Universität Bielefeld

Temperature dependence of the electrical conductivity and dilepton rates from hot quenched lattice QCD

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in collaboration with
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Lattice 2013
Mainz
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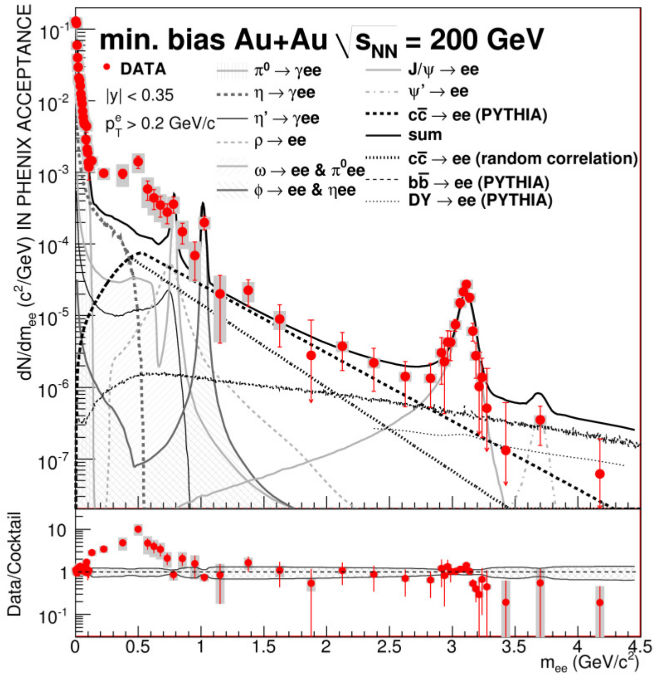
Motivation – PHENIX/STAR results for the low-mass dilepton rates

pp-data well understood by hadronic cocktail

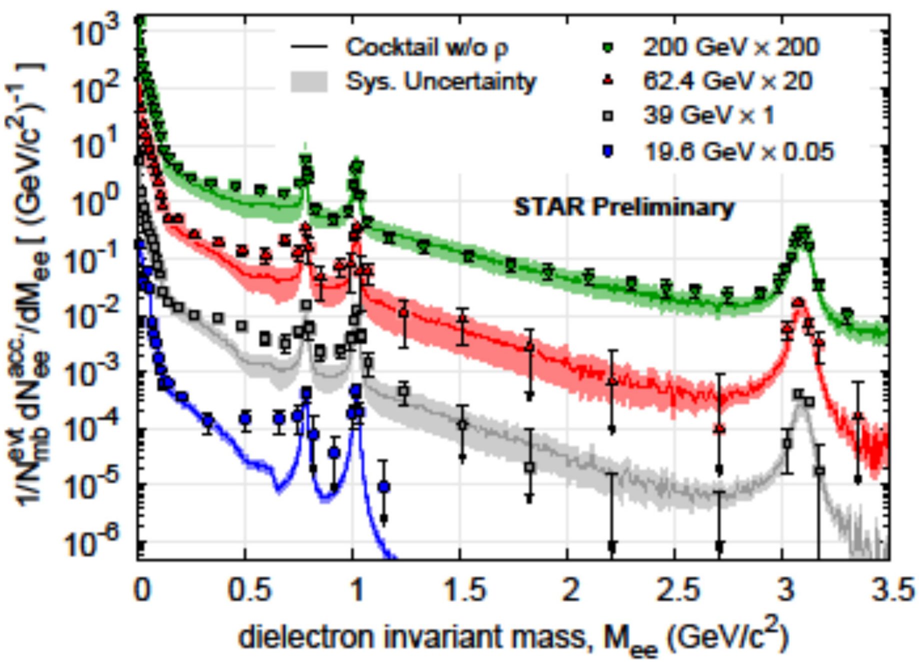
large enhancement in Au+Au between 150-750 MeV

indications for thermal effects!?

Need to understand the contribution from QGP→ spectral functions from lattice QCD



[PHENIX PRC81, 034911 (2010)]



[STAR preliminary, arXiv:1210.5549]

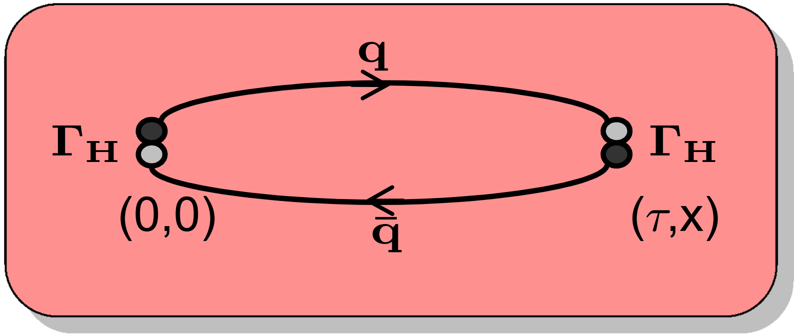
Dileptonrate directly related to vector spectral function:

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3} \frac{1}{(\omega^2 - \vec{p}^2)(e^{\omega/T} - 1)} \rho_{\mathbf{V}}(\omega, \vec{\mathbf{p}}, \mathbf{T})$$

Vector correlation functions at high temperature

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\rho(\omega,\vec{p},T)K(\tau,\omega,T)$$

$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau-\frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$



Lattice observables:

$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x})J_\nu^\dagger(0,\vec{0})\rangle$$

$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V\bar{\psi}(\tau,\vec{x})\Gamma_\mu\psi(\tau,\vec{x})$$

← local, non-conserved current, needs to be renormalized

$$G_{\mu\nu}(\tau,\vec{p}) = \sum_{\vec{x}} G_{\mu\nu}(\tau,\vec{x})e^{i\vec{p}\vec{x}}$$

← only $\vec{p}=0$ used here

How to extract spectral properties from correlation functions?

