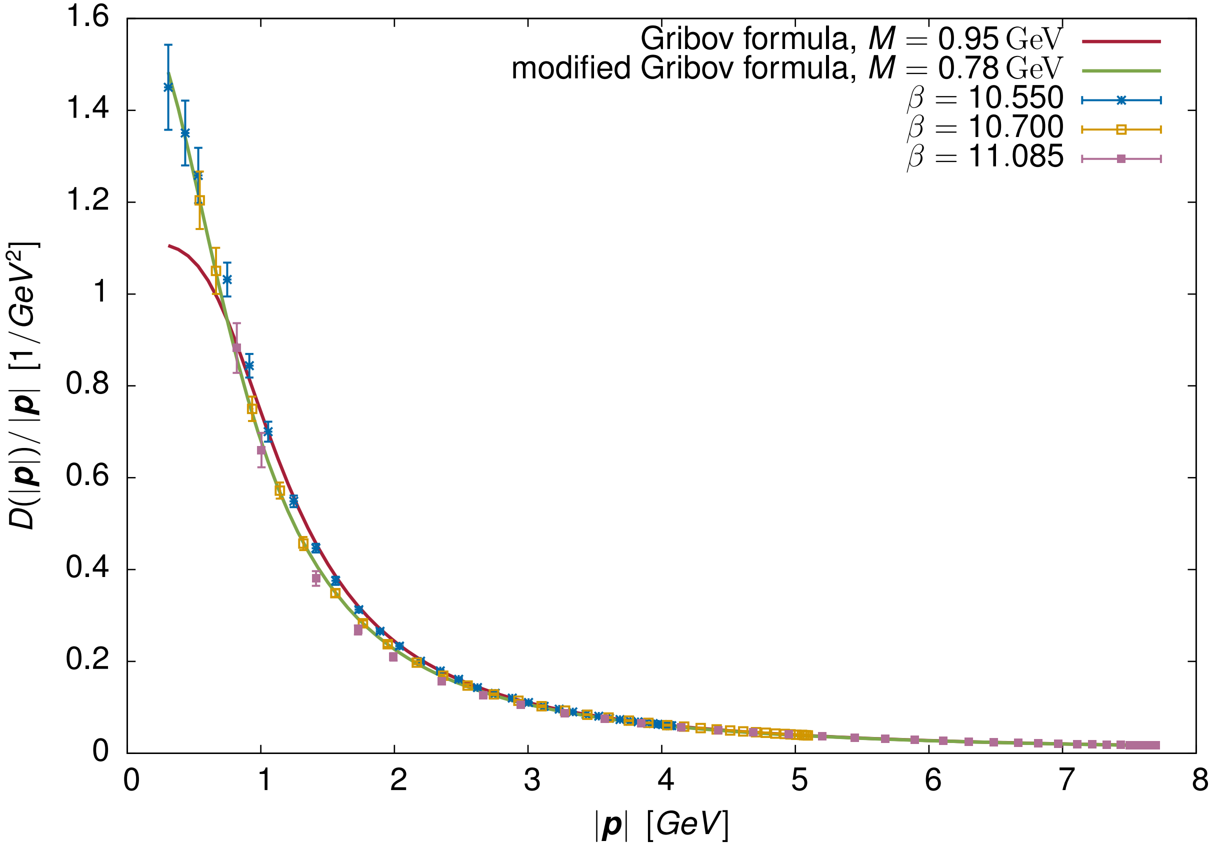
- On anisotropic lattices:

$$\frac{D(|\boldsymbol{p}|)}{|\boldsymbol{p}|} = \frac{1}{\sqrt{|\boldsymbol{p}|^4 + \gamma M^2 |\boldsymbol{p}|^2 + \alpha M^3 |\boldsymbol{p}| + M^4}}$$

SU(3) gluon propagator, $V = 24^4$



SU(4) gluon propagator, $V = 24^4$



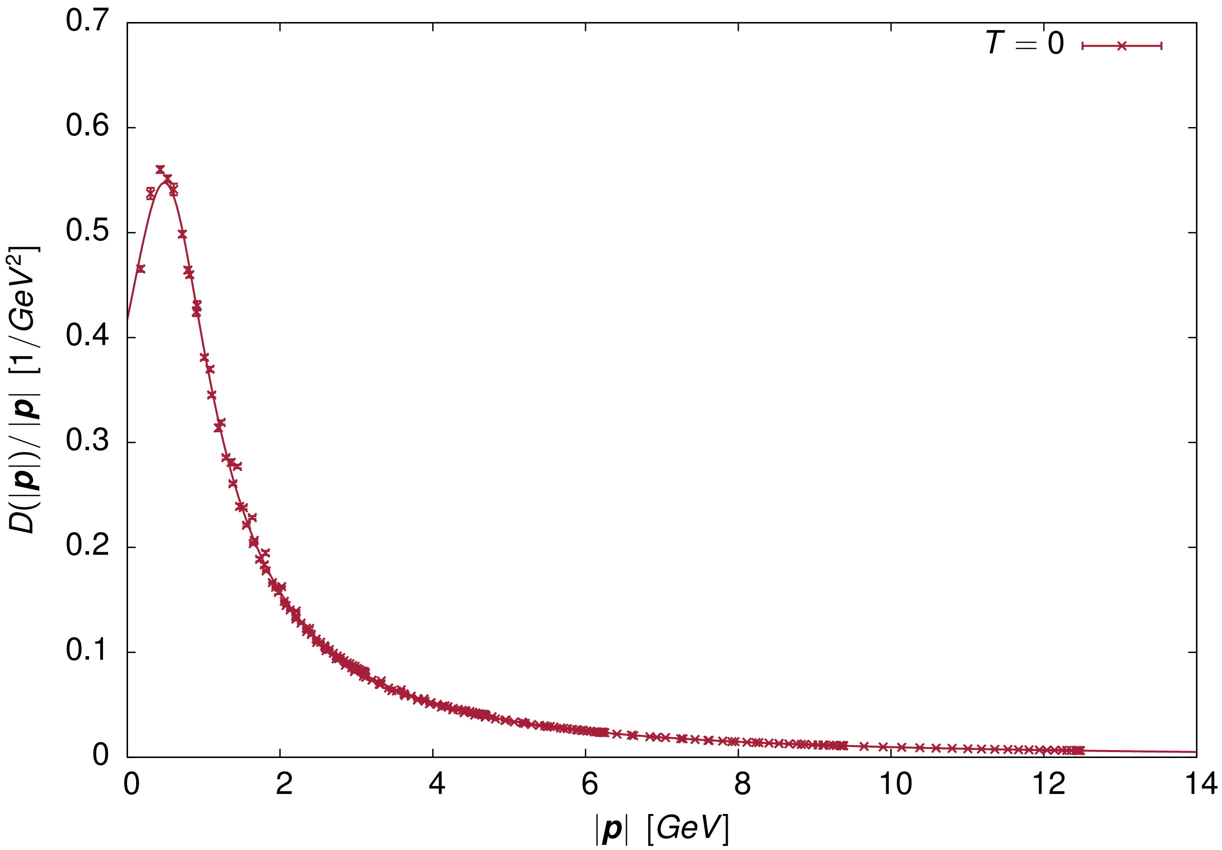
Finite Temperature & anisotropic lattices

- Temperature given by $T = \frac{1}{N_t a_t}$
- IR analysis on isotropic lattices limited to low temperature ($T < 1.5 T_c$)
- Solution: anisotropic lattices $\xi = a_s / a_t$
  - up to $T = 6 T_c$ at $\xi = 4$ possible

- Setup:
  - SU(2)
  - anisotropy $\xi = a_s / a_t = 4$
  - lattice volume $V = N_t \times 32^3$
  - 100 configurations
  - gauge fixing: Simulated annealing and Overrelaxation with varying number of copies

