Lattice computation of the 2nd order transport coefficient κ in pure Yang-Mills theory

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Outline

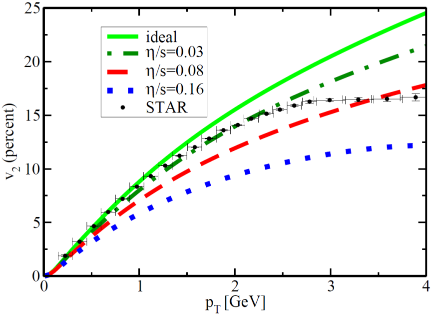
1. The Quark Gluon plasma and transport coefficients

2. Computation of 2nd order transport coefficient κ in Lattice QCD
   - Connection of κ to Lattice QCD
   - Issues concerning lattice computation of κ
   - κ from lattice perturbation theory
   - Temperature dependence of κ

3. Summary

Quark Gluon plasma (QGP) as an almost ideal fluid

- Measurement of elliptical flow v_2 in heavy ion collisions at RHIC or LHC



- QGP behaves like an almost ideal fluid Romatschke: arXiv:0706.1522
- Description via relativistic hydrodynamics using transport coefficients as underlying parameters

Relativistic hydrodynamics and transport coefficient κ

- Energy-momentum-tensor acts as the central quantity in hydrodynamics

$$T^{\mu\nu} = T_0^{\mu\nu} + \Pi^{\mu\nu}$$
$$\Pi^{\mu\nu} = \pi^{\mu\nu} + \Delta^{\mu\nu}\Pi$$

- Traceless part $\pi^{\mu\nu}$ can be expanded in gradients
- Second order gradient expansion in $\mathcal{N} = 4$ SYM theory Baier, Romatschke, Son, Starinets & Stephanov: arXiv:0712.2451

$$\pi^{\mu\nu} = \ldots + \kappa \left(R^{\langle\mu\nu\rangle} - 2u_\alpha u_\beta R^{\alpha\langle\mu\nu\rangle\beta} \right) + \ldots$$

- Includes transport coefficients such as shear viscosity η and κ

Connection of κ to Lattice QCD (LQCD)

- Response of fluid to a metric perturbation in linear response theory

  Romatschke, Son: arXiv:0903.3946

$$G^{\mathrm{R}}(\omega = 0, \vec{q}) = G(0) + \frac{\kappa}{2}|\vec{q}\,|^2 + \mathcal{O}(|\vec{q}\,|^4)$$

- G^{R}: Retarded correlator of the energy-momentum tensor

$$G^{\mathrm{R}}(x, y) = \langle T_{12}(x) T_{12}(y) \rangle$$

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$$G^{\mathrm{R}}(x, y) = \langle T_{12}(x) T_{12}(y) \rangle$$

- Retarded and Euclidean correlator coincide for $\omega = 0$ up to a contact term B

$$\boxed{G^{\mathrm{R}}(\omega, \vec{q}) \;=\; G^{\mathrm{E}}(\vec{q}, \omega + \mathrm{i}\epsilon) + B}$$

- Operator product expansion: B is momentum independent Kohno, Asakawa & Kitazawa: 1112.1508

Connection of κ to LQCD

- Connection between κ and Euclidean correlator G^{E}

$$\boxed{G^E(\omega = 0, \vec{q}) = \mathrm{const.} + \frac{\kappa}{2}|\vec{q}\,|^2 + \ldots}$$

- G^{E} can be extracted from LQCD
  $\Rightarrow$ **$\boldsymbol{\kappa}$ is directly accessible via LQCD**
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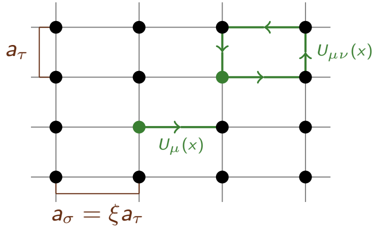
- Computation procedure for κ:

$G^{\mathrm{E}}(x, y) = \langle T_{12}(x) T_{12}(y) \rangle$ $\longrightarrow$ $G^{\mathrm{E}}(x, y) \rightarrow G^{\mathrm{E}}(q_z)$ $\longrightarrow$ fit $G^{\mathrm{E}}(q_z)$ versus q_z^2 $\longrightarrow$ slope $\kappa/2$