Spectral functions at high temperature

Free theory (massless case):

free non-interacting vector spectral function (infinite temperature):

$$\begin{aligned}\rho_{00}^{free}(\omega) &= 2\pi T^2 \omega \delta(\omega) \\ \rho_{ii}^{free}(\omega) &= 2\pi T^2 \omega \delta(\omega) + \frac{3}{2\pi}\,\omega^2 \tanh(\omega/4T)\end{aligned}$$

δ-functions exactly cancel in $\rho_V(\omega)$=-$\rho_{00}(\omega)$+$\rho_{ii}(\omega)$

With interactions (but without bound states):

while ρ_{00} is protected, the δ-funtion in ρ_{ii} gets smeared:

$\kappa = \frac{\alpha_s}{\pi}$ at leading order

Ansatz:

$$\begin{aligned}\rho_{00}(\omega) &= 2\pi\chi_q \omega \delta(\omega) \\ \rho_{ii}(\omega) &= 2\chi_q c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\,\tanh(\omega/4T)\end{aligned}$$

Ansatz with 3-4 parameters: $(\chi_q), c_{BW}, \Gamma, \kappa$

[“Thermal dilepton rate and electrical conductivity…”, H.T.-Ding, OK et al., PRD83 (2011) 034504]

Electrical Conductivity

Electrical Conductivity ⟷ slope of spectral function at ω=0 (Kubo formula)

$$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p}=0, T)}{\omega T}$$

$$C_{em} = e^2 \sum_{f=1}^{n_f} Q_f^2 \quad = \quad \begin{array}{lll} 5/9\ e^2 & \text{for} & n_f = 2 \\ 6/9\ e^2 & \text{for} & n_f = 3 \end{array}$$

Using our Ansatz for $\rho_{ii}(\omega)$:

$$\frac{\sigma}{T} = \frac{2}{3} \frac{\chi_q}{T^2} \frac{T}{\Gamma} c_{BW}\ C_{em}$$

Vector correlation function on large & fine lattices

[H.T.-Ding, OK et al., PRD83 (2011) 034504]

Quenched SU(3) gauge configurations at $T/T_c=1.5$ (separated by 500 updates)

Lattice size $N_\sigma^3 N_\tau$ with $N_\sigma = 32 – 128$
$N_\tau = 16, 24, 32, 48$ Temperature: $T = \frac{1}{aN_\tau}$

Non-perturbatively O(a) clover improved Wilson fermions

Non-perturbative renormalization constants

Quark masses close to the chiral limit, $\kappa \simeq \kappa_c \Leftrightarrow m_{\overline{MS}}/T[\mu=2\text{GeV}] \approx 0.1$

Volume dependence

N_τ	N_σ	β	c_{SW}	κ	Z_V	$1/a$[GeV]	a[fm]	#conf
16	32	6.872	1.4124	0.13495	0.829	6.43	0.031	60
16	48	6.872	1.4124	0.13495	0.829	6.43	0.031	62
16	64	6.872	1.4124	0.13495	0.829	6.43	0.031	77
16	128	6.872	1.4124	0.13495	0.829	6.43	0.031	129
24	128	7.192	1.3673	0.13440	0.842	9.65	0.020	156
32	128	7.457	1.3389	0.13390	0.851	12.86	0.015	255
48	128	7.793	1.3104	0.13340	0.861	19.30	0.010	431

cut-off dependence & continuum extrapolation

close to continuum

New results at 1.1 and 1.2 T_c

[M.Müller et al., arXiv:1301.7436]

PRACE-Project:
Thermal Dilepton Rates and
Electrical Conductivity in the QGP
(JUGENE Bluegene/P in Jülich)

	$1.1\ T_c$	$1.2\ T_c$					
N_σ	N_τ	N_τ	β	κ	$1/a$[GeV]	a[fm]	#Confs
96	32	28	7.192	0.13440	9.65	0.020	250
144	48	42	7.544	0.13383	13.21	0.015	300
192	64	56	7.793	0.13345	19.30	0.010	240

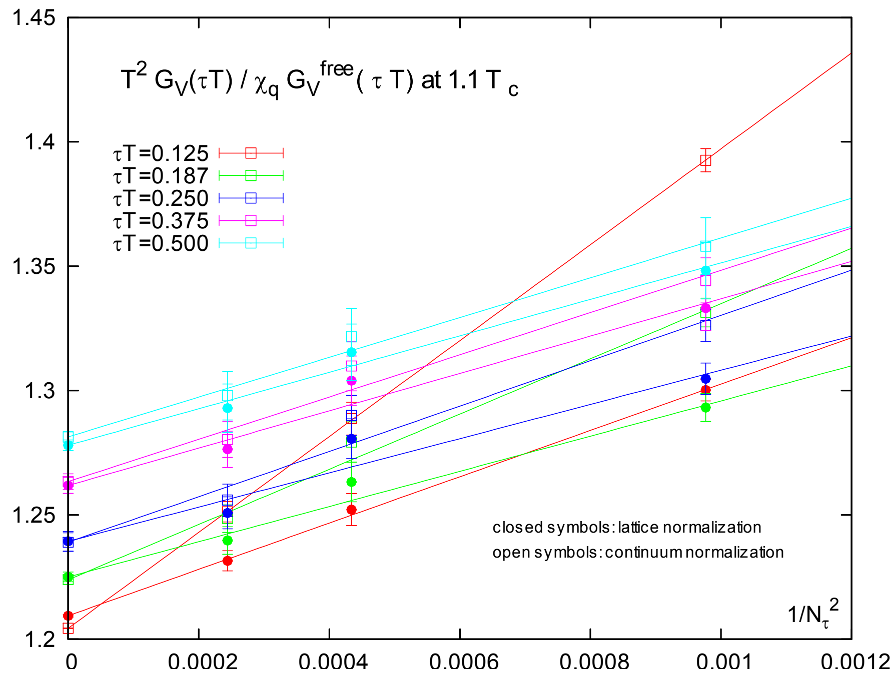
study of T-dependence of dilepton rates and electrical conductivity