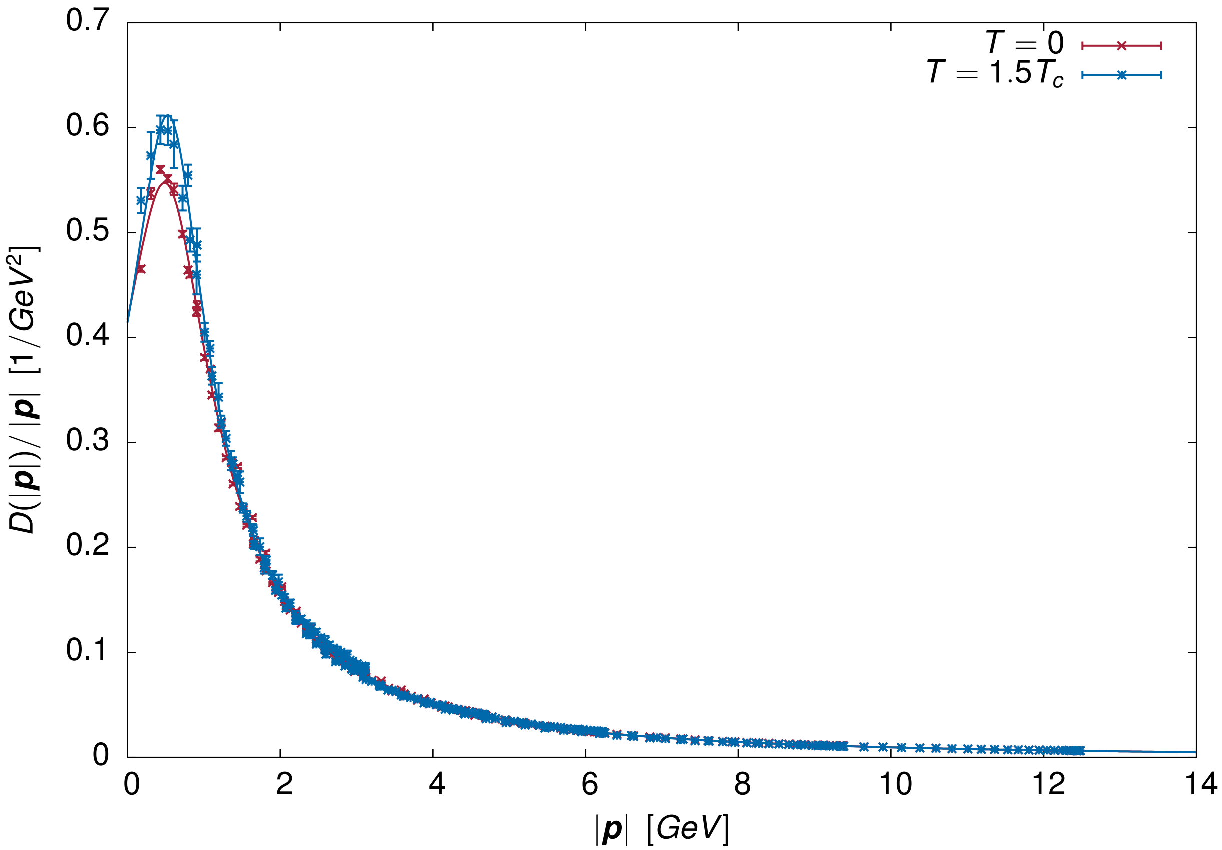
Anisotropic gluon propagator up to $T = 3T_c$

5 sets of configurations for each T ($\beta \in [2.25, 2.64]$)



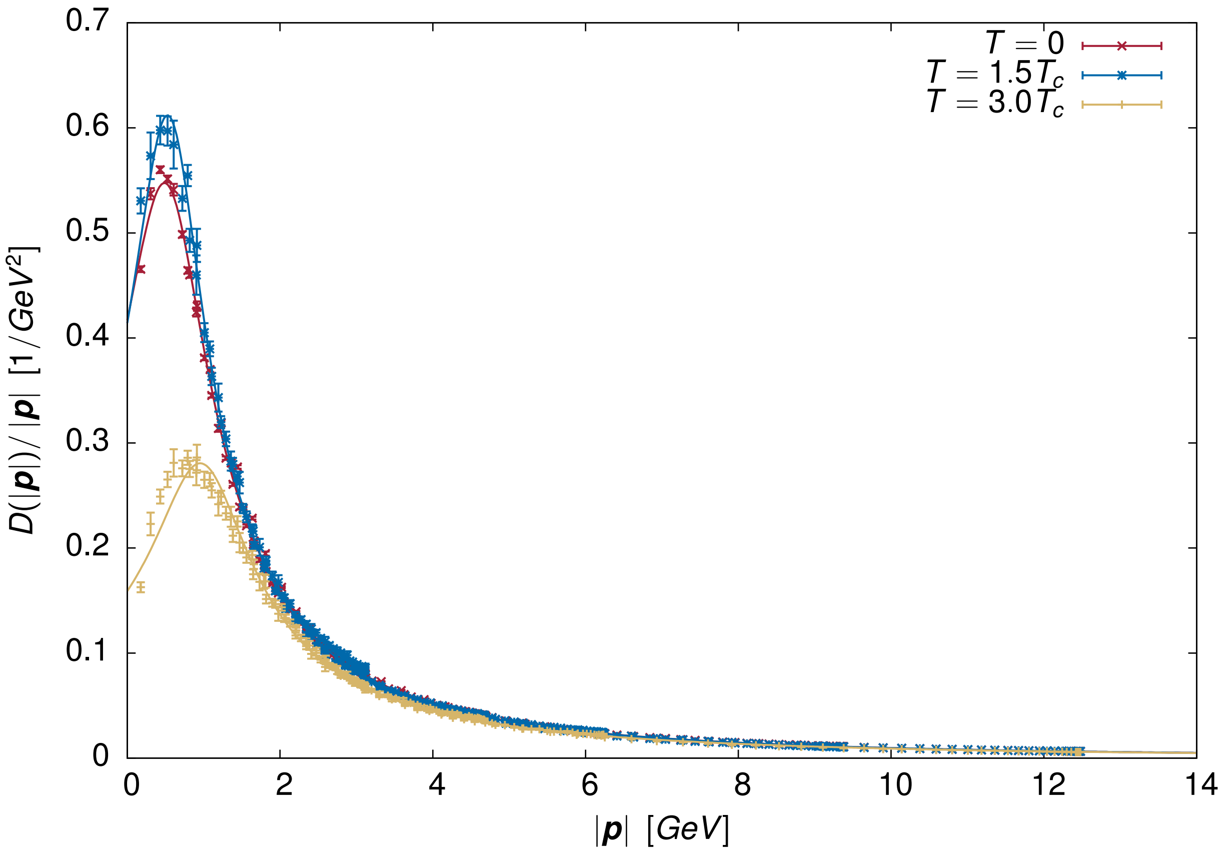
Anisotropic gluon propagator up to $\mathbf{T = 3T_c}$

5 sets of configurations for each T ($\beta \in [2.25, 2.64]$)



