Cutoff effects on Lattice Nuclear Forces

Takumi Doi

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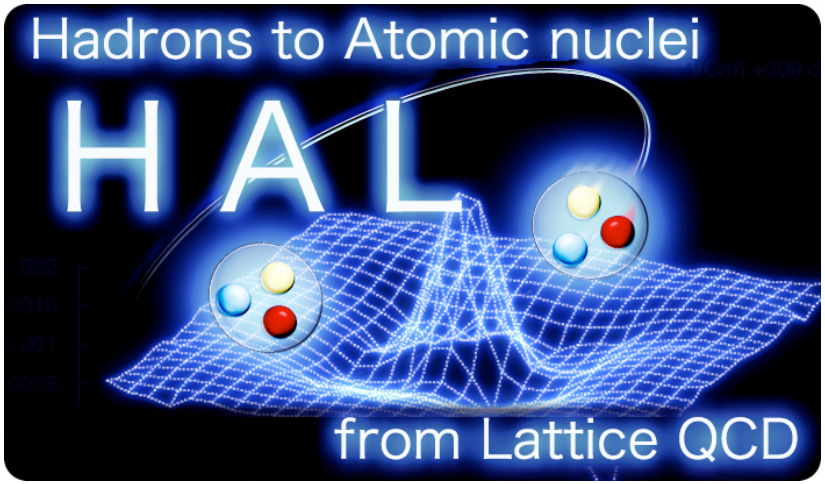
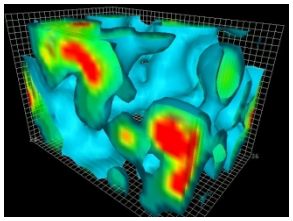
for HAL QCD Collaboration



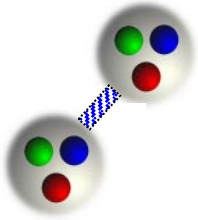
S. Aoki, K. Murano (YITP)
N. Ishii, H. Nemura, K. Sasaki,
M. Yamada, F. Etminan (Univ. of Tsukuba)
B. Charron (Univ. of Tokyo)
T. Hatsuda , Y. Ikeda (RIKEN)
T. Inoue (Nihon Univ.)

Nuclear Physics and Astrophysics from Lat QCD

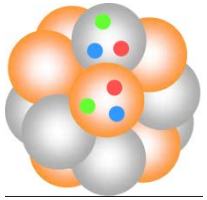


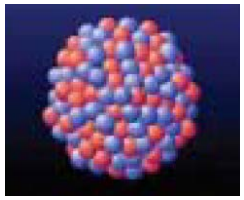
QCD Vacuum

Baryon

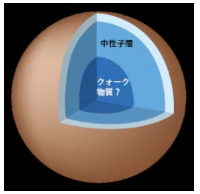


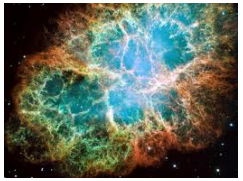
Few-Body / Light Nuclei





Heavy Nuclei





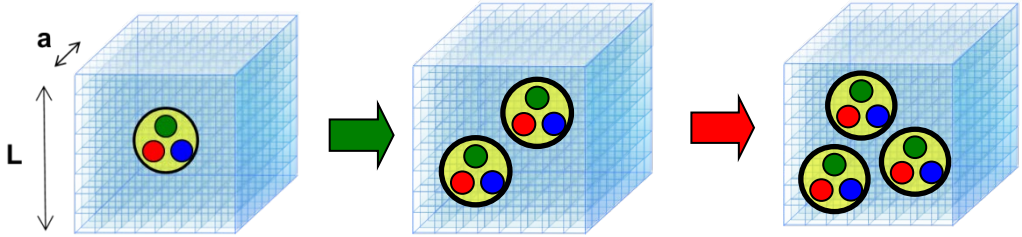
Neutron Star / Supernova

Standard Model

1st-principle lat calc.

Nuclear Forces

Ab-initio nuclear calc.



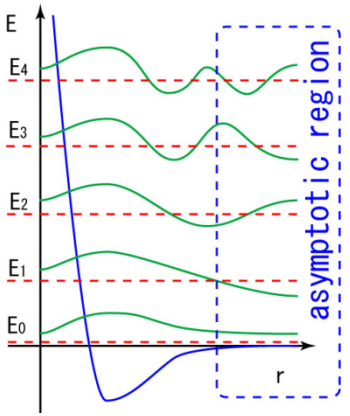
Lattice QCD predictions play a crucial role

Nuclear Forces from Lattice QCD [HAL QCD method]

- Potential is constructed so as to reproduce the NN phase shifts (or, S-matrix)
- Nambu-Bethe-Salpeter (NBS) wave function