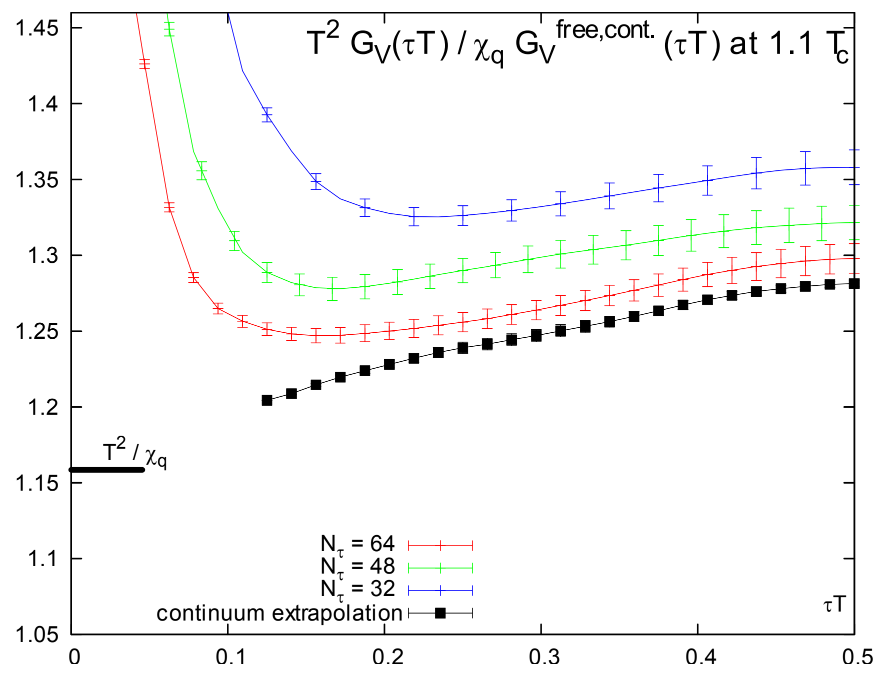
fixed aspect ratio $N_\sigma/N_\tau = 3$ to allow continuum limit at finite momentum:

$$\frac{\vec{p}}{T} = 2\pi \vec{k} \frac{N_\tau}{N_\sigma}$$

constant physical volume $(1.9\text{fm})^3$

Continuum extrapolation





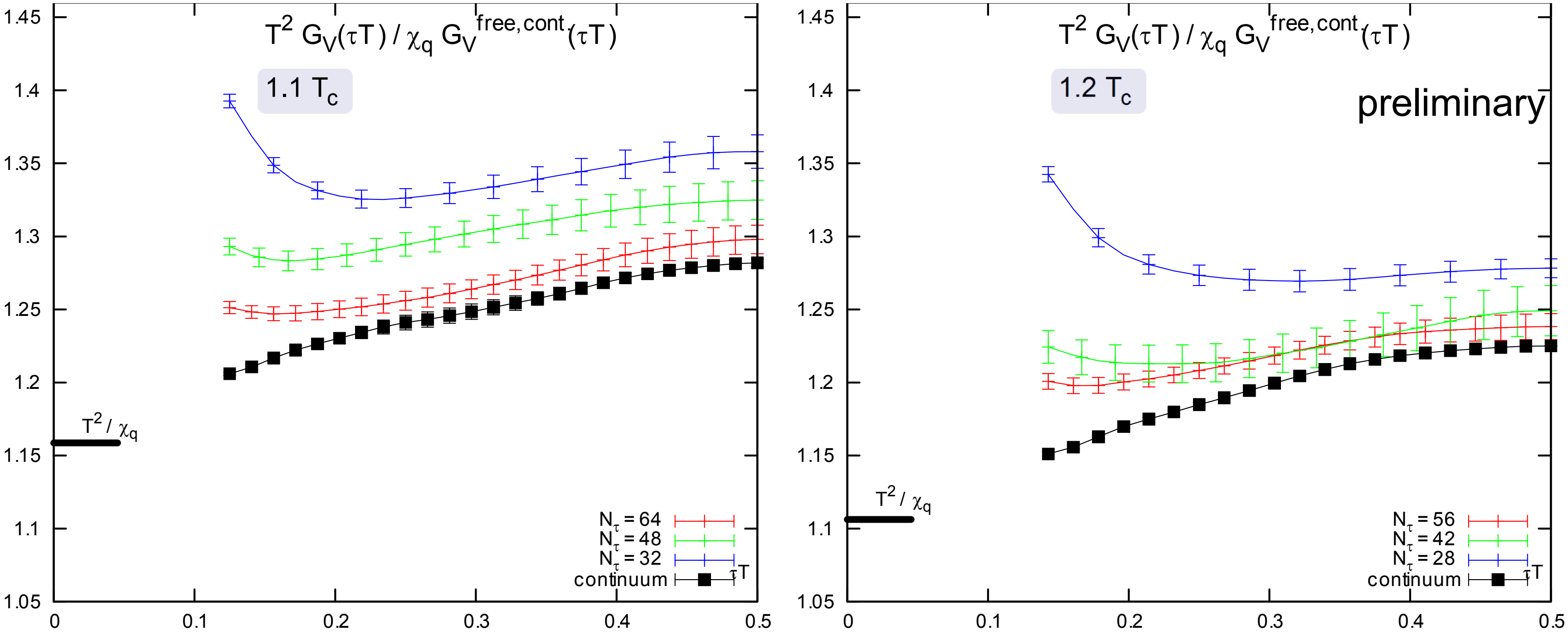
cut-off effects visible at all distances but

