Lattice 2013
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Mainz

Multi-nucleon bound states in $N_f=2+1$ lattice QCD

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- Aims and motivations
- Issues
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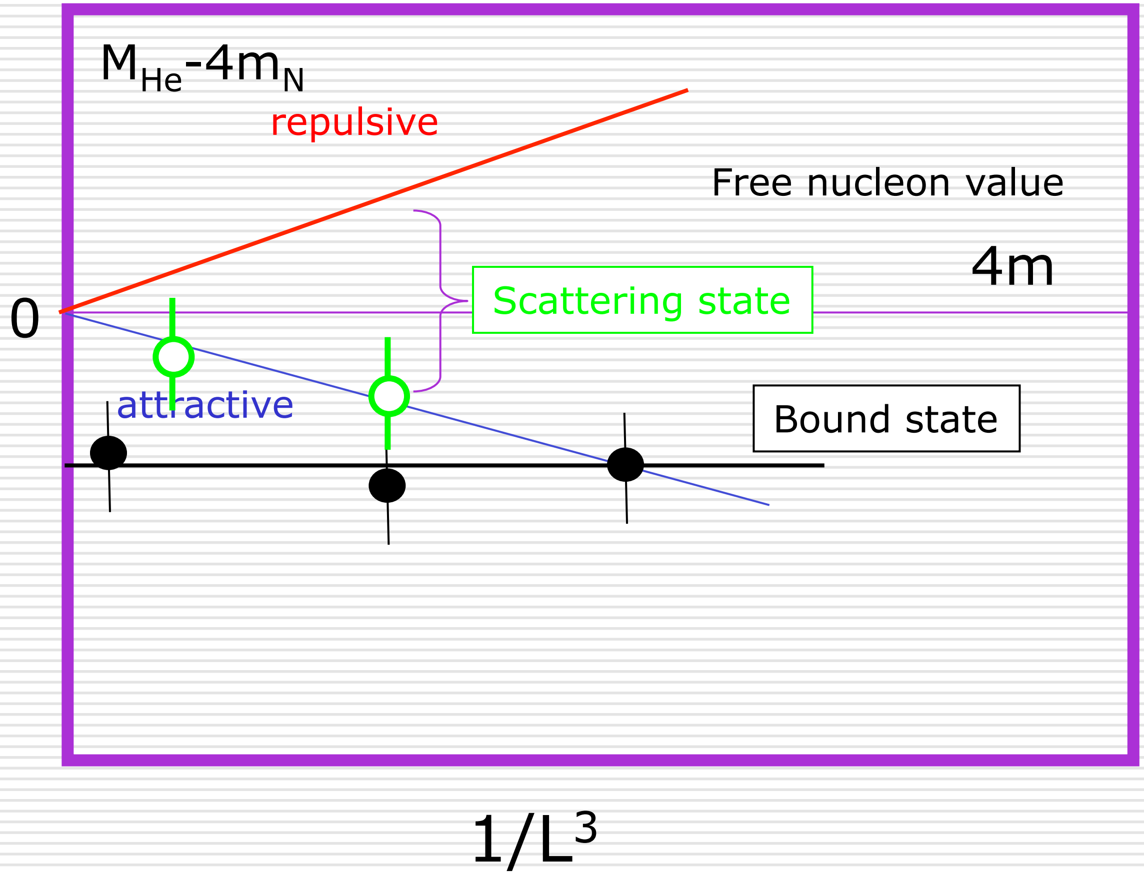
- Results
- Summary

Aim and motivation

- *Direct calculation of nuclear properties from quarks, gluons, and lattice QCD*
- *Only method with true reliability to discuss unnatural nuclei with large neutron/proton ratio*
- *Only method with true reliability to discuss the fate of nuclei if the standard model parameters (coupling constants, quark masses, etc) were different from what they are in Nature*

Issue (I): Bound state or scattering state?

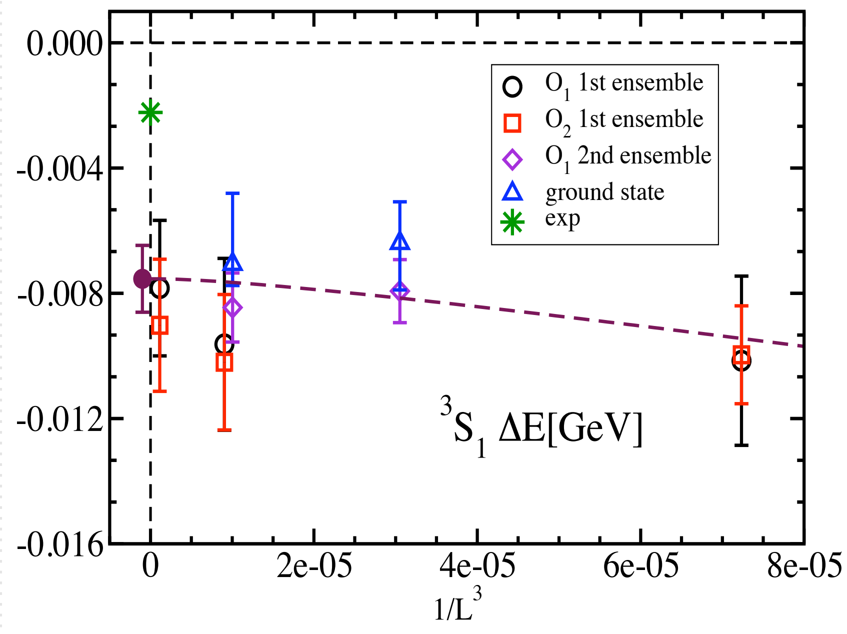
- Measurement for a single spatial volume cannot distinguish a bound state from a scattering state
- Use multiple volumes to distinguish the infinite volume limit
- Can also make a cross check with excited state