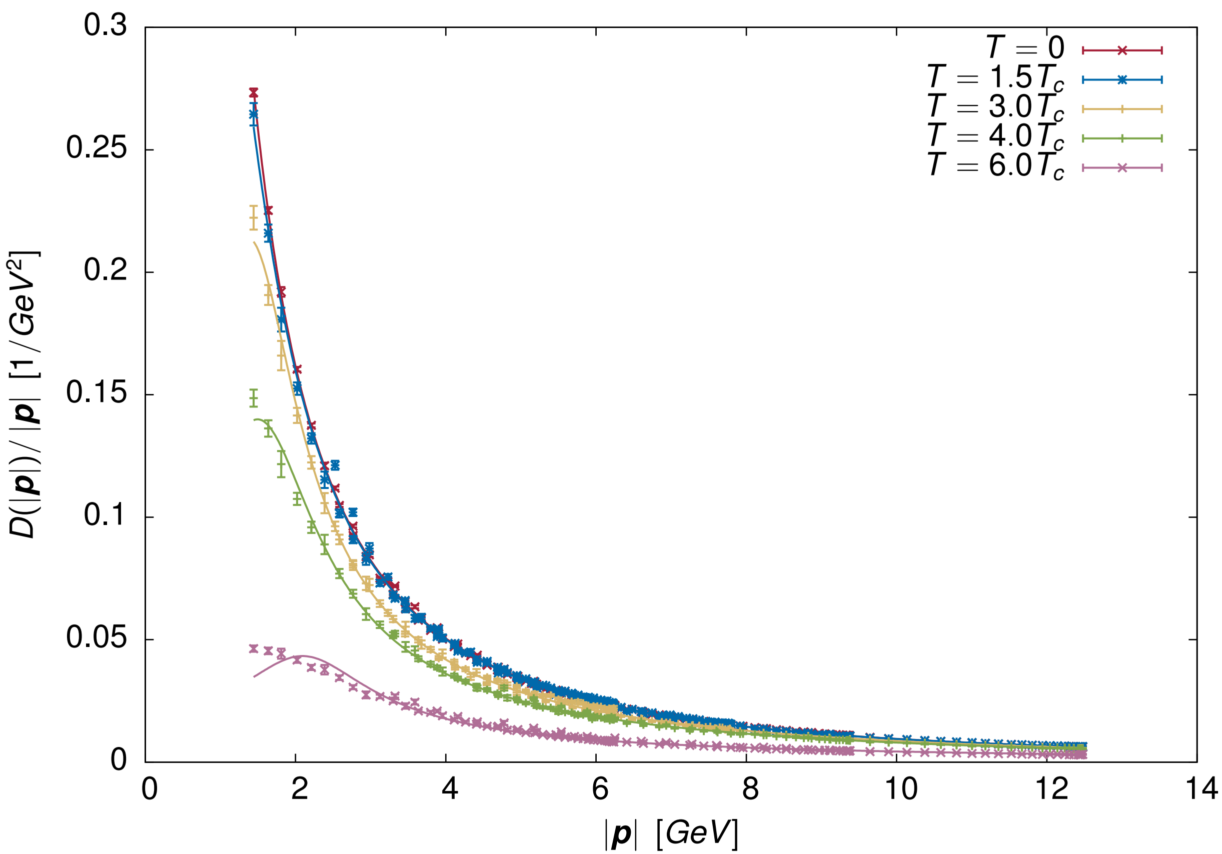
Anisotropic gluon propagator up to $T = 3T_c$

5 sets of configurations for each T ($\beta \in [2.25, 2.64]$)



Anisotropic gluon propagator up to $T = 6T_c$

3 or 4 sets of configurations for each T ($\beta \in [2.49, 2.64]$)



The ghost propagator

$$G(\boldsymbol{p}) = \frac{1}{3N_s} \left\langle \sum_{\boldsymbol{x},\boldsymbol{y}} \mathrm{e}^{\mathrm{i}\boldsymbol{p}(\boldsymbol{x}-\boldsymbol{y})} \left[M^{-1}\right]^{aa}(\boldsymbol{x},\boldsymbol{y}) \right\rangle$$

- ghost dressing function $d(|\boldsymbol{p}|) = |\boldsymbol{p}|^2 \, G(|\boldsymbol{p}|)$
- continuum results: IR power-law

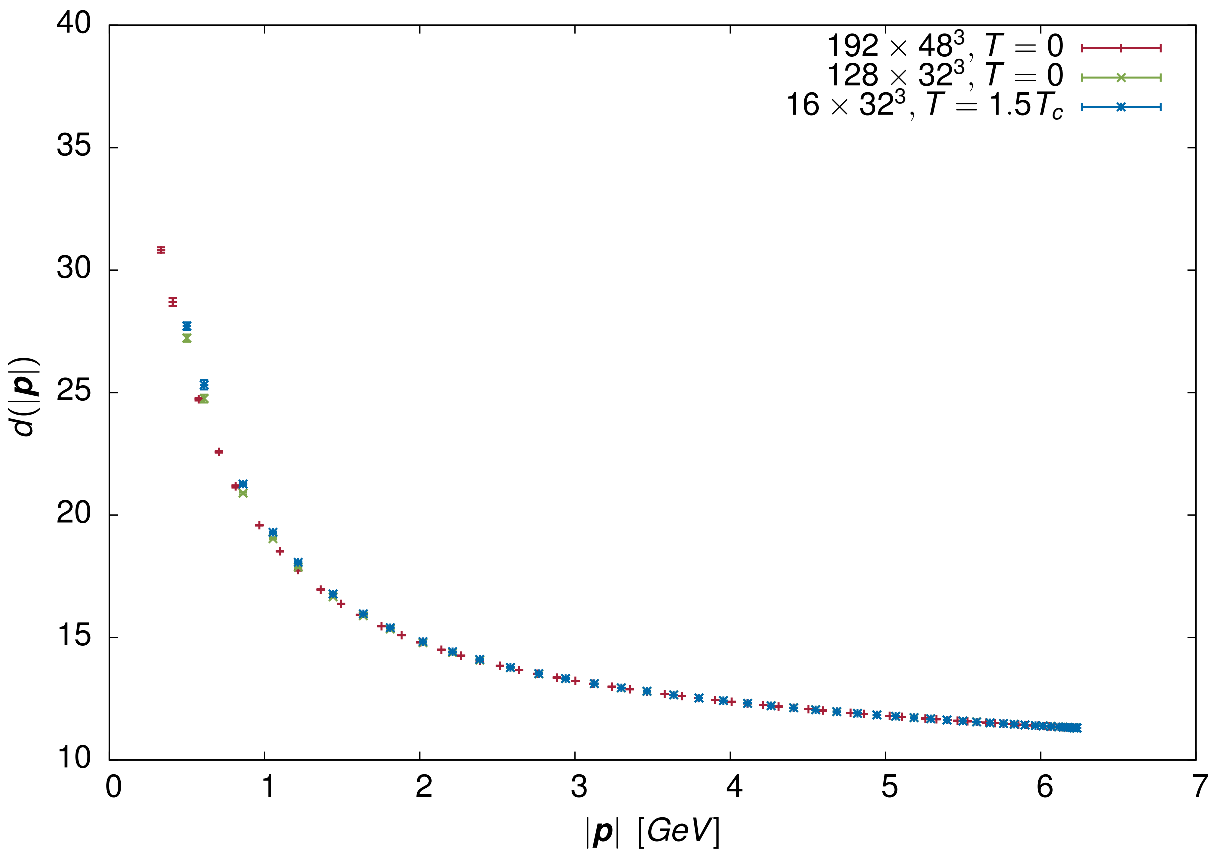
$$d(|\boldsymbol{p}|) \sim \frac{1}{|\boldsymbol{p}|^{\kappa}}$$

- asymptotic freedom: UV power-law with log corrections:

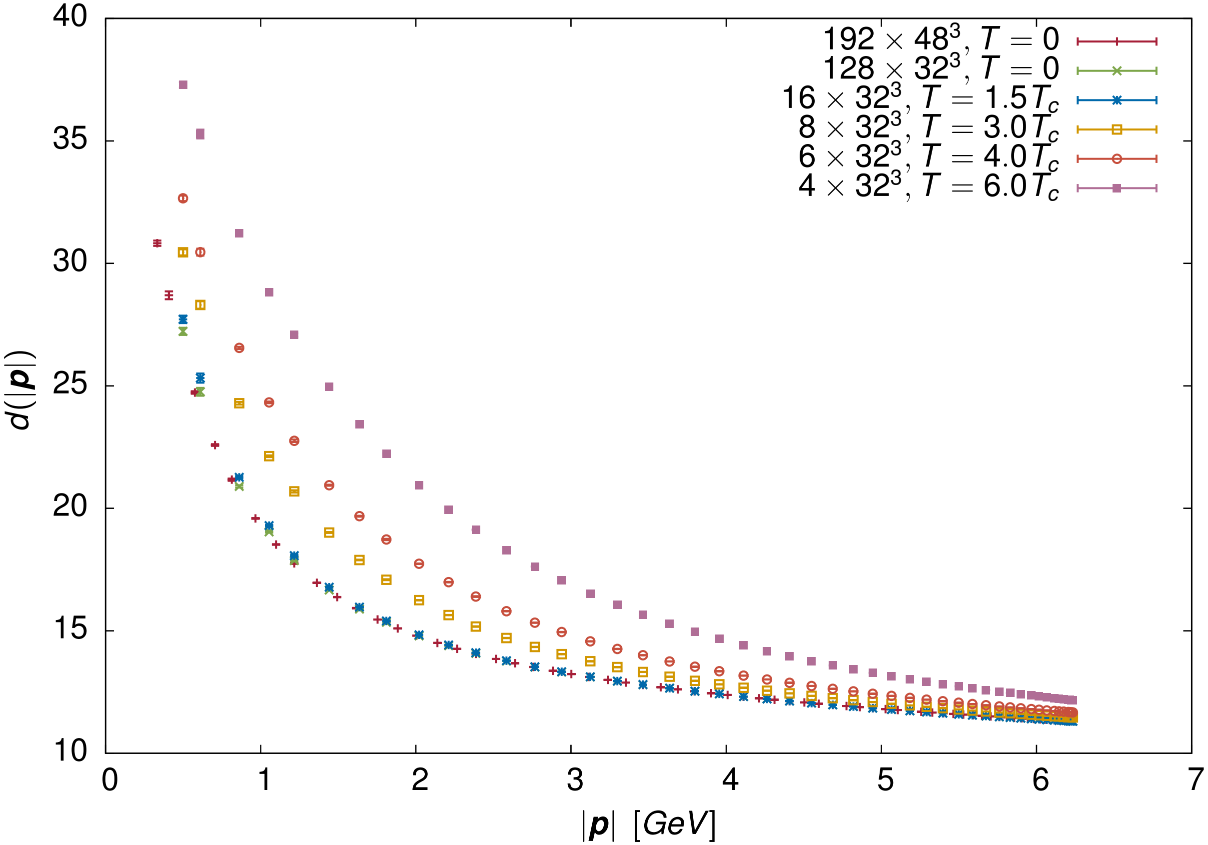
$$d(|\boldsymbol{p}|) \sim \frac{1}{\log^{\gamma} \frac{|\boldsymbol{p}|}{m}}$$

- continuum sum rules violated

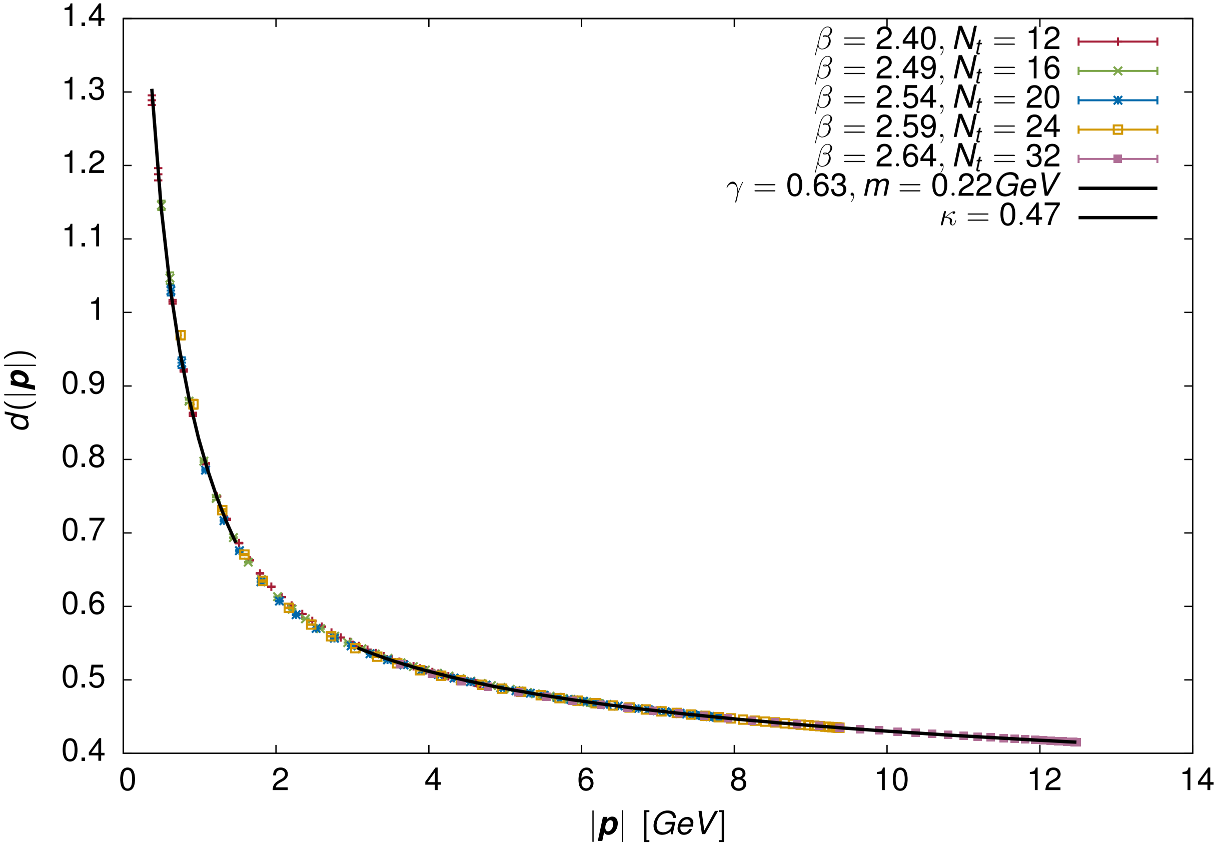
Ghost dressing function at fixed $\beta = 2.5$ and ξ_0