Framework for pure gauge theory

- Discretisation of spacetime by an $L^3 \times N_\tau$ hypercubic lattice with anisotropy $\xi = {}^{a_\sigma}/_{a_\tau}$
- Discrete momenta

$$q_i = \frac{2\pi}{L a_\sigma} n_i, \quad n_i = 0, \ldots, L-1$$



- Wilson action for an anisotropic lattice Namekawa et. al.: arXiv:0105012

$$S[U] = \frac{\beta}{N_\mathrm{c}} \operatorname{Re} \operatorname{Tr} \left[\frac{1}{\xi_0} \sum_{x,i,j} (1 - U_{ij}(x)) + \xi_0 \sum_{x,i} (1 - U_{i0}(x)) \right]$$

- Bare anisotropy $\xi_0(\beta)$ and temperature $T(\beta) = [a_\tau(\beta) N_\tau]^{-1}$
- Clover discretisation of energy-momentum tensor $T_{\mu\nu}$ Meyer: arXiv:0904.1806

Issues concerning lattice computation of κ (i)

Momentum expansion of G^{E}

- Momentum expansion of $G^{\mathrm{E}}(q)$ only valid for small momenta

$$\frac{q_i}{T} = \frac{2\pi N_\tau}{\xi L} n_i < 1$$

- Computation requires large spatial lattice extents L
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Additive renormalisation

- Subtraction of vacuum part

$$\langle T_{12} T_{12} \rangle_0 = \langle T_{12} T_{12} \rangle_T - \langle T_{12} T_{12} \rangle_{T=0}$$

- Cheaper: Replace $T = 0$ with some $T < T_c$

Issues concerning lattice computation of κ (ii)

Multiplicative renormalisation

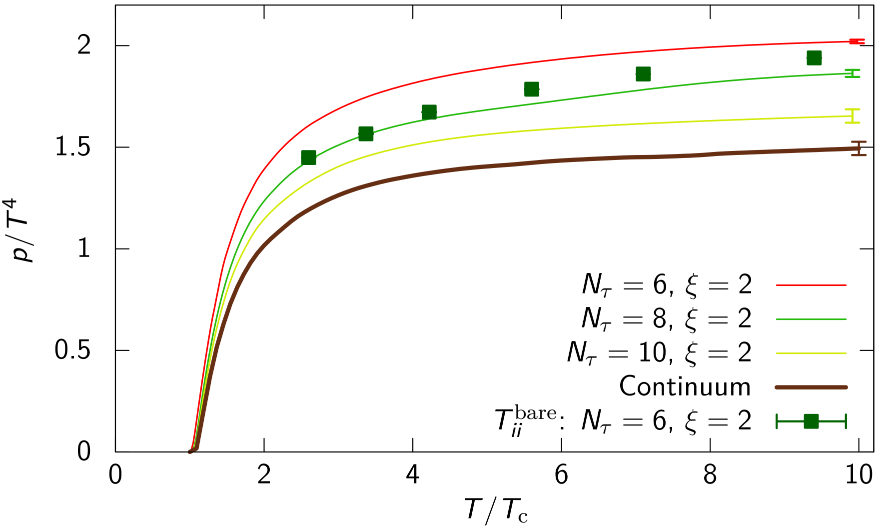
- Required due to reduced translational symmetry on the lattice
- Utilise cubic symmetry

$$\langle T_{12}(x) T_{12}(y) \rangle = \frac{1}{2} \Big[\langle T_{11}(x) T_{11}(y) \rangle - \langle T_{11}(x) T_{22}(y) \rangle \Big]$$

- Match energy-momentum tensor against pressure p Borsanyi, Endrodi, Fodor, Katz & Szabo: arXiv:1204.6184

$$\langle T_{ii}^{\text{renorm}} \rangle = Z(\beta, \xi) \langle T_{ii} \rangle = p_{\mathrm{cont}}$$

- Anisotropic lattice requires two renormalisation constants $Z^{\sigma}(\beta, \xi)$ and $Z^{\tau}(\beta, \xi)$
- Compute ratio Z^{τ}/Z^{σ} numerically by renormalisation group invariant quantities Meyer: arXiv:0809.5202



2
1.5
1
0.5
0
0
2
4
6
8
10
p/T^4
T/T_c
$N_\tau = 6,\ \xi = 2$
$N_\tau = 8,\ \xi = 2$
$N_\tau = 10,\ \xi = 2$
Continuum
T_{ii}^{bare}: $N_\tau = 6,\ \xi = 2$

