Lattice study of quark distribution amplitudes in the pion and its excitations

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Objectives

- We want to be able to compute the bound states of QCD, and to test these predictions against high-quality experimental data.
- Within this project, we want to go beyond the spectrum to compute the properties of the excited states. In particular, we probe the structure of the pion and its excitations through the computation of the quark distribution amplitudes on improved anisotropic lattices.

Motivation

- Recent progress aimed at extracting the spectrum of excited states (both for mesons and for baryons):

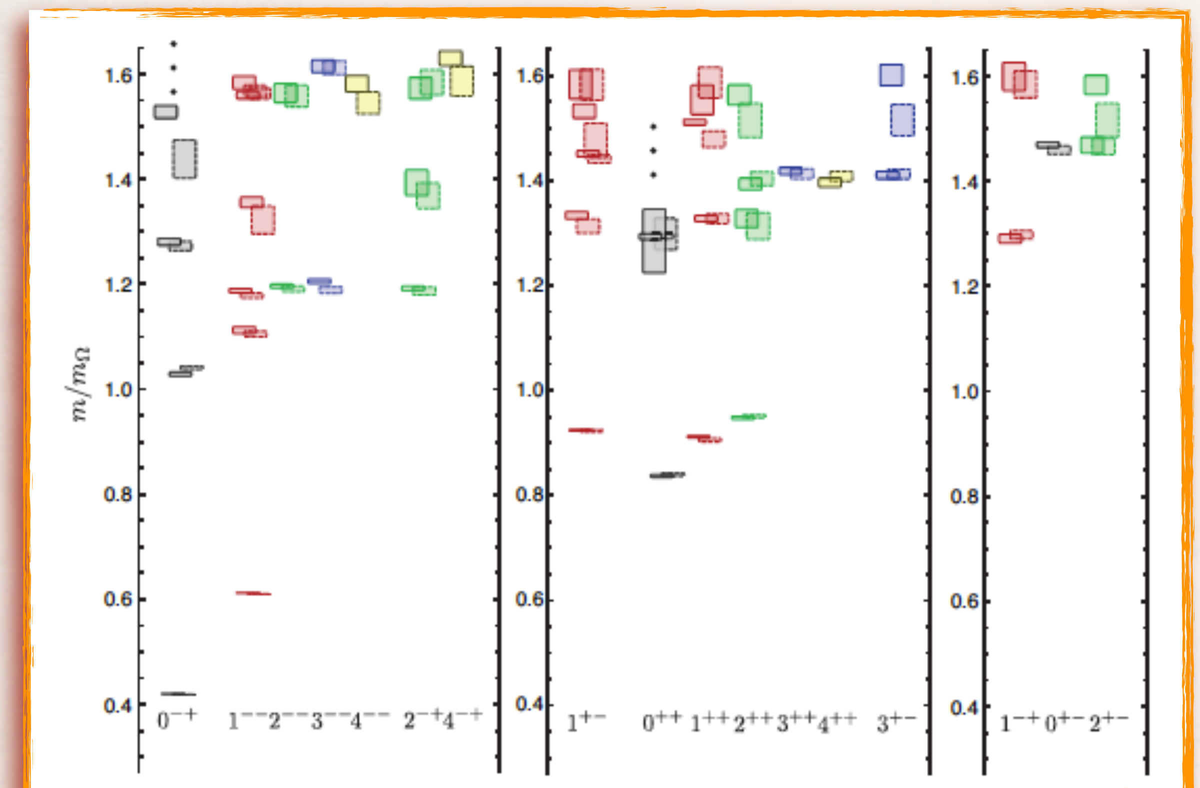

Fig. 1. Spin-identified spectrum of isovector (octet) mesons from the $m_\pi = 524\, MeV$ lattices [1].