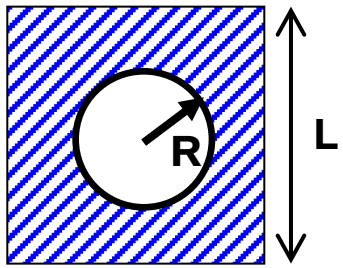
$$\psi(\vec{r}) = \langle 0|\ N(\vec{r})N(\vec{0})\ |N(\vec{k})N(-\vec{k}); in\rangle$$

$$(\nabla^2 + k^2)\psi(\vec{r}) = 0, \quad r > R$$



– Wave function ⇐⇒ phase shifts

$$\psi(r) \simeq A\frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$



M.Luscher, NPB354(1991)531

C.-J.Lin et al., NPB619(2001)467

CP-PACS Coll., PRD71(2005)094504

Ishizuka, PoS LAT2009 (2009) 119

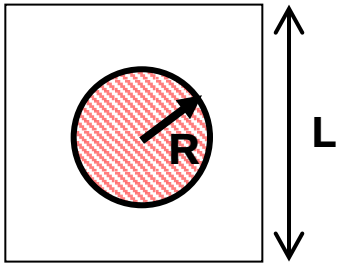
S.Aoki et al. (HAL Coll.), arXiv:1303.2210

⇐ Extension to multi-particle systems (talk by S.Aoki (Tue.))

“Potential” as a representation of S-matrix [HAL QCD method]

- Consider the wave function at “interacting region”

$$(\nabla^2 + k^2)\psi(\boldsymbol{r}) = m \int d\boldsymbol{r}' U(\boldsymbol{r}, \boldsymbol{r}')\psi(\boldsymbol{r}'), \quad r < R$$



  - U(r,r’): faithful to the phase shift by construction
    - U(r,r’): E-independent, while non-local in general
      - Non-locality ➔ derivative expansion

Aoki-Hatsuda-Ishii PTP123(2010)89

S.Aoki et al., PRD87(2013)034512

- Further improvement
  - **Time-dependent HAL QCD method w/o g.s. saturation**
    - U(r,r’): E-indep ➔ excited states give signal !

N.Ishii et al., PLB712(2012)437

  - **Unified contraction algorithm (UCA)**

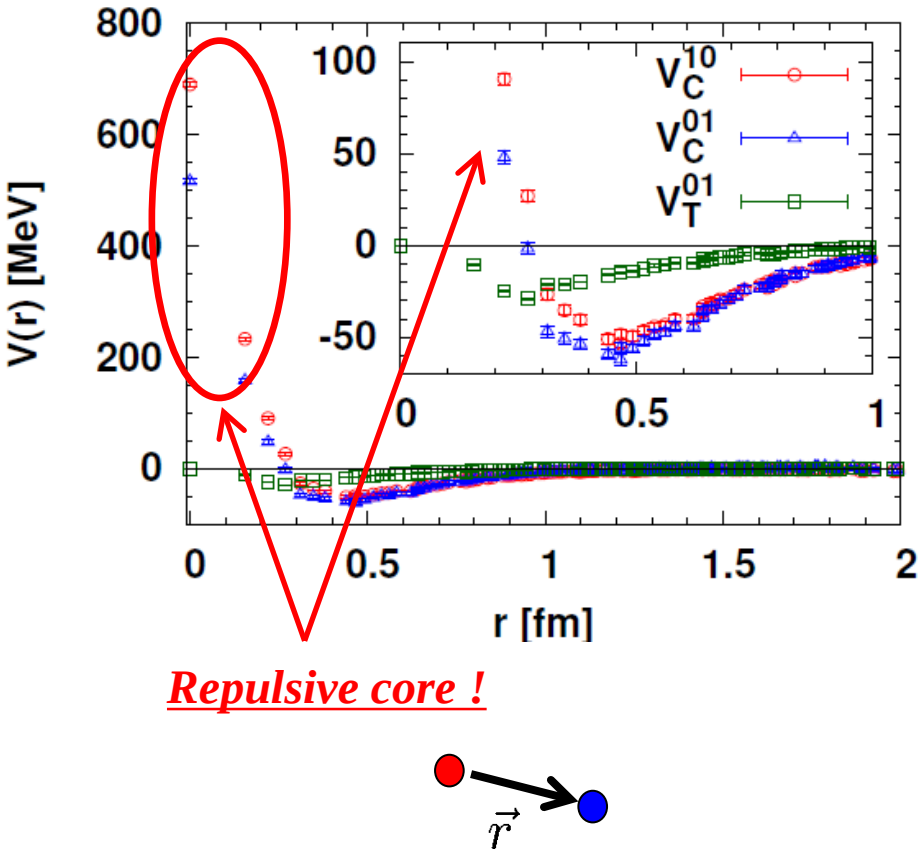
TD, M.Endres, Comput.Phys.Comm184(2013)117

    - Drastic speedup: $\times 192$ for ^{3}H/^{3}He, $\times 20736$ for ^{4}He, $\times 10^{11}$ for ^{8}Be

(Extension: next talks)