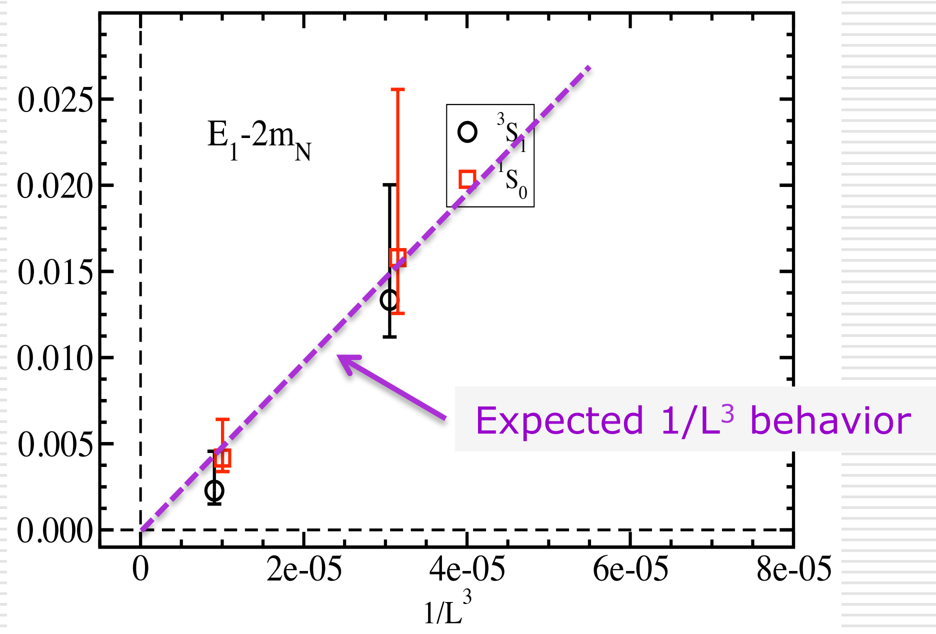
Bound ground state and 1st excited state

T. Yamazaki, Y. Kuramashi, A. Ukawa, PRD84, 054506 (2011)

Deuteron is a bound state (quenched QCD, m_π=0.8GeV)



1st excited state is a scattering state just above the threshold



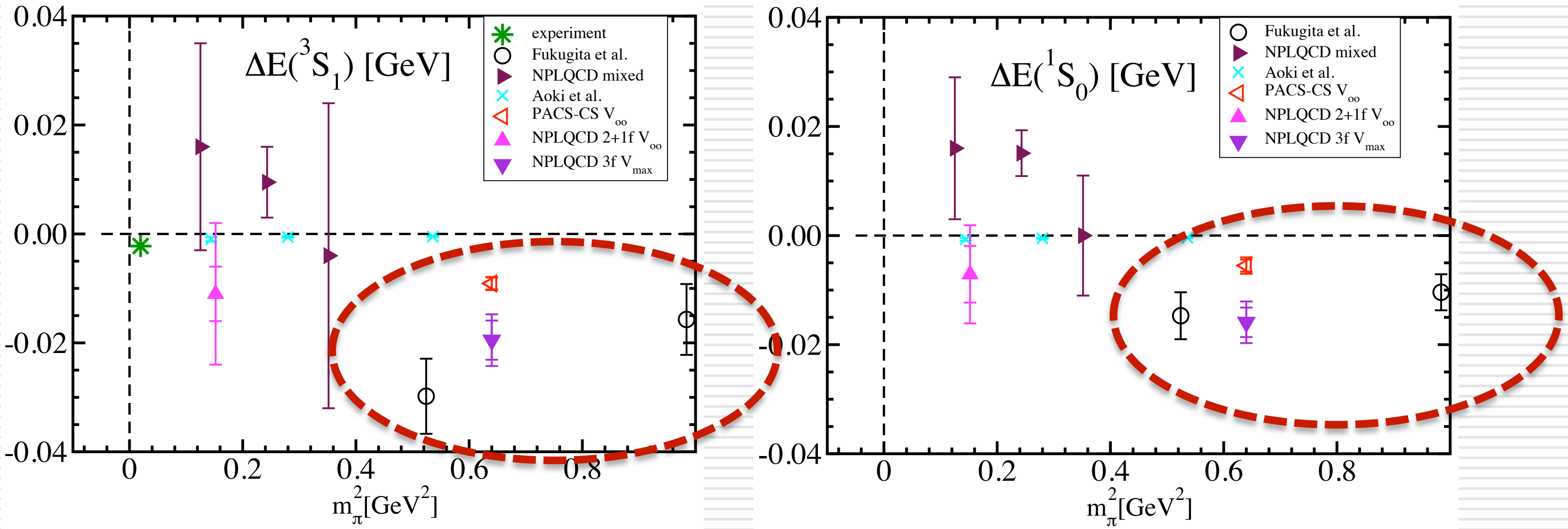
$a_0 < 0$ evaluated from the 1st excited state energy consistent with bound state formation