Ghost dressing function at fixed $\beta = 2.5$ and ξ_0

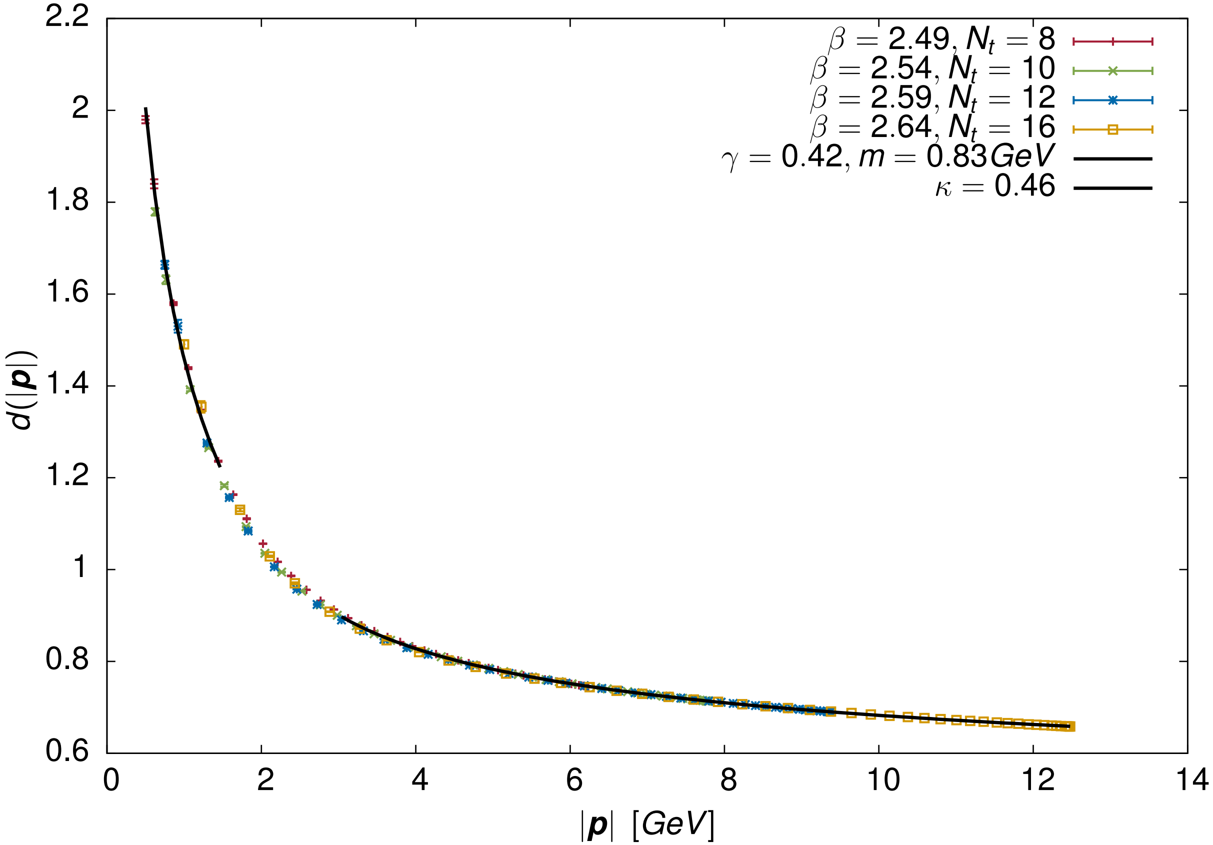


Ghost dressing function at $T = 1.5T_c$



UV: equally well fitted with fixed $\gamma = 0.5$ ($m = 0.44 GeV$)

Ghost dressing function at $T = 3.0T_c$



Coulomb Potential

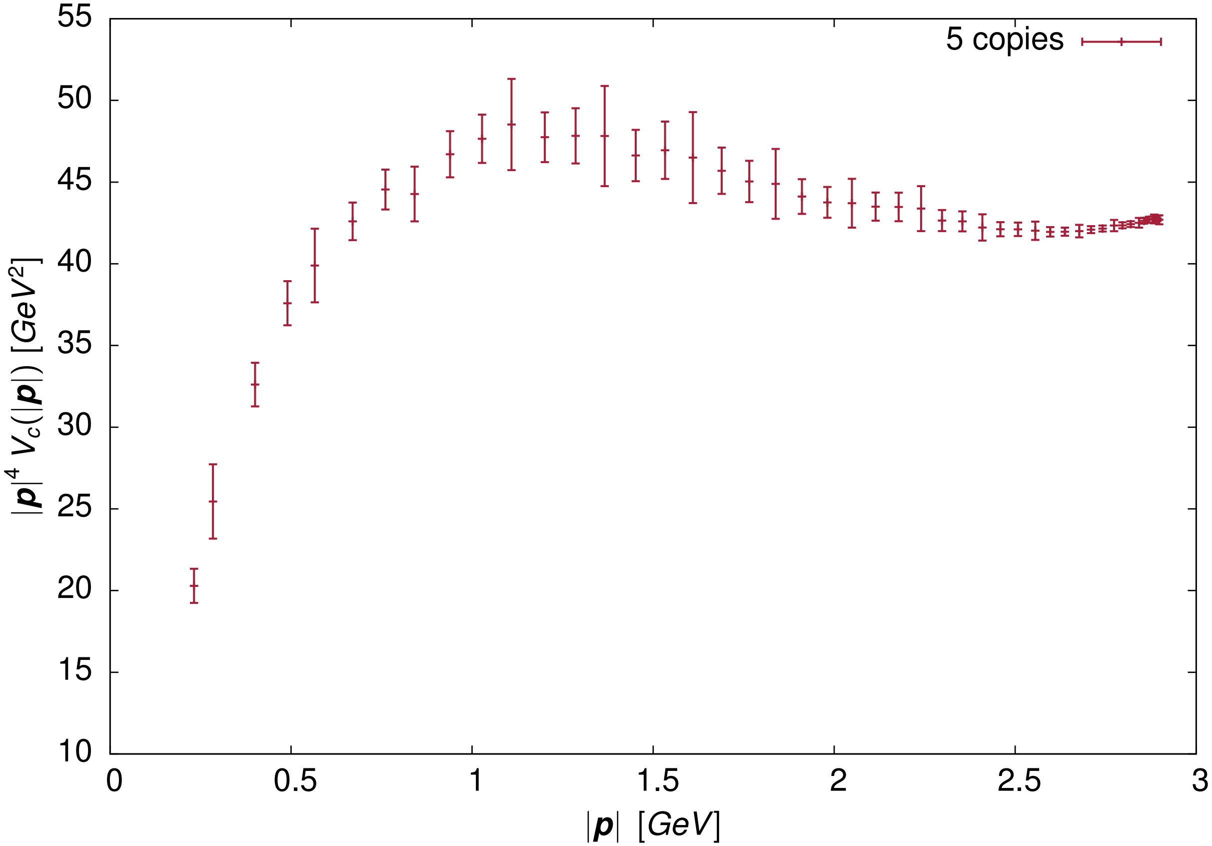
$$V_c(\boldsymbol{p}) = \frac{1}{3N_s}\left\langle \sum_{\boldsymbol{x},\boldsymbol{y}} \mathrm{e}^{\mathrm{i}\boldsymbol{p}(\boldsymbol{x}-\boldsymbol{y})} \left[M^{-1}(-\Delta)M^{-1}\right]^{aa}(\boldsymbol{x},\boldsymbol{y}) \right\rangle$$

$$V_c(|\boldsymbol{p}|) \sim |\boldsymbol{p}|^4 \text{ at } |\boldsymbol{p}| \to 0 \quad \Longleftrightarrow \quad V_c(r) \sim \sigma_c r \text{ at } r \to \infty$$

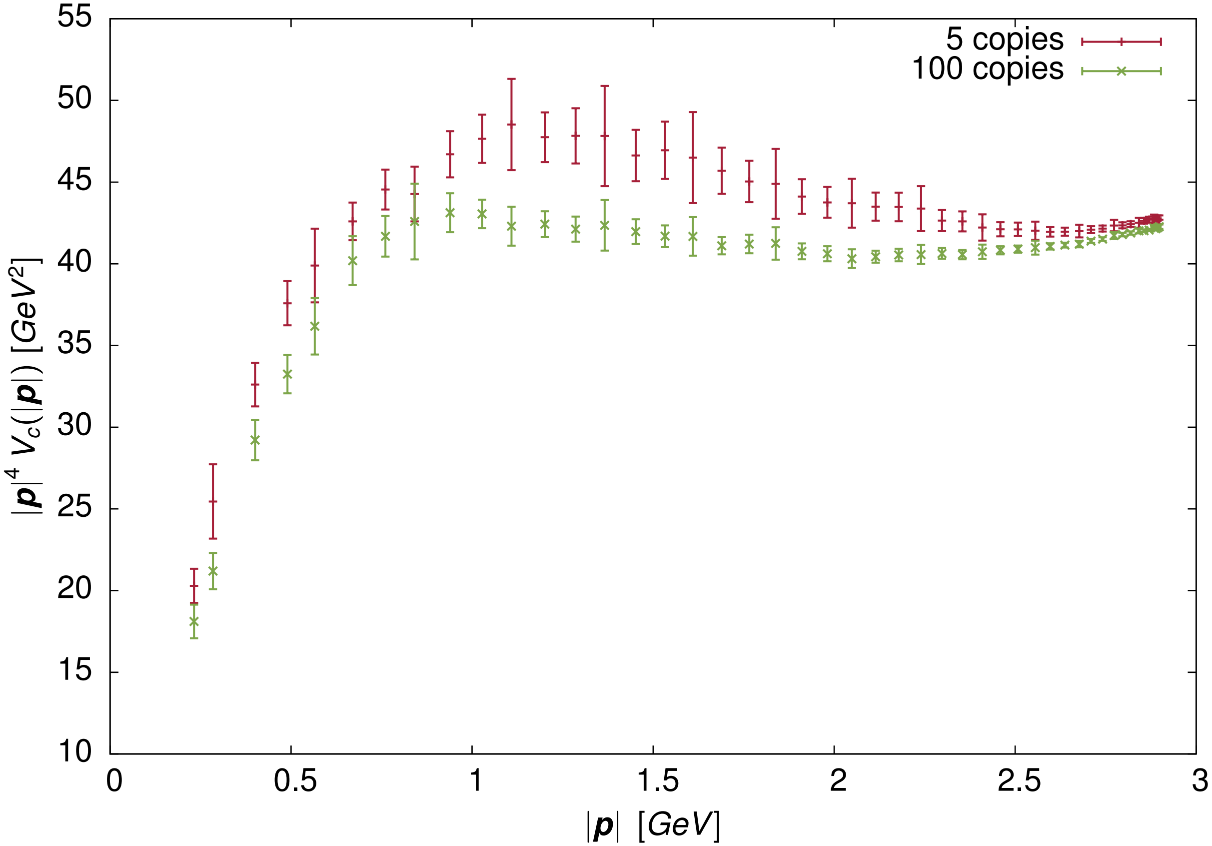
- Problems:
  - strong Gribov copy effects
  - Conjugate Gradient inversion is costly
  - extrapolation to the string tension based on few datapoints