well defined continuum limit on the correlator level

well behaved continuum correlator down to small distances

approaching the correct asymptotic limit for $\tau \to 0$

Continuum extrapolation



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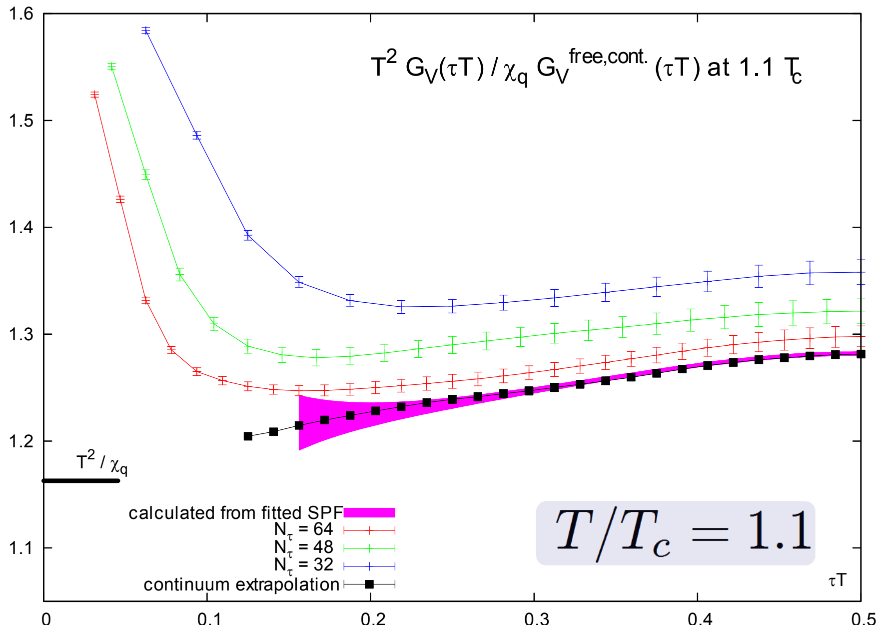
Spectral function and electrical conductivity

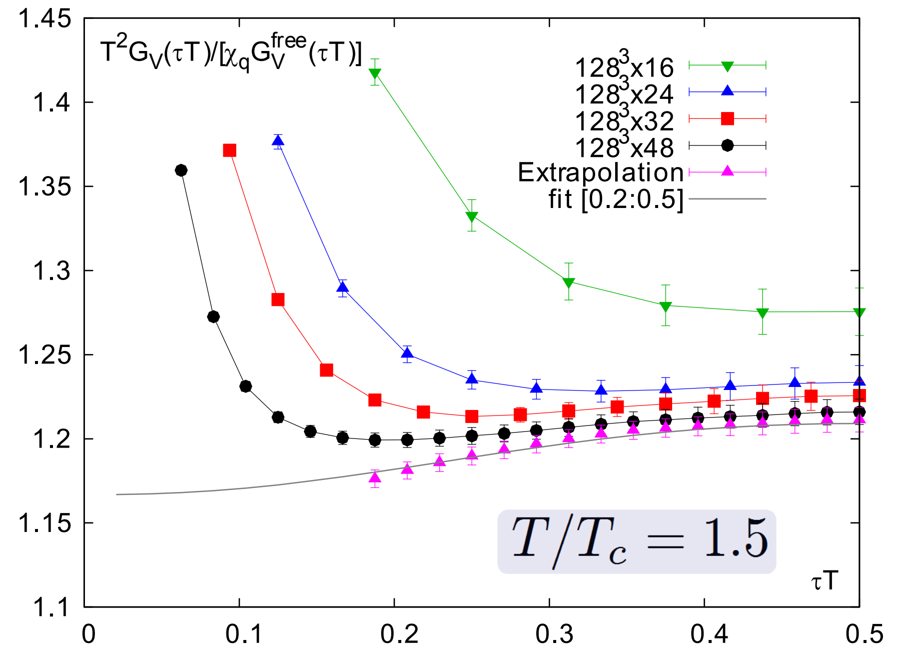
Use our Ansatz for the spectral function