$a_0[\mathrm{fm}](^3\mathrm{S}_1)$	$a_0[\mathrm{fm}](^1\mathrm{S}_0)$
$-1.05(24)(^{+0.05}_{-0.65})$	$-1.62(24)(^{+0.01}_{-0.75})$

also confirmed recently by NPLQCD N_f=3 m_π=0.81GeV, arXiv:1301.5790

Issue (II): quark mass dependence

- Lattice results at heavy quark masses tend to indicate bound states *irrespective of the nucleon number and spin*
- Absence of bound states in some channels, e.g., di-neutron, requires dynamical explanation



- Raises the question *if nuclei are bound for zero quark mass?*

Issue (III): quark contractions

- Factorial growth for larger nuclei

$$N_u! \times N_d!$$

- up to A=4 (Helium), manageable by reduction through symmetries and other techniques

  e.g., T. Yamazaki, Y. Kuramashi, A. Ukawa, PRD81, 111504(R) (2010)

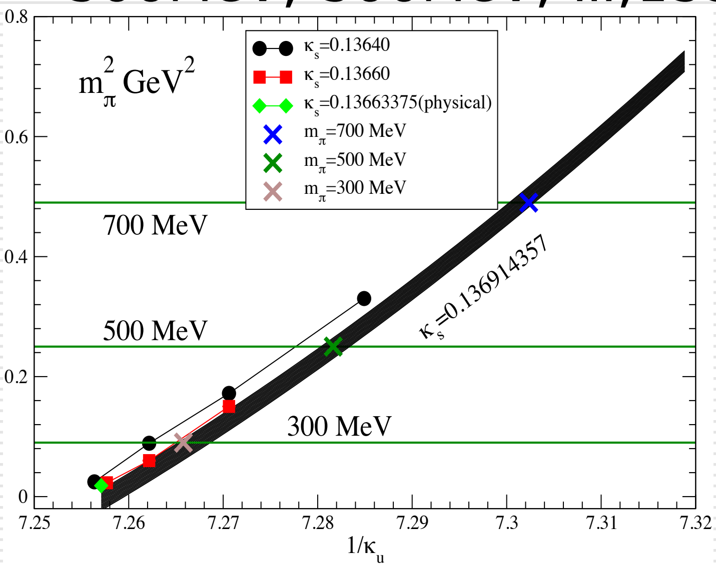
He	$6! \times 6! = 512,000$	→	$1,107$
He^3	$5! \times 4! = 2,880$		93

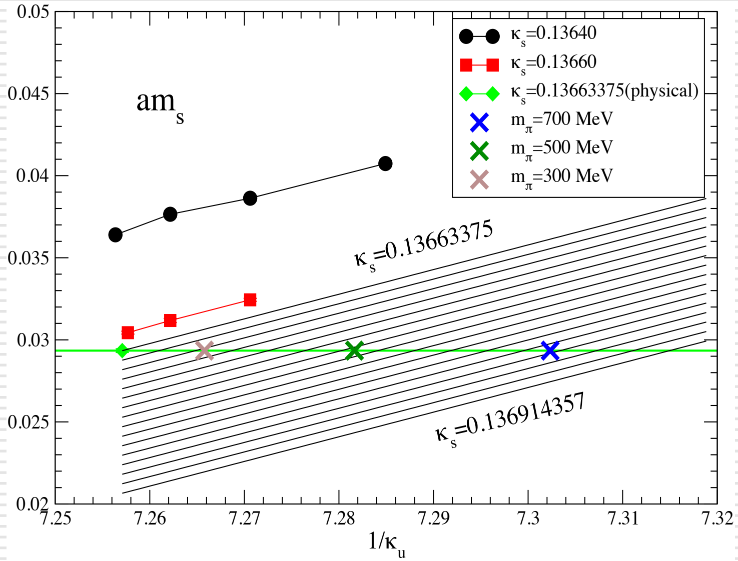
- For larger A, requires more systematic approach such as recursive counting

  T. Doi and M. Endres, CPC184, 117 (2013);
  W. Detmold and K. Orginos, PRD87, 114512 (2013);
  J. Gunther, B. Toth, and L. Varnhorst, PRD87, 094513 (2013)