$$\lim_{|\boldsymbol{p}|\to 0} V_c(|\boldsymbol{p}|) = 8\pi\sigma_c$$

Gribov problem ($T = 0, \beta = 2.2$)



Gribov problem ($T = 0, \beta = 2.2$)



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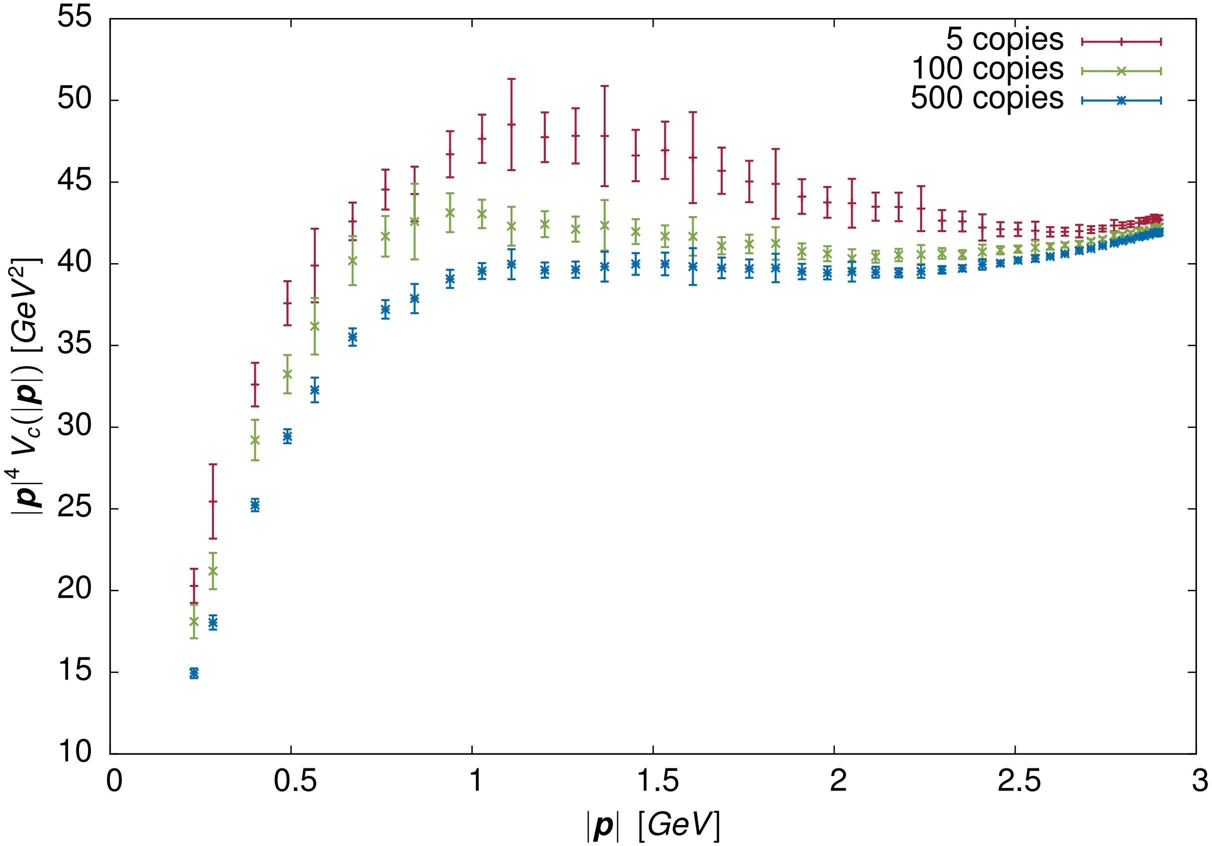
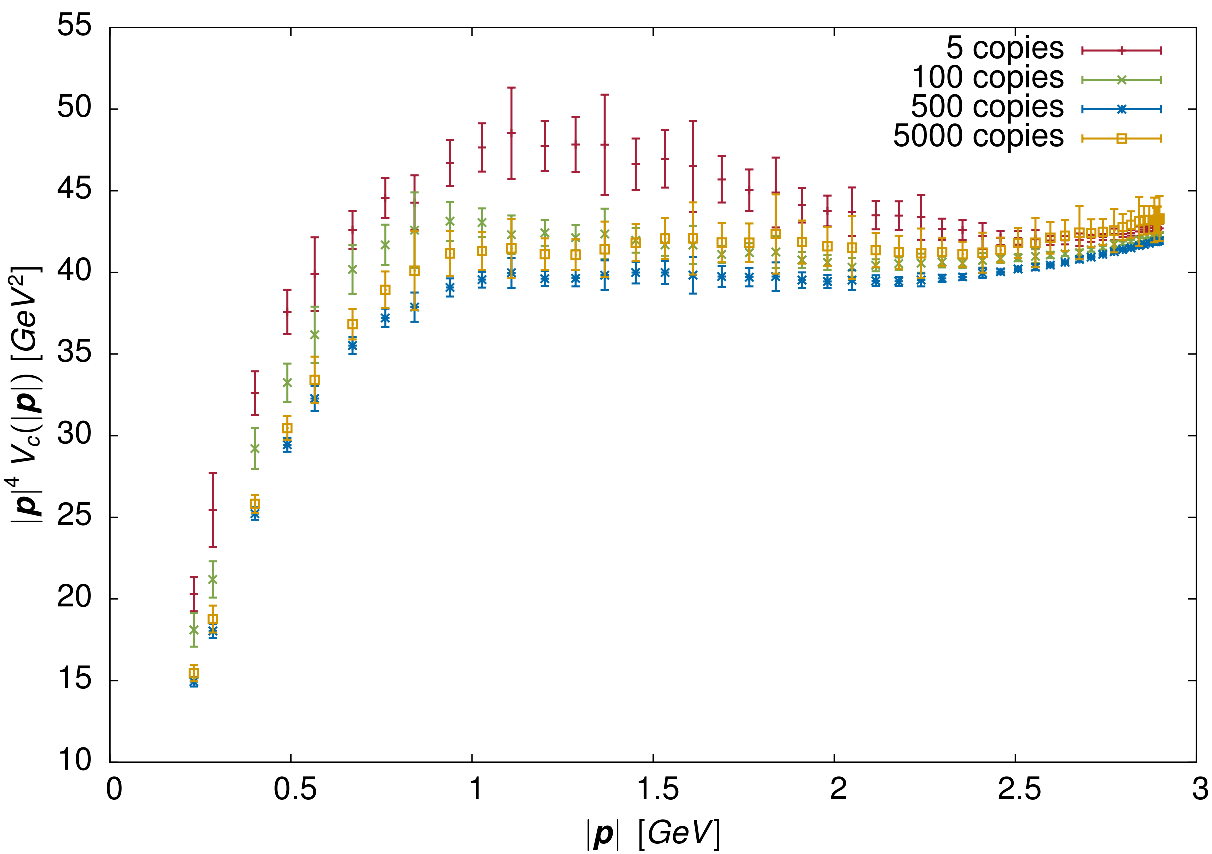