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$

$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\ \omega^2\ \tanh(\omega/4T)$$

and fit to the continuum extrapolated correlators





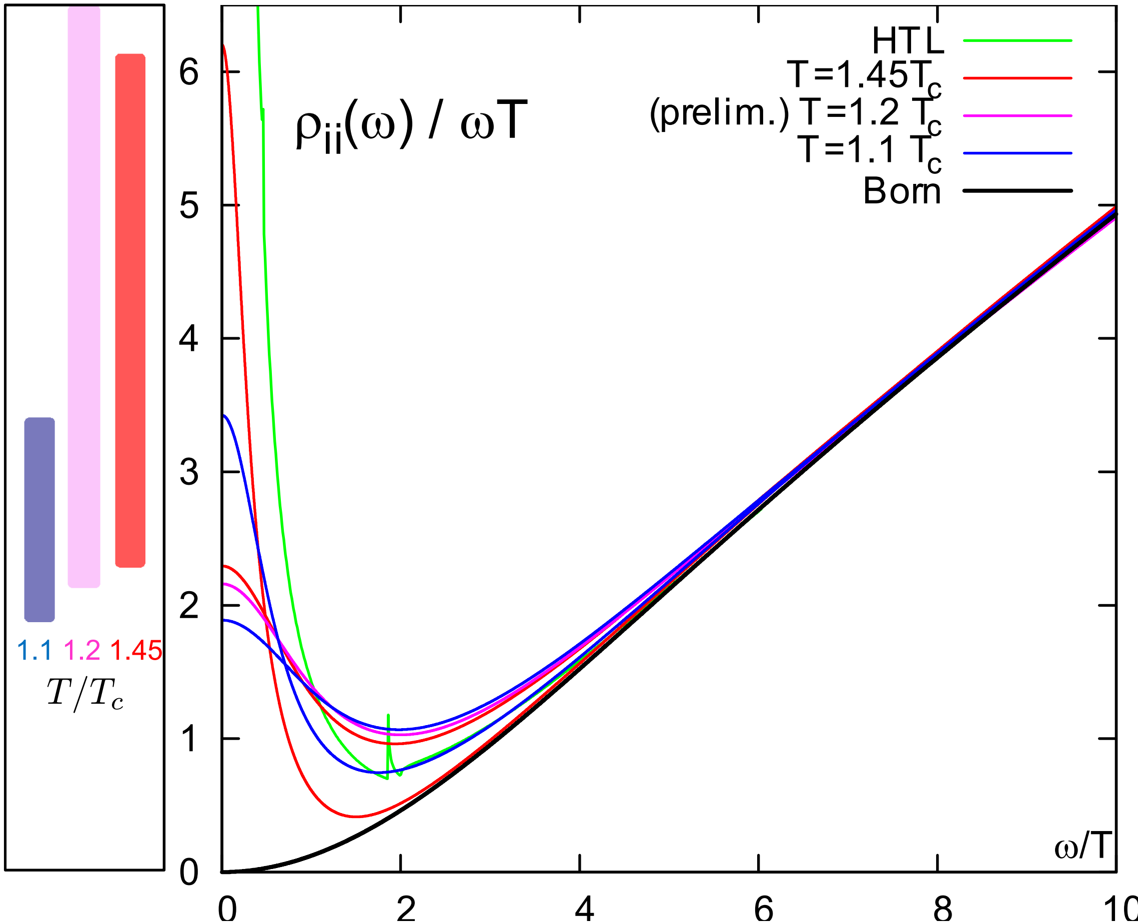
[H.T.-Ding, OK et al., PRD83 (2011) 034504]

all three temperatures are well described by this rather simple Ansatz

Spectral function and electrical conductivity

Use our Ansatz for the spectral function

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$
$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\ \omega^2\ \tanh(\omega/4T)\times\Theta(\omega_0,\Delta_\omega)$$



Analysis of the systematic errors

using truncation of the large ω contribution

$$\Theta(\omega_0,\Delta_\omega) = \left(1+e^{(\omega_0^2-\omega^2)/\omega\Delta_\omega}\right)^{-1}$$

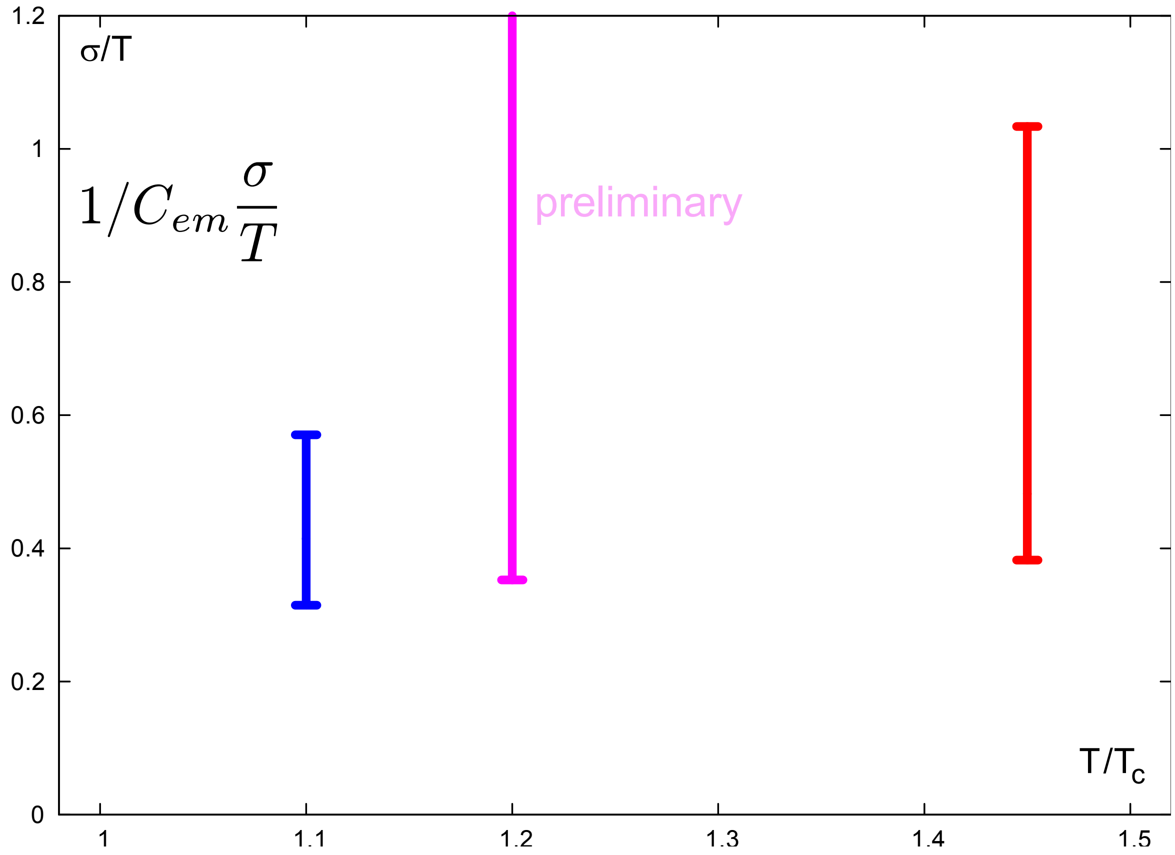
$$\frac{\sigma}{T} = \frac{C_{em}}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