Strategy of simulation

- N_f=2+1 QCD simulation
  - Wilson-clover quark action and Iwasaki gluon action
  - strange quark mass fixed at physical value
  - up/down quark mass gradually reduced, m_π=700MeV, 500MeV, 300MeV, …,135MeV





  - Several spatial sizes e.g., L=32, 40, 48, 64, at each pion mass to distinguish bound/scattering states

Some details of measurements

- Smeared quark source
  - $\psi(r) = A\exp(-Br)$
  - Smearing parameter B adjusted to ensure an early and good plateau for nucleon (examples shown later)

- Multiple source locations for each configuration to increase statistics
  - Multiple times slices
  - Multiple spatial locations at each time slices
  - Use all four directions as time for *space-time symmetric L^4 lattices*

Run statistics

- m_π=0.51GeV(done)

L	#conf	#sep	#bin	#meas/conf	M_π(GeV)	m_N(GeV)
32	200	20	10	192	0.5109(16)	1.318(4)
40	200	10	10	192	0.5095(8)	1.314(4)
48	200	10	20	192	0.5117(9)	1.320(3)
64	190	10	19	256	0.5119(4)	1.318(2)

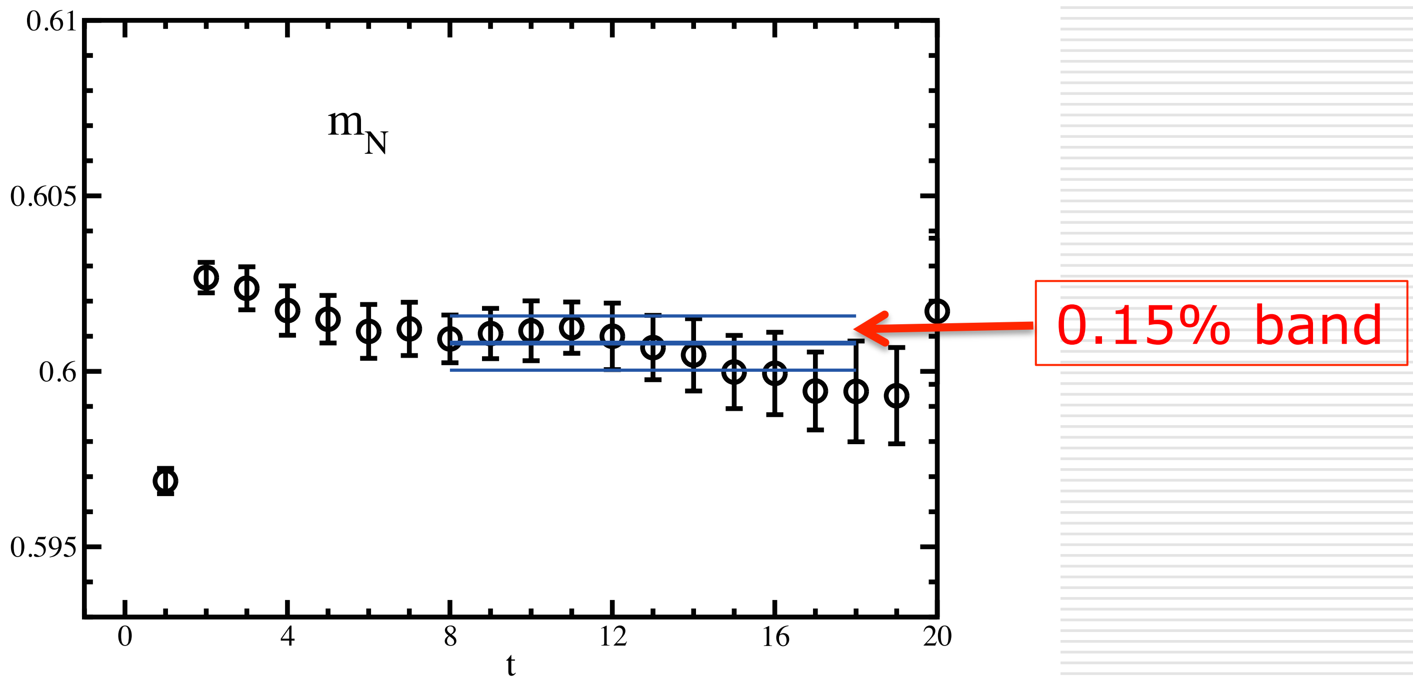
- m_π=0.30GeV (still running)

L	#conf	#sep	#bin	#meas/conf	M_π(GeV)	m_N(GeV)
48	360	10	20	576	0.3004(15)	1.058(2)
64	160	10	10	384	0.2985(8)	1.057(2)

Results at m_π=0.51GeV (I)