Results and Open Issues

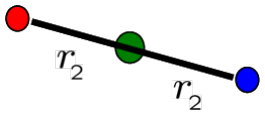
NN forces (central & tensor)

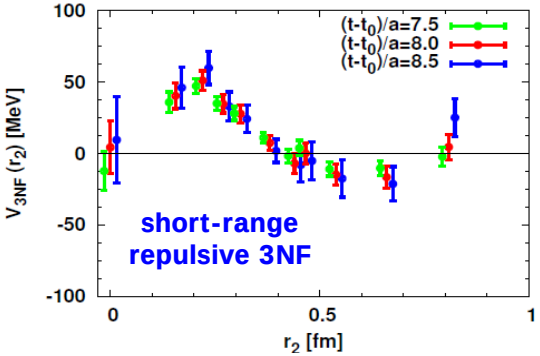


Repulsive core !

Nf=2 clover (CP-PACS), 1/a=1.27GeV,
L=2.5fm, $m\pi$=1.1GeV, m_N=2.1GeV

Short-range forces in 3N forces





Origin of repulsive core ?

Generalized BB forces in SU(3) limit on Lat

(T. Inoue et al. (HAL))

➔ Quark Pauli-blocking (+OGE)

(Oka-Yazaki)

OPE (Operator Product Expansion) study

(Aoki-Balog-Weisz)

➔ Divergent repulsive core

Lattice: How much is discretization error ?

➔ No study for cutoff effects

TD et al. (HAL Coll.) PTP127(2012)723 + t-dep method updates

Lattice simulation setup

- a^{-1}=0.9178GeV, a=0.2150fm (beta=1.80)
  - 12^3 X 24 lattice, L=2.6fm
  - 642 configs x 24 measurements , t+1=[1,12]
  - beta=1.80, Kappa(ud)=0.14090
    - Masses: (π, N, Δ) = (1.06, 2.08, 2.22) GeV
- a^{-1}=1.269GeV, a=0.1555fm (beta=1.95)
  - 16^3 X 32 lattice, L=2.5fm
  - 599 configs x 32 measurements, t+1=[5,13]
  - Kappa(ud)=0.13750
    - Masses: (π, N, Δ) = (1.13, 2.15, 2.31) GeV
- a^{-1}=1.834GeV, a=0.1076fm (beta=2.10)
  - 24^3 X 48 lattice, L=2.6fm
  - 798 configs x 48 measurements, t+1=[7,18]
  - Kappa(ud)=0.13570
    - Masses: (π, N, Δ) = (1.16, 2.17, 2.34) GeV