electrical conductivity

electrical conductivity

T-dependence of the electrical conductivity:

$$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p}=0, T)}{\omega T}$$



similar studies using dynamical clover Wilson (w/o continuum limit):

A.Amato et al., arXiv:1307.6763

B.B.Brandt et al., JHEP 1303 (2013) 100

previous studies using staggered fermions (need to distinguish ρ_{even} and ρ_{odd}):

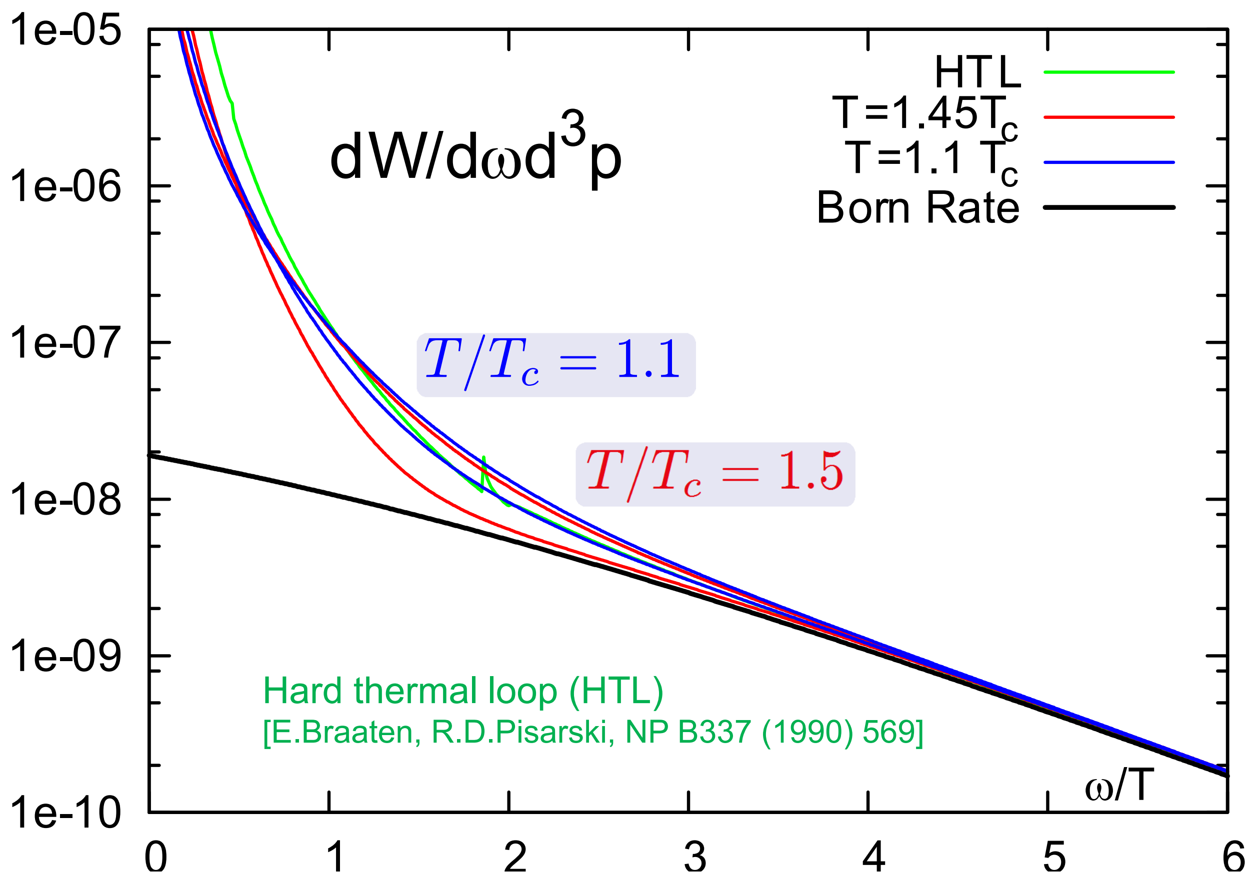