Gribov problem ($T = 0, \beta = 2.2$)



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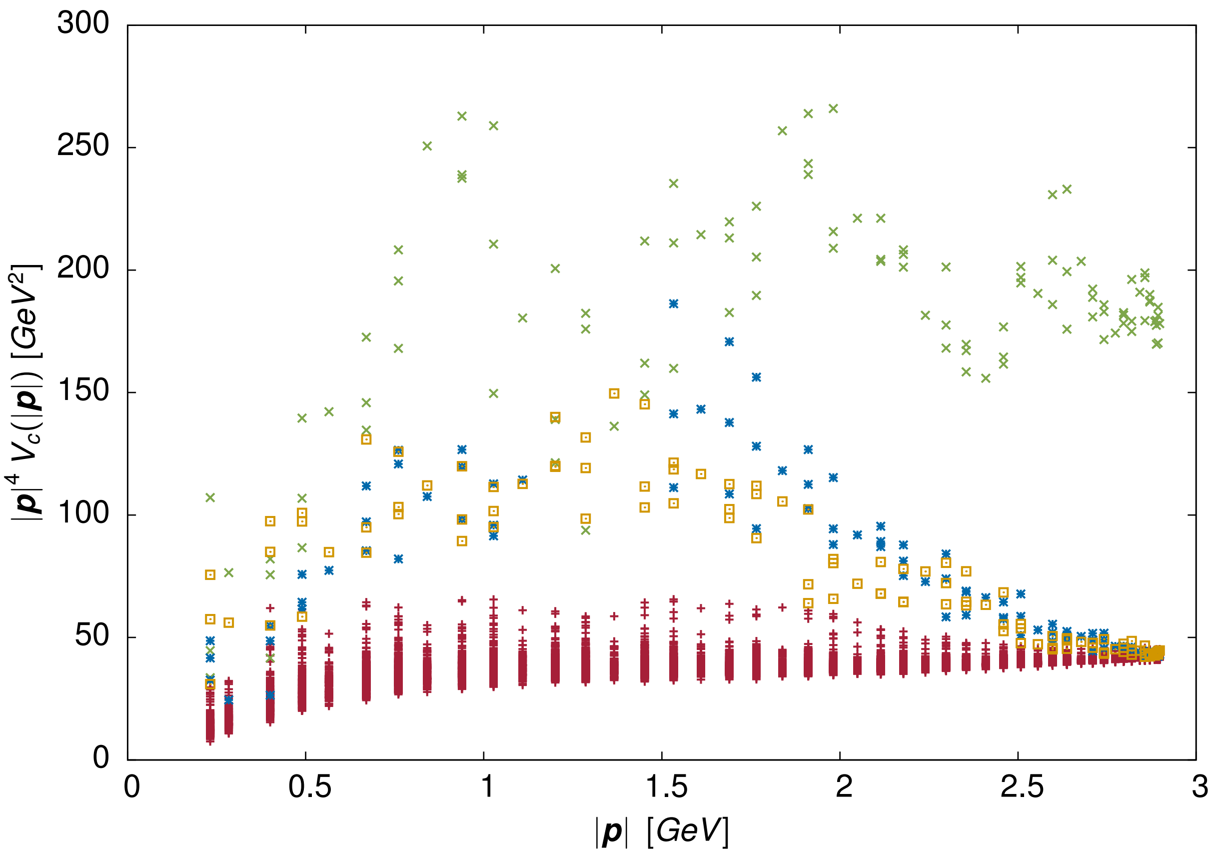
Gribov problem

raw data, configurations with highest contribution separated



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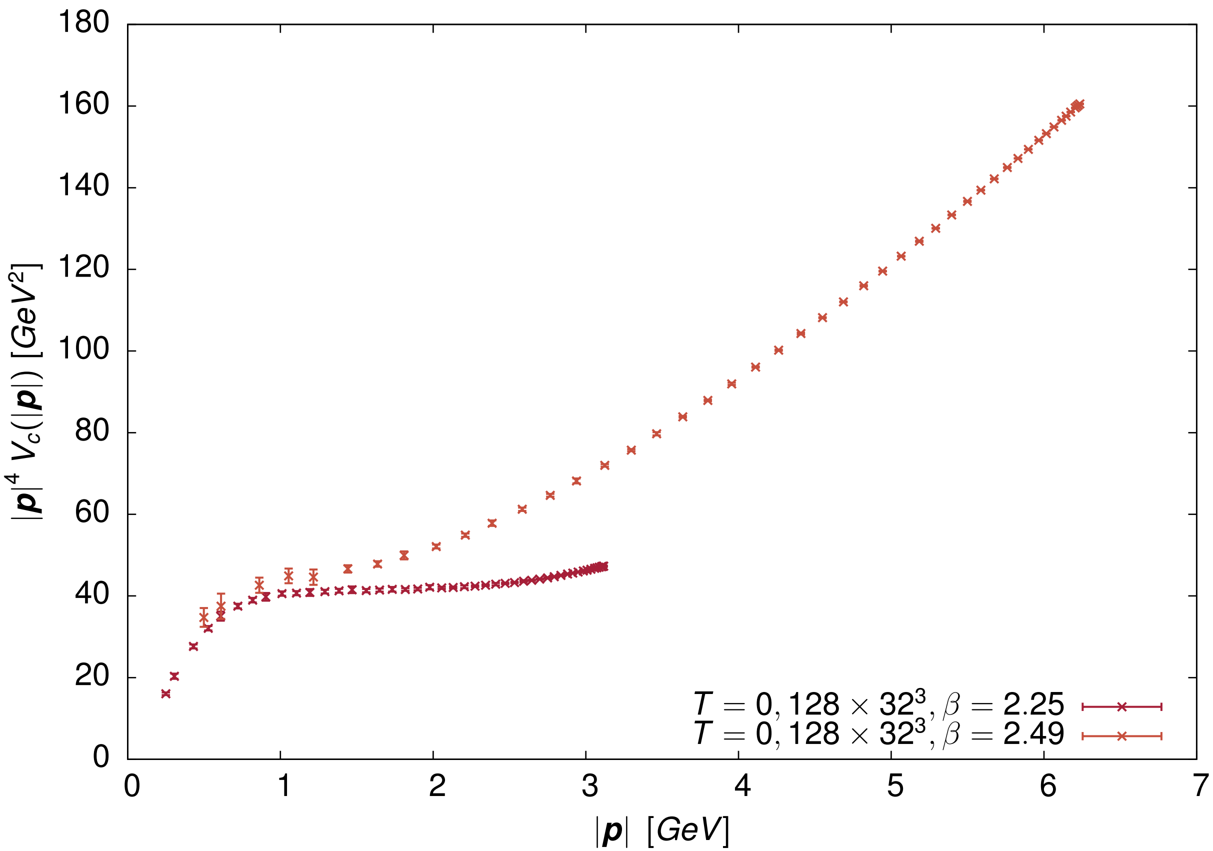
Remarks on gauge fixing and the Coulomb potential

Compare the functional value $F^g[U]$ to $V_c(\boldsymbol{p} = (1, 1, 1))$ at fixed timeslice in 10,000 gauge copies