Nf=2 dynamical clover fermion + RG improved gauge

CP-PACS Coll. S. Aoki et al., Phys. Rev. D65 (2002) 054505

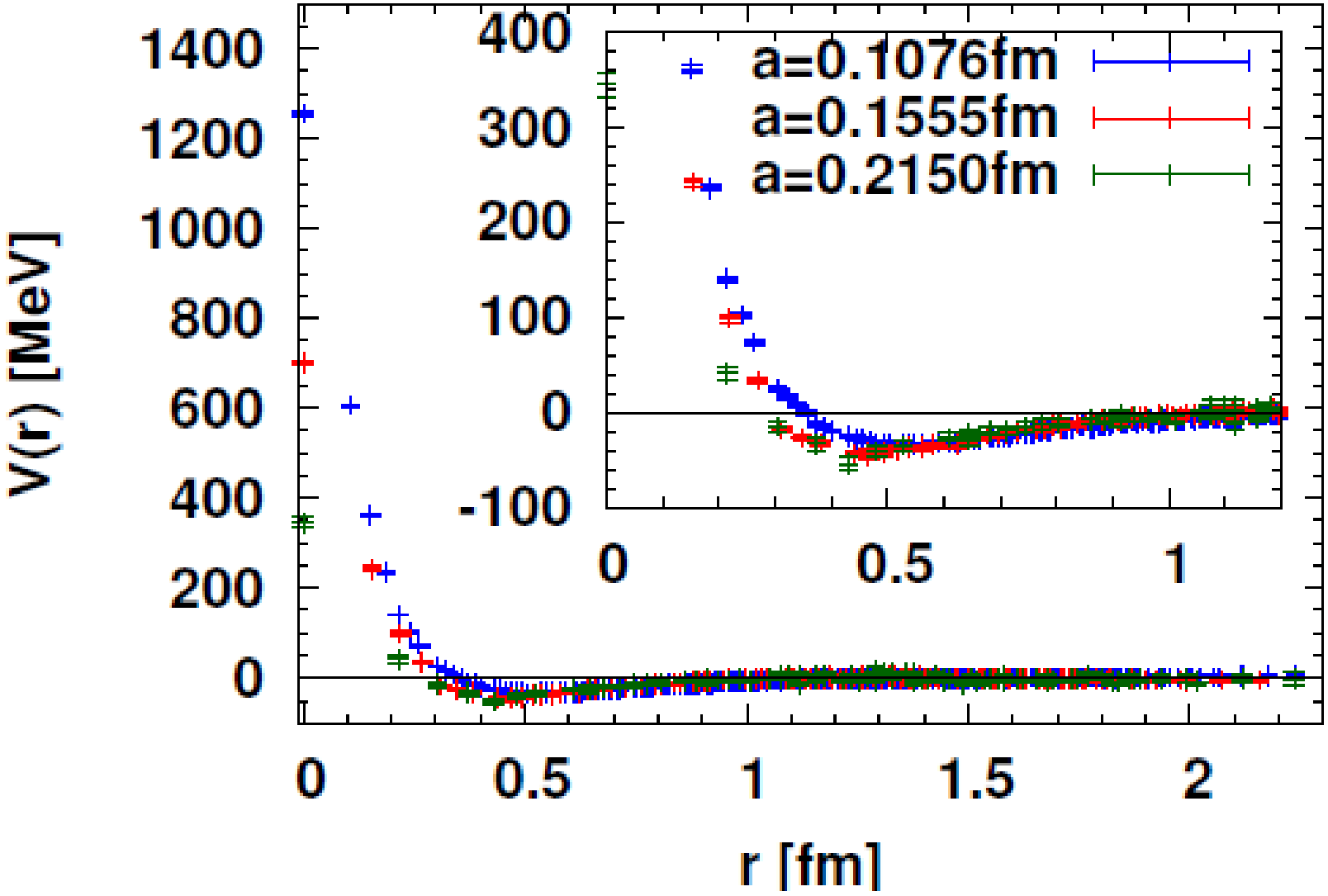
~40% finer

(Same volume & mass)