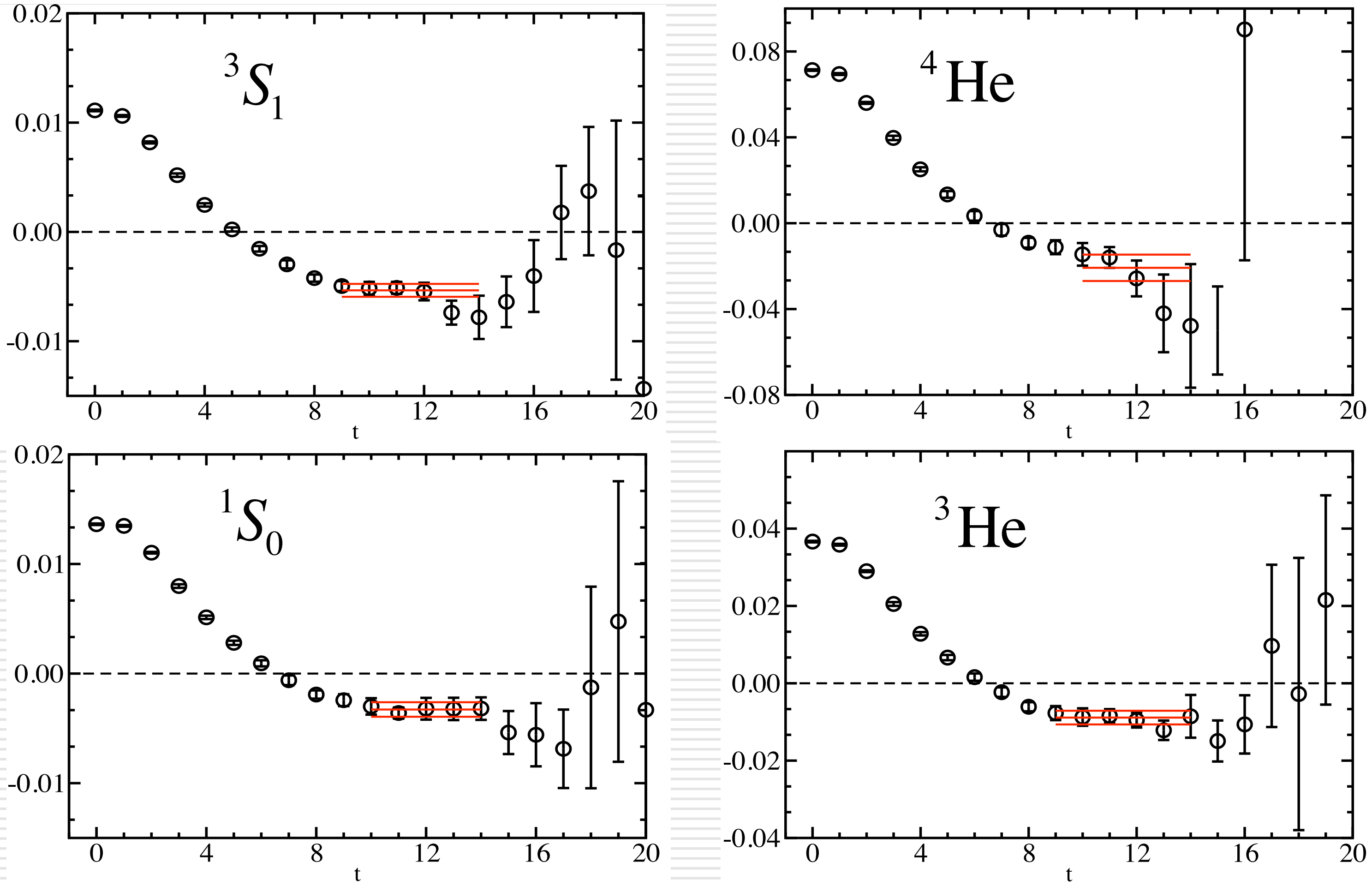
- Tuning of smearing parameter to ensure a good nucleon effective mass plateau