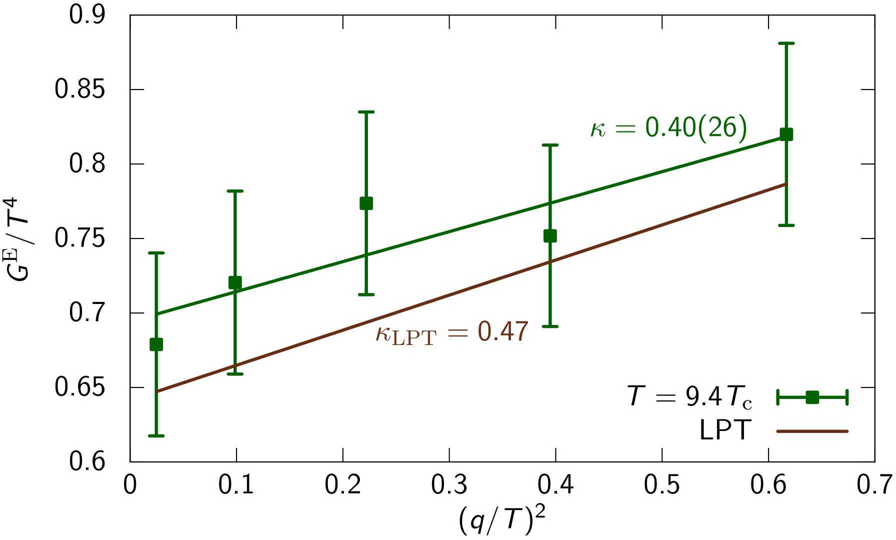
κ in lattice perturbation theory (LPT)

- Computation of correlator G^E in free LPT ($g = 0$)

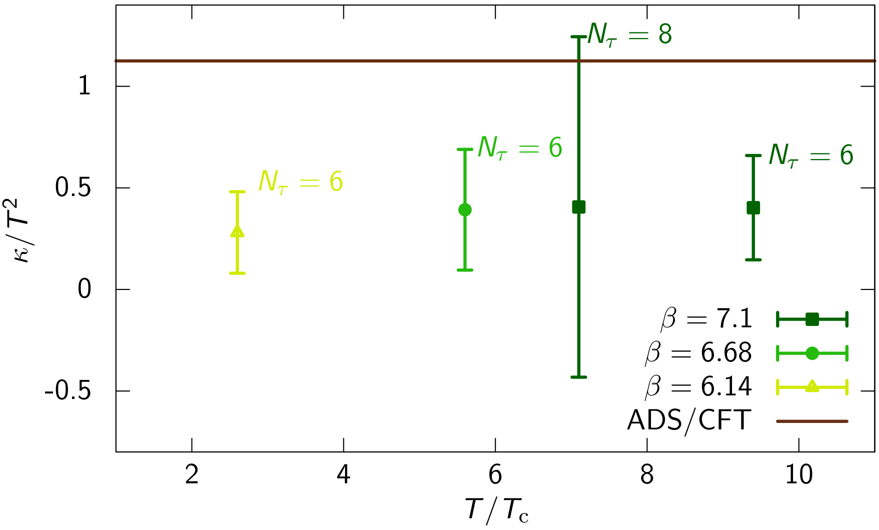
$$\frac{G^E(q)}{T^4} = (N_c^2 - 1)\left\{\frac{\pi^4}{N_\tau^2}\left(\frac{2\xi^2}{945} + \frac{4}{189}\right) + \frac{q^2}{T^2}\left[\frac{1}{36} + \frac{\pi^2}{N_\tau^2}\left(-\frac{\xi^2}{240} + \frac{49}{2160}\right)\right]\right\}$$

- Reproduce weak coupling regime in LQCD by increasing temperature

Comparison of LPT and simulation



Temperature dependence of κ



Summary

- Description of QGP in relativistic hydrodynamics with transport coefficients as underlying parameters
- 2nd order transport coefficient κ directly accessible from LQCD

$$G^E(\omega = 0, \vec{q}) = \frac{\kappa}{2}|\vec{q}\,|^2 + \mathrm{const.}$$

- Even pure gauge lattice simulations expensive
- First determination of κ from LQCD and reproduction of LPT result
- First result for temperature dependence of κ

Appendix

Simulation parameters

β	7.1	7.1	6.68	6.14
N_τ	6	8	6	6
N_σ	120	120	120	120
N_τ^{vac}	72	72	42	24
ξ	2	2	2	2
a_σ [fm]	0.026	0.026	0.044	0.094
T/T_{c}	9.4	7.1	5.6	2.6
$T^{\mathrm{vac}}/T_{\mathrm{c}}$	0.8	0.8	0.8	0.7

Simulation results κ

T/T_{c}	9.4	7.1	5.6	2.6
a_σ [fm]	0.026	0.026	0.044	0.094
κ/T^2	0.40(26)	0.41(84)	0.39(30)	0.28(20)