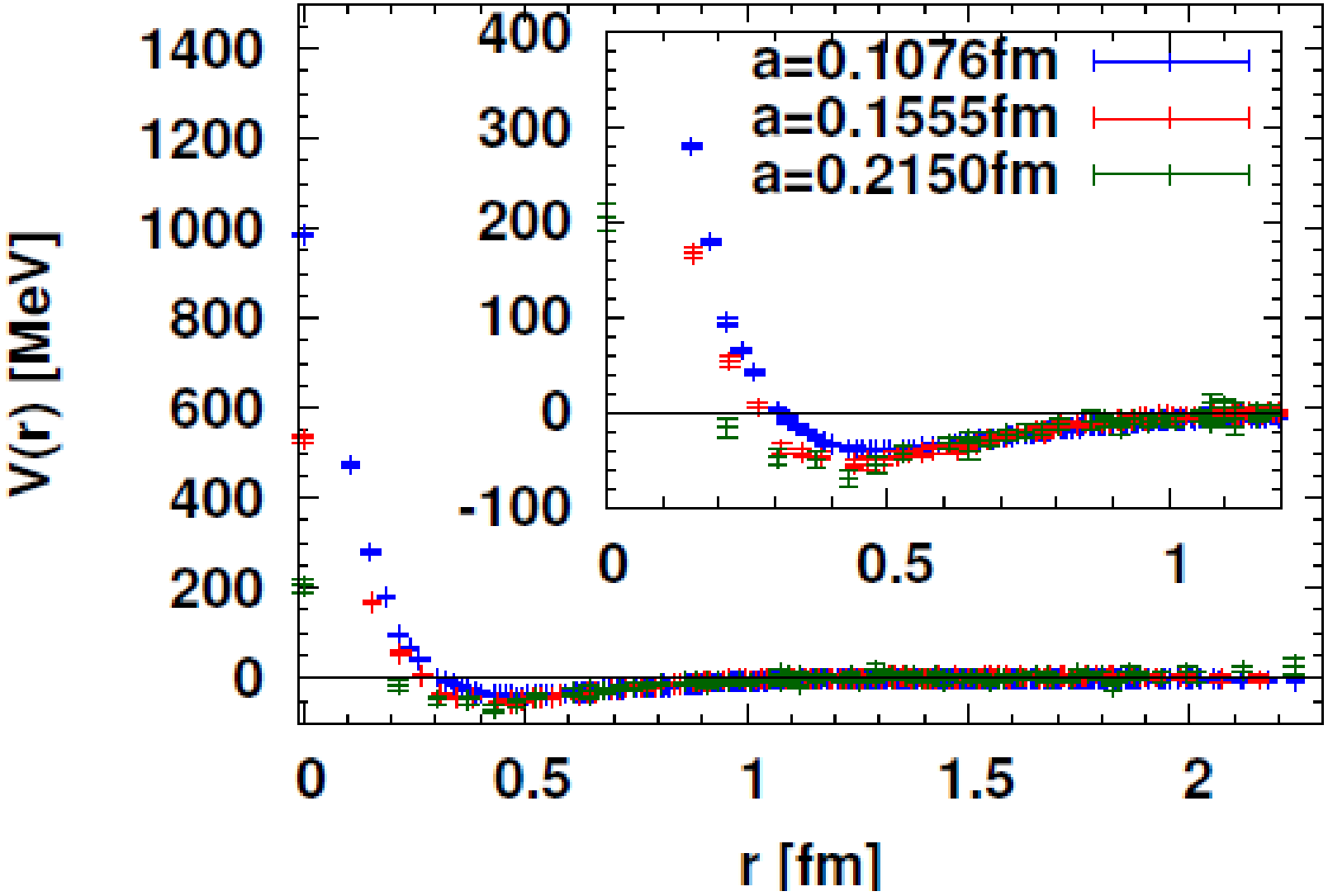
~40% finer

Wall source w/ Coulomb gauge

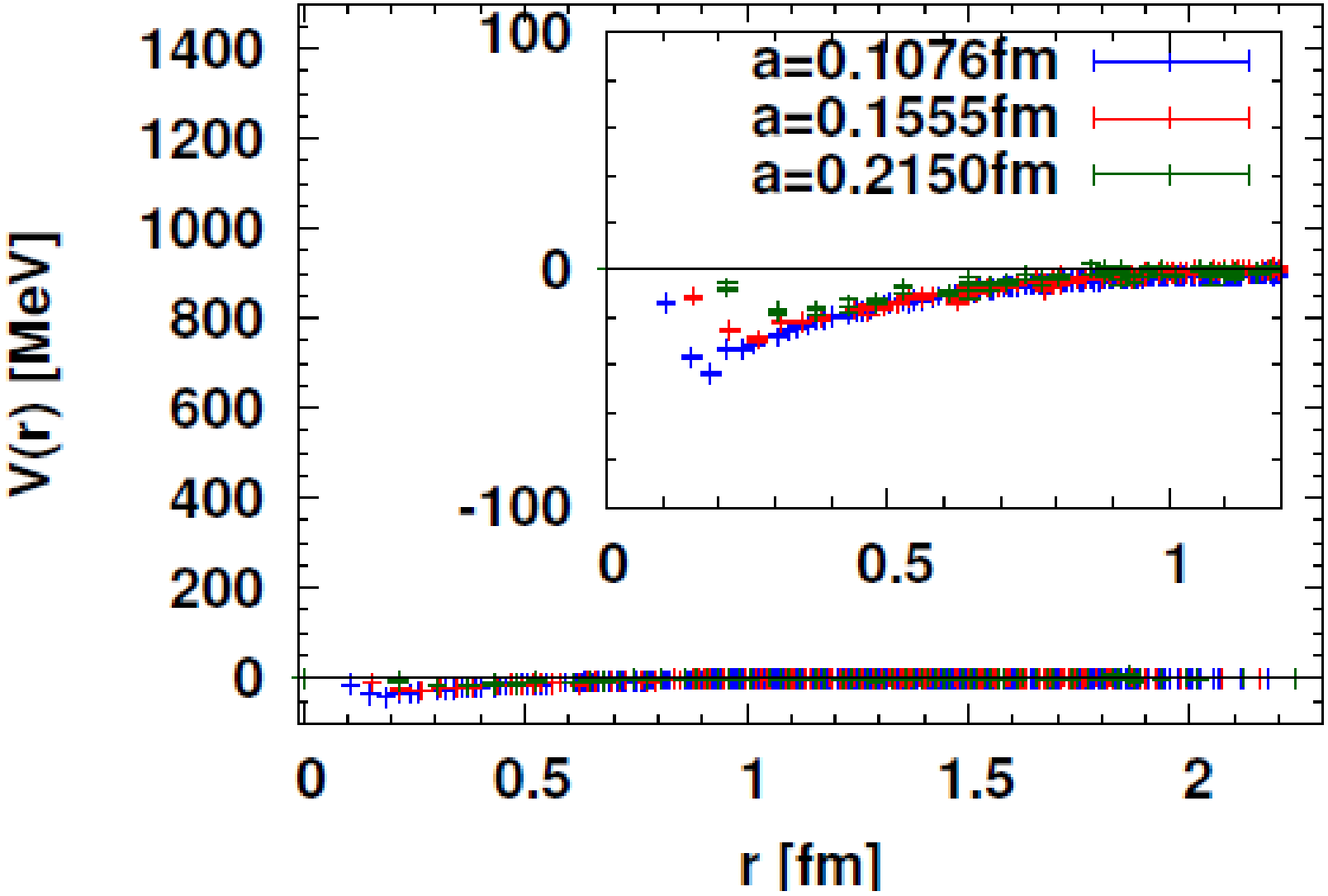
1S0 central