L=64 lattice, 190 configs, 256 sources/config



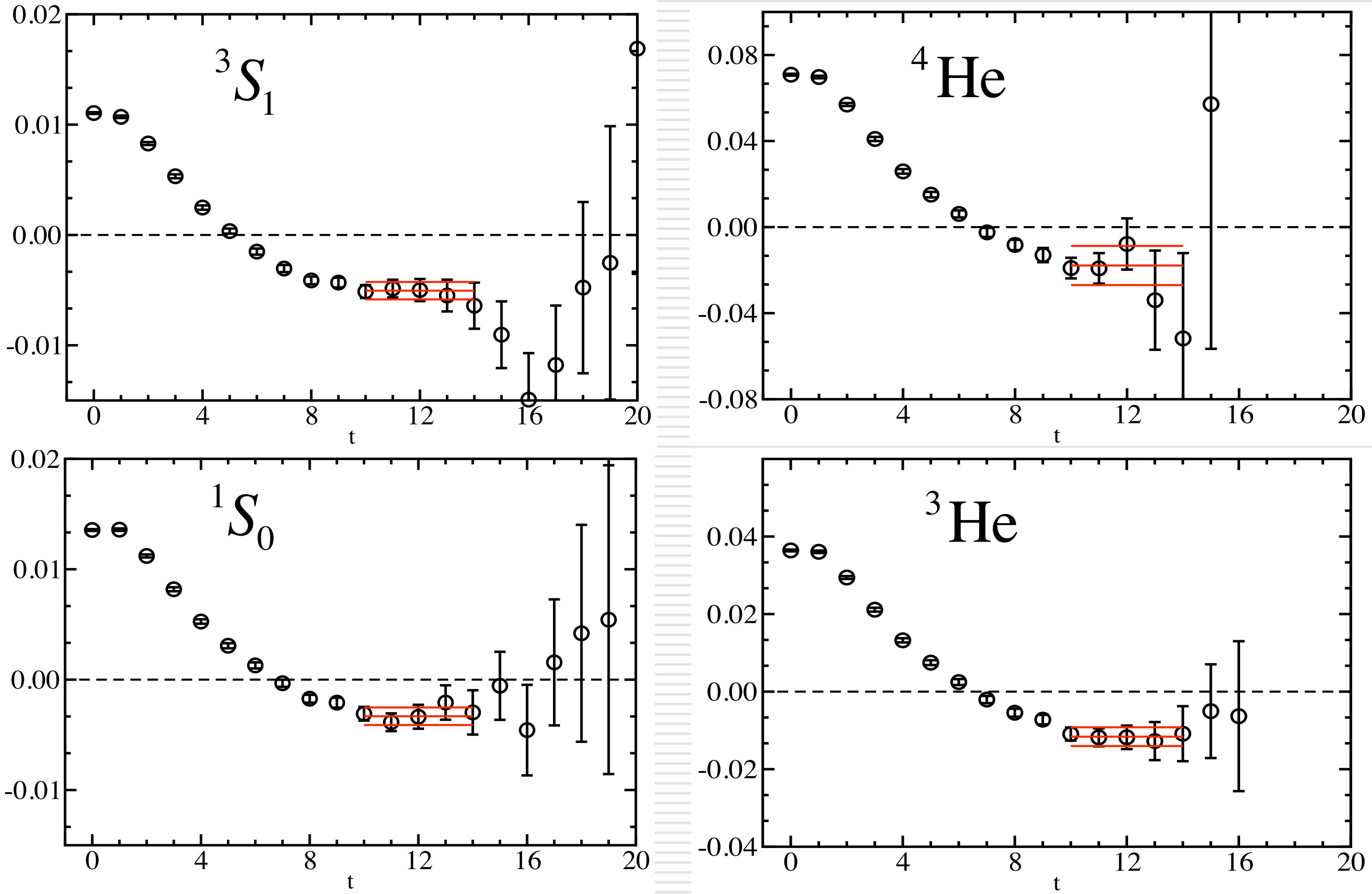
Results at m_π=0.51GeV (II)

- L=64 effective masses



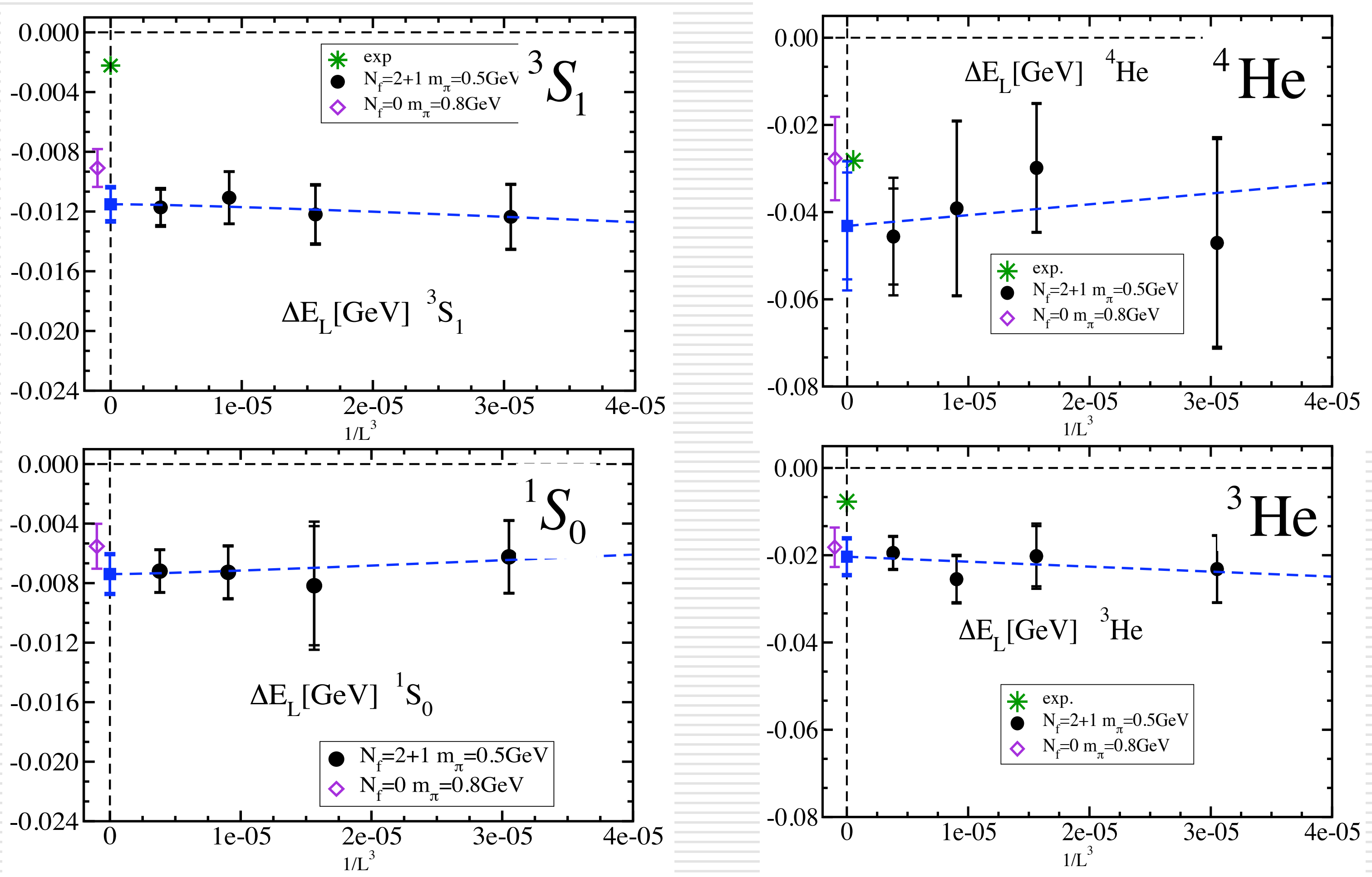
Results at m_π=0.51GeV (III)