S.Gupta, PLB 597 (2004) 57

G.Aarts et al., PRL 99 (2007) 022002

Dilepton rates

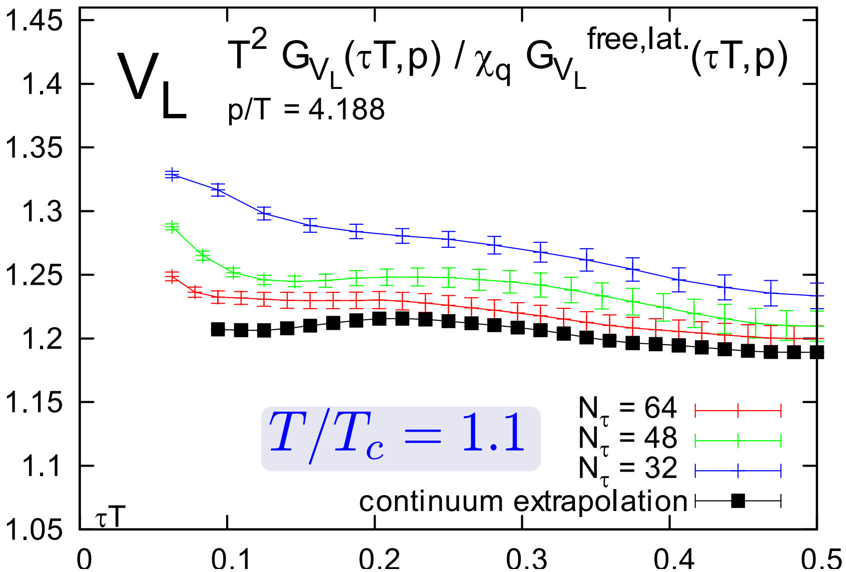
Dileptonrate directly related to vector spectral function:

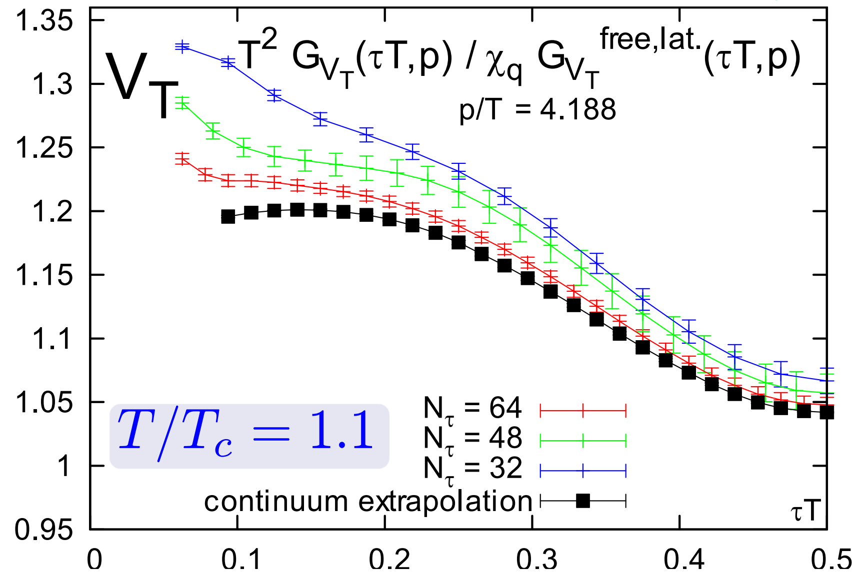
$$\frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{54\pi^3}\frac{1}{\omega^2(e^{\omega/T}-1)}\rho_{\mathbf{V}}(\omega,\mathbf{T})$$



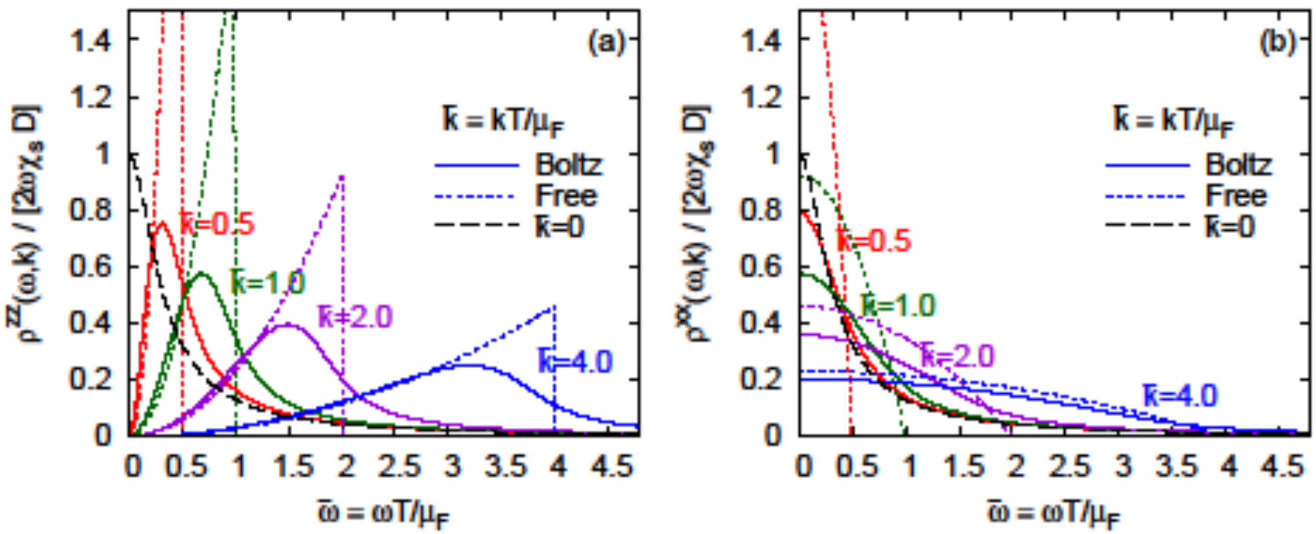
Non-zero momentum

[M.Müller et al., preliminary 2012]