3S1-3D1 central

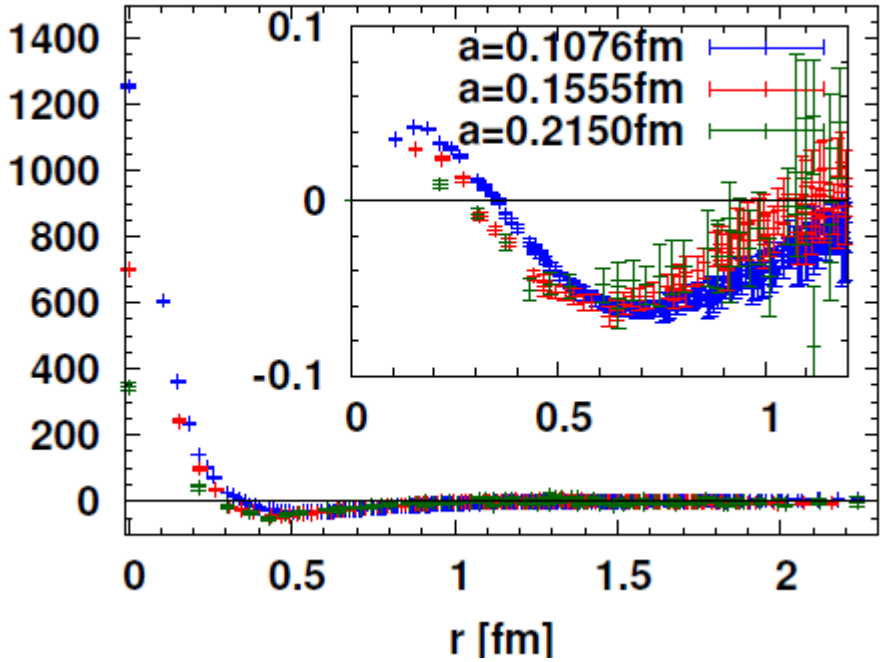


3S1-3D1 tensor

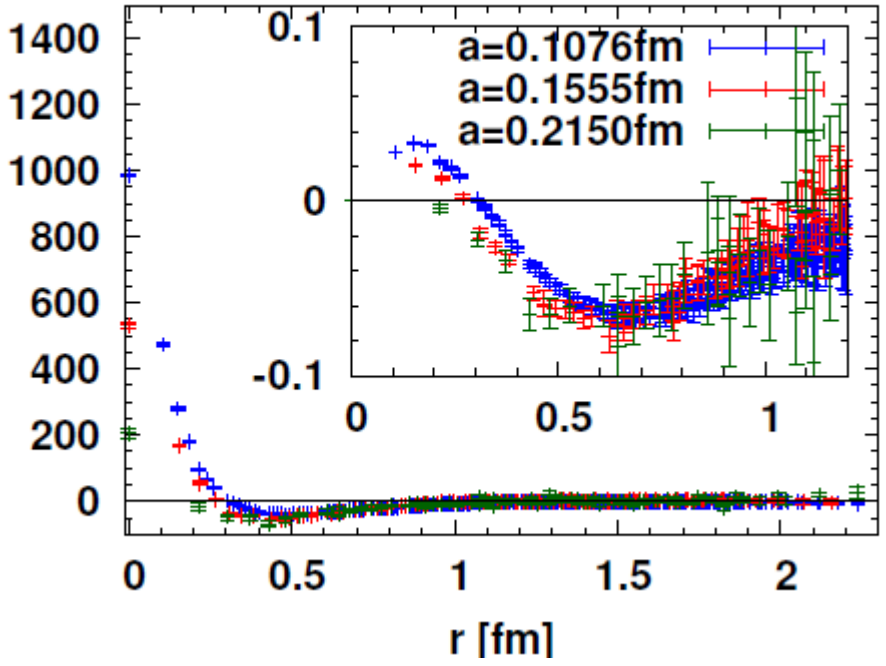


V(r) vs. r^2 V(r)