- L=48 effective masses



Results at m_π=0.51GeV (IV)

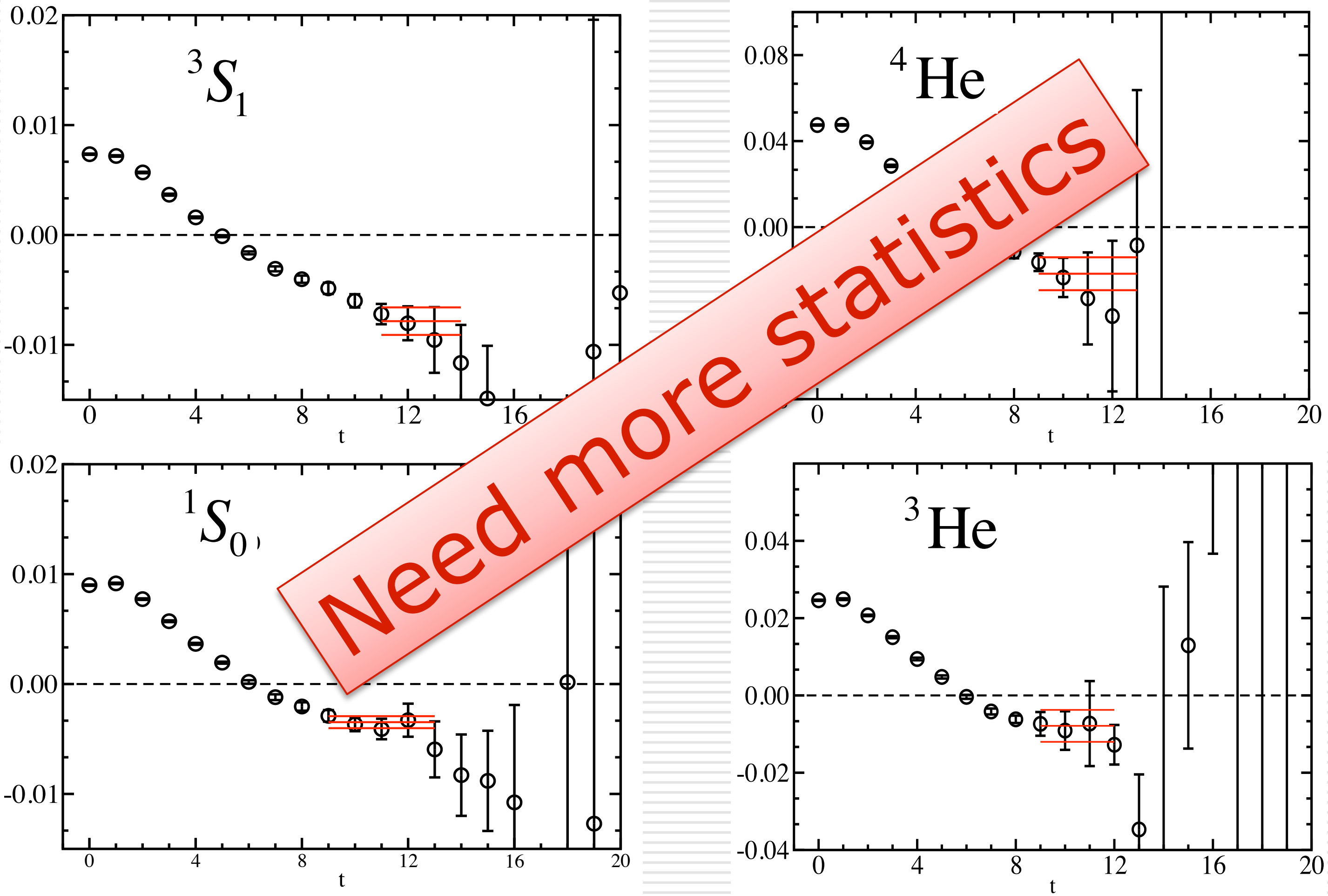
- Volume extrapolations



Still bound state in all channels

Results at m_π=0.30GeV (preliminary)