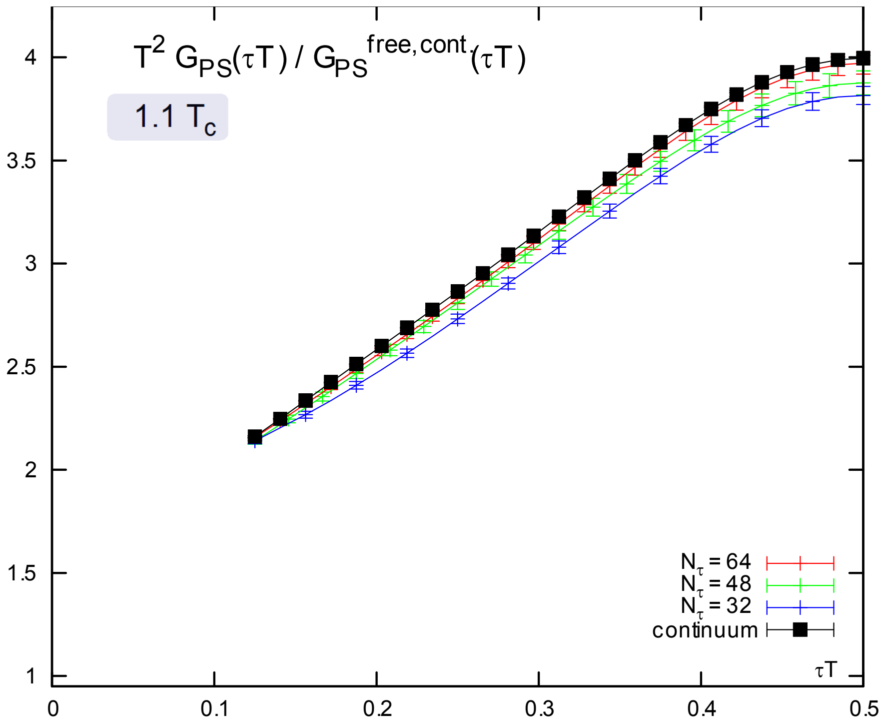


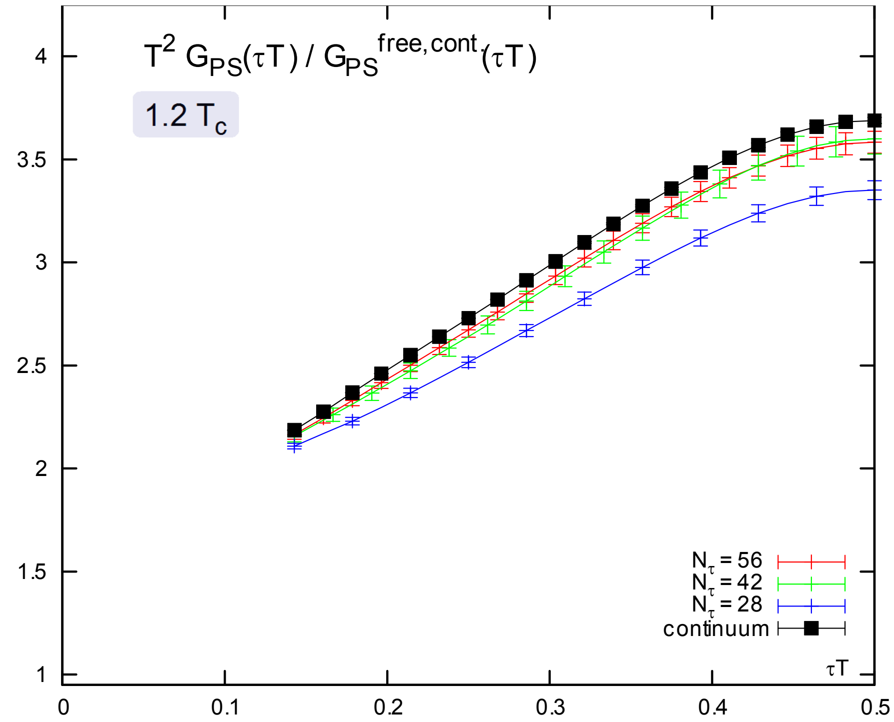
indications for non-trivial behavior of spectral functions at small frequencies:



[Hong+Teaney, PRC82 (2010)044908]

Pseudo-scalar channel





in contrast to the vector channel

no transport peak expected in the pseudo-scalar channel

still strong correlations visible in the pseudo-scalar channel

spectral function still needs to be determined!

Conclusions and Outlook

Conclusions:

Detailed knowledge of the **vector correlation function** in the region $1.1 \leq T/T_c \leq 1.5$

→ **continuum extrapolation** of correlation function and thermal moments

continuum $G_V(\tau T)$ well reproduced by **Breit-Wigner plus continuum** Ansatz for $\sigma_V(\omega)$ in the temperature region $1.1 \leq T/T_c \leq 1.5$

→ **electrical conductivity** σ/T shows small temperature effects

→ **Dilepton rate** approaches leading order Born rate for $\omega/T \geq 4$
enhancement at small ω/T

Outlook:

include HTL result for $\sigma_V(\omega)$ at large ω/T in the Ansatz

vector correlation function at **non-zero momentum**

especially close to T_c effects of dynamical quarks need to be included