1S0 Central



3S1-3D1 Central

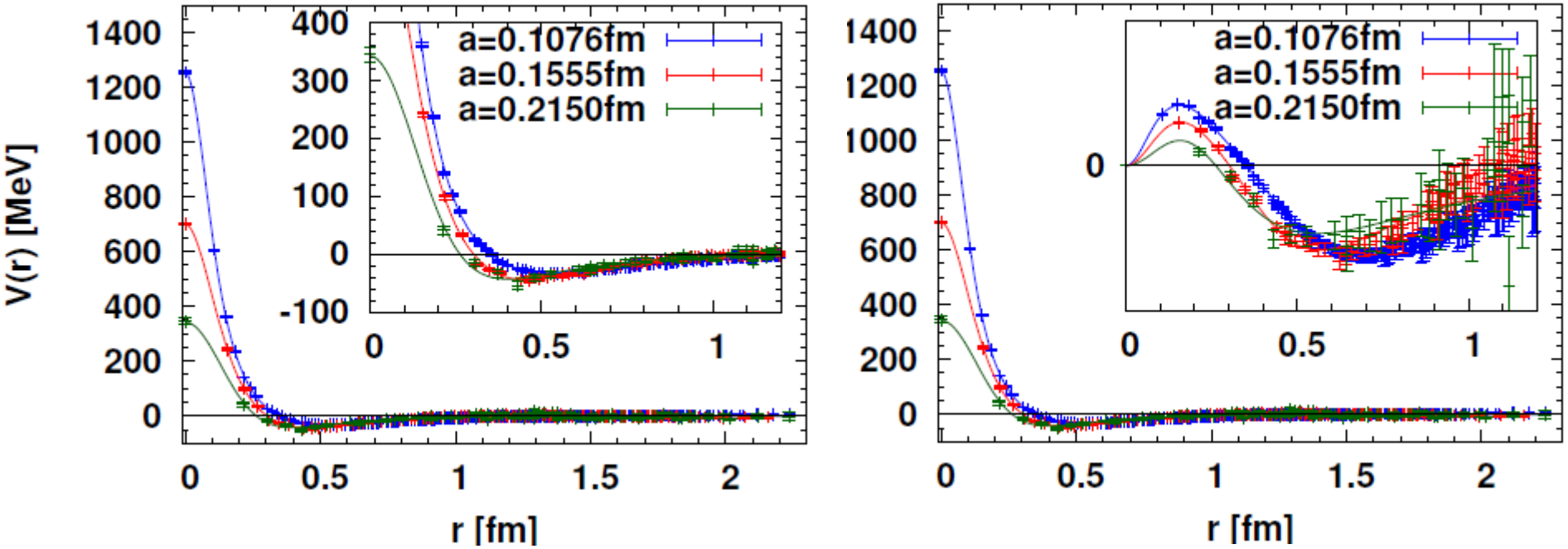


Outer figs: V(r) [MeV]
Inner figs: r^2 V(r) [fm]

Cutoff effects suppressed by phase-space, but still non-negligible

Effect on phase shifts

1S0 Central



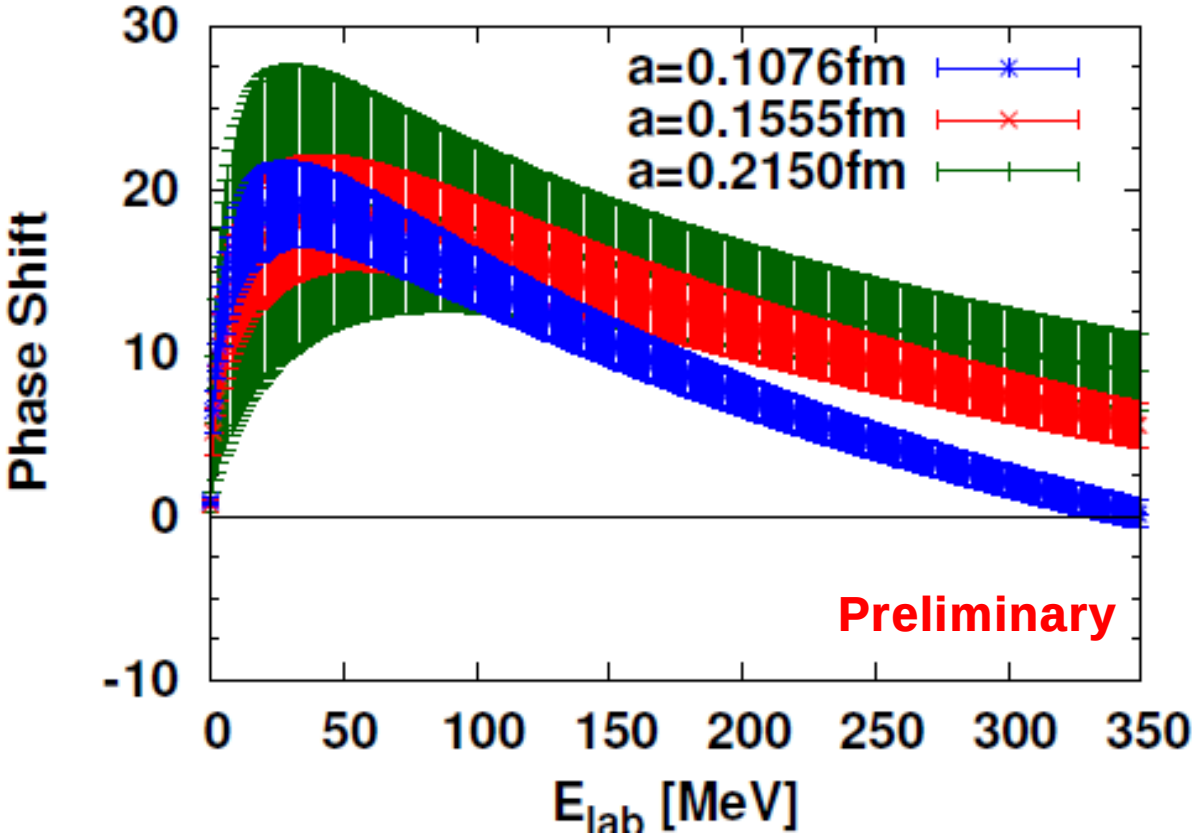
Fit function: 2-range Gauss + Yukawa^2

$$V(r) = b_1 e^{-b_2 r^2} + b_3 e^{-b_4 r^2} + b_5 (1 - e^{-b_6 r^2})^2 \left(\frac{e^{-b_7 r}}{r} \right)^2$$