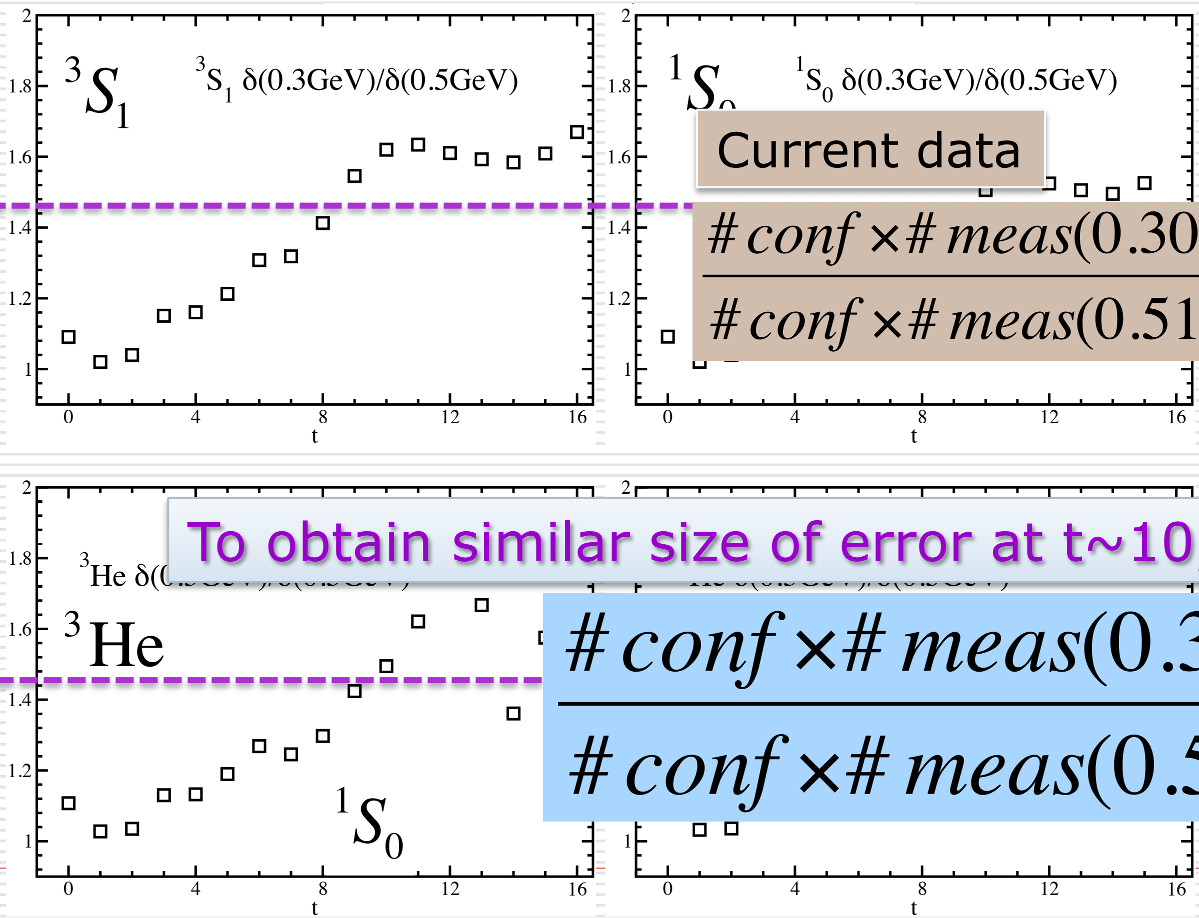
- L=48 effective masses



Results at m_π=0.30GeV (preliminary) (II)

- Comparison of relative errors for m_π=0.30GeV and m_π=0.51GeV (L=48)

$$\delta_H(t) = \frac{\delta C_H(t)}{C_H(t)}$$



Current data

$$\frac{\#conf \times \# meas(0.30GeV)}{\#conf \times \# meas(0.51GeV)} \approx 8$$

statistics

To obtain similar size of error at t~10, we need

$$\frac{\#conf \times \# meas(0.30GeV)}{\#conf \times \# meas(0.51GeV)} \approx 16$$

Summary

- *Attempting a direct calculation of light nuclear properties from lattice QCD*
- *Strategy toward the physical point:*
  - *step-wise reduction of up/down quark mass while keeping strange quark mass at the physical point*
  - *Volume analysis at each quark mass*
- *Successful calculation at m_π=0.51GeV*
- *Now running at m_π=0.30GeV; significant increase in statistics appears needed*