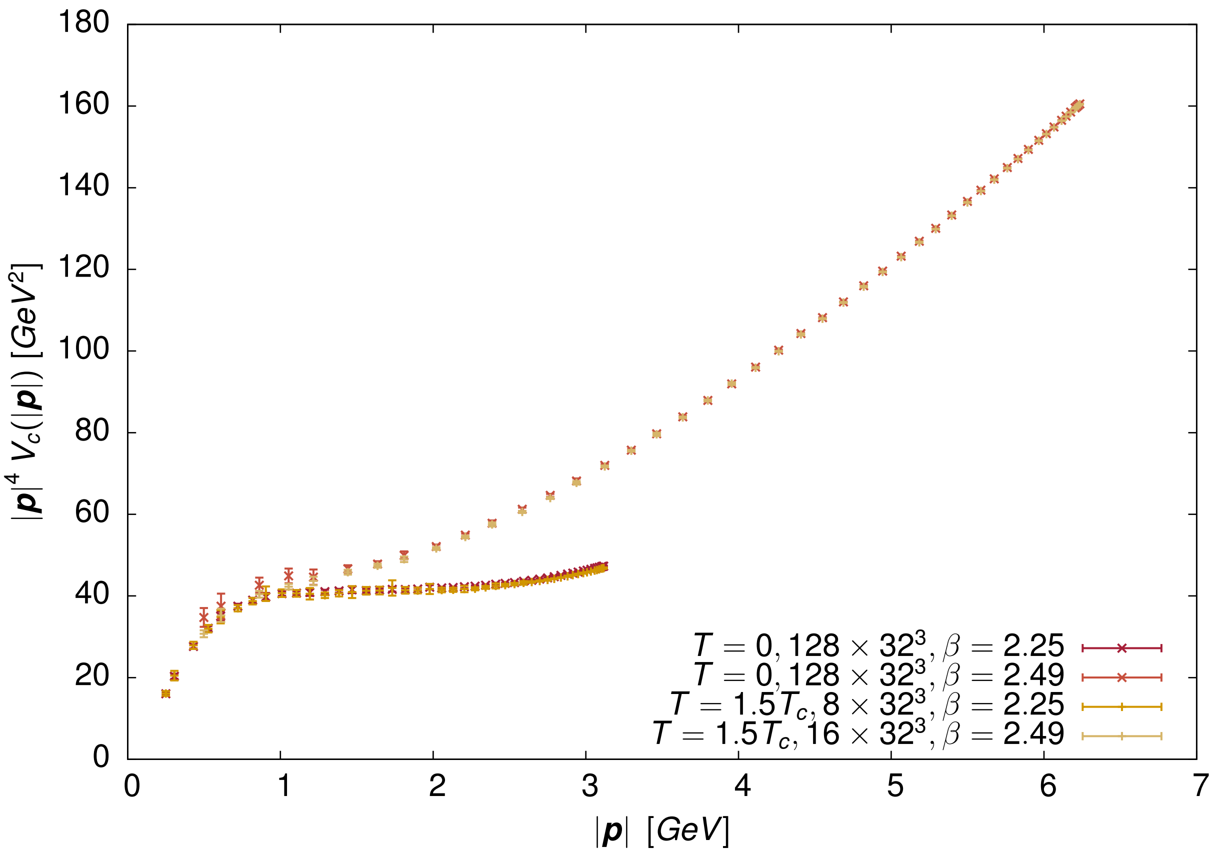
- extreme values of V_c not correlated to $F^g[U]$
- in average V_c gets smaller at increasing $F^g[U]$
- the highest value of $F^g[U]$ is found in more than 10 copies
- exceptional configurations are found in $\approx 1\%$ of the copies

Want to try yourself? http://www.culgt.com

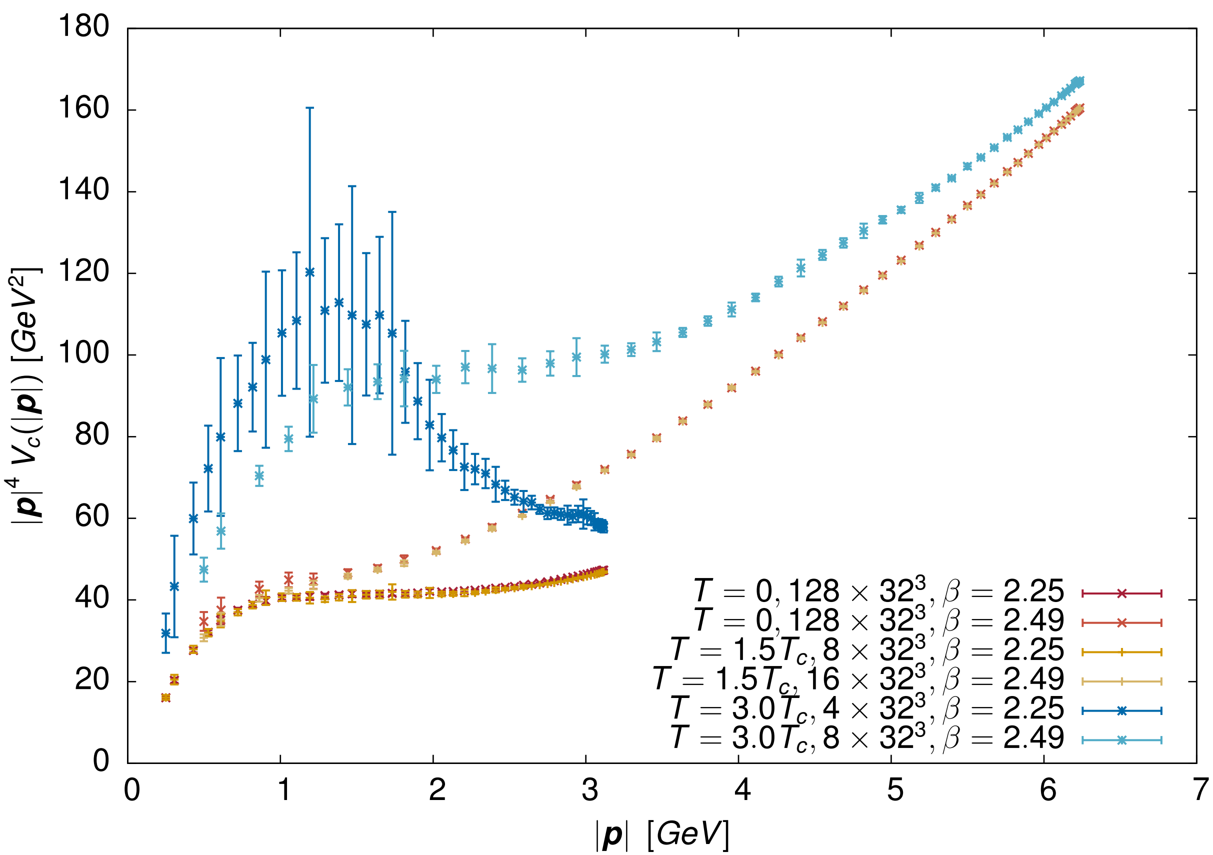
Coulomb potential at T $\geq$ 0



Coulomb potential at T $\geq$ 0



Coulomb potential at T $\geq$ 0



Summary

- The static gluon propagator obtained by integrating out the p_0 dependence is multiplicatively renormalizable
- At finite temperature:
  - No quantity is sensitive to deconfinement up to $1.5T_c$
  - Differences set in at larger T
  - The gluon propagator is further IR-suppressed at higher temperature
- Extrapolation of σ_c from V_c has large uncertainties:
  - Large Gribov copy effect
  - Large outliers need much higher statistics
  - UV scaling is defective: go to weak coupling limit

Outlook

- Control the exceptional configuration in ghost and V_c inversion $\rightarrow$ deflation techniques
- T-dependence of V_c calculated from the temporal gluon propagator $\langle A_0 A_0 \rangle$ or partial Polyakov lines $[\log\langle U_0 U_0^\dagger \rangle]$

Thank you.

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