(Other fit function works as well)

Effect on phase shifts

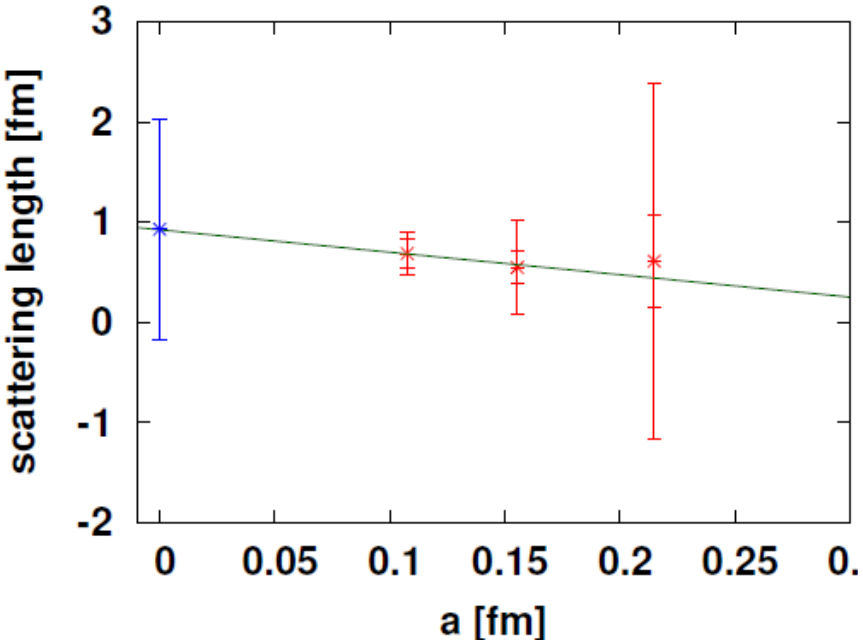
1S0 Central

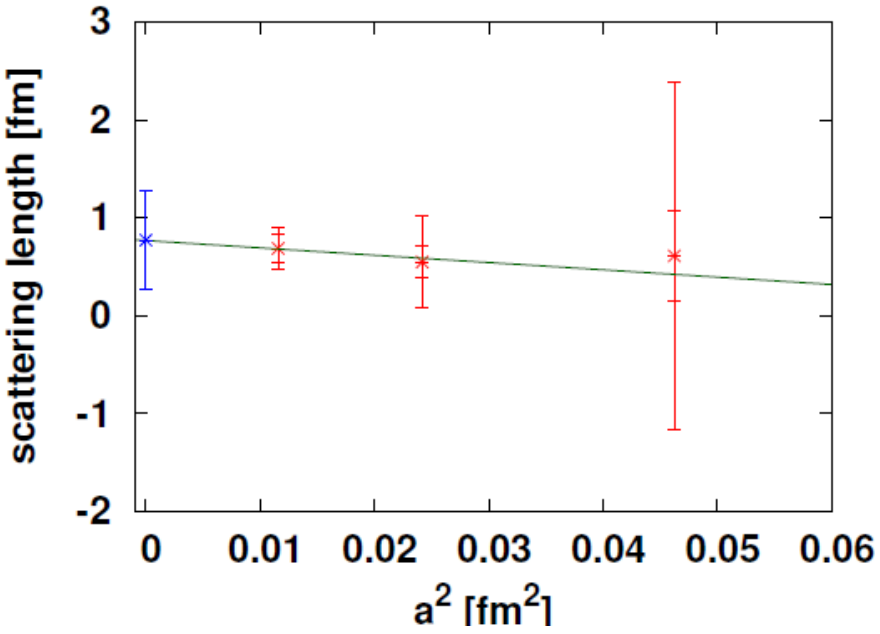


➔ Cutoff dependence appears at higher energies

Effect on scattering length

1S0 Central





Preliminary

➔ Cutoff dependence on scatt length is mild

c.f. I=2 pipi : O(a)-dependence
T.Yamazaki et al., PRD70(2004)074513

Summary

- **We have studied cutoff effects on NN-forces on the lattice**
  - Nf=2 dynamical clover fermion at $m\pi >=\sim$ 1.1 GeV
  - Three cutoffs: 1/a = 0.92, 1.27, 1.83 GeV
  - Potentials themselves have large cutoff dependence at very short range
  - Effects on phase shifts get milder by phase space suppression, but still non-negligible at higher energies
- Outlook
  - Refined analysis w/ dispersion relation breaking effect (at lighter masses)
  - Other channels (3S1-3D1, etc.)
  - Effect on Three-nucleon forces