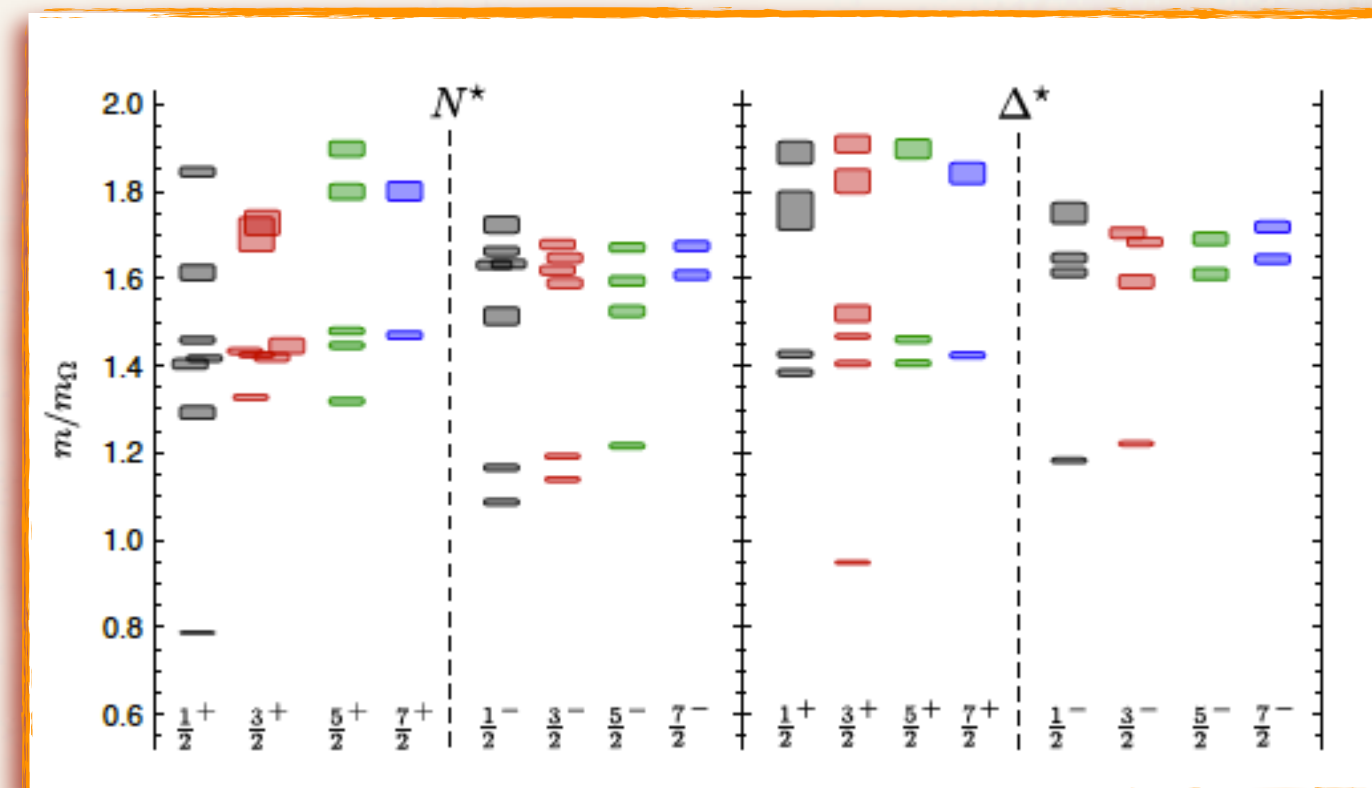

Fig. 2. Spin-identified spectrum of Nucleons and Deltas from the $m_\pi \sim 700\, MeV$ lattices [2].

[1] J. Dudek, R. Edwards, M. Peardon, D. Richards, C. Thomas, arXiv:1004.4930v1 [hep-ph]

[2] R. Edwards , J. Dudek, D. Richards, S. Wallace, arXiv:1104.5152v2 [hep-ph]

Anisotropic lattices

- In Euclidean space-time, the excited-state spectrum can be computed by observing the behavior of correlation functions formed from appropriately constructed operators:

$$\langle \mathcal{O}_n(t)\mathcal{O}_n^\dagger(0)\rangle = \langle 0|\hat{\mathcal{O}}_n|n\rangle\langle n|\hat{\mathcal{O}}_n^\dagger|0\rangle e^{-m_n t} + \langle 0|\hat{\mathcal{O}}_n|n'\rangle\langle n'|\hat{\mathcal{O}}_n^\dagger|0\rangle e^{-m'_n t} + \ldots \quad (1)$$

These correlation functions decay faster than those for ground state, and at large times propagation of noise swamps signals.

Resolution of the excited states is not a trivial problem!

Anisotropic lattices

- To overcome this difficulty, we use anisotropic lattices with finer temporal discretization

  (this also let us avoid the computational cost that would come from reducing the spacing in all directions)

- We use a discretization in which the spatial lattice spacing (a_s) and temporal lattice spacing (a_t) are related through:

$$\xi = \frac{a_s}{a_t} \approx 3.5 \qquad (2)$$

The lattice action

We use dynamical anisotropic lattices generated by the Hadron Spectrum Collaboration [3, 4]:

- $N_f = 2 + 1$ flavor (2 dynamical light quarks and a dynamical strange quark) 'clover' action with stout-link smearing;
- Symanzik- and tadpole-improved gauge action.

Volume	m_π	N_{cnfg}
$16^3 \times 128$	700 MeV	115

Table 1. Gauge-field ensembles: lattice volume, pion mass and number of gauge configurations

[3] R. Edwards, B. Joo, H.-W. Lin, Phys. Rev. D78 (2008) 054501.

[4] H.-W. Lin et al., Phys. Rev. D79 (2009) 043502.

Meson spectroscopy on the lattice

To extract the spectrum of the excited states from the exponentially suppressed signals, we apply the variational method [5, 6].

- Let us extract more information by analyzing a whole matrix of correlators for each irrep:

$$C_{ij}(t) = \langle \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)\rangle \quad (3)$$

- To determine the physical observables from this matrix, we solve generalized eigenvalue problem:

$$C_{ij}(t)v_j^{(n)} = \lambda(t)^{(n)} C_{ij}(t_0)v_j^{(n)} \quad (4)$$

- The ordered eigenvalues (principal correlators) behave as:

$$\lambda^{(n)}(t) = e^{-m_n(t-t_0)}\left[1+\mathcal{O}\left(e^{-(t-t_0)\Delta_n}\right)\right] \quad (5)$$

[5] C. Michael, Nucl. Phys. B259 (1985) 58.

[6] M. Luscher, U. Wolf, Nucl. Phys. B339 (1990) 222.

Basis of interpolators and distillation technique

- It is essential to use a "good" basis of interpolators which would generate states from the vacuum that have large overlap with the physical state we are interested in.

  ➡ To achieve this, we use the distillation technique [7]. It defines a smearing function

$$\Box_{xy}(t) = \sum_{k=1}^{N_{vec}} F(\lambda^{(k)})\xi_x^{(k)}(t)\xi_y^{(k)*}(t) \qquad (6)$$

  and provides an efficient method which allow us to calculate correlation functions with large basis of operators.

- Smeared quark fields are constructed by applying this distillation operator (6) to each quark field appearing in the interpolating operators.

[7] M. Peardon et al., Phys. Rev. D80 (2009) 054506.

Operator construction

- To obtain a large basis of operators that we can use in variational method, we apply a derivative-based construction for operators described by Hadron Spectrum collaboration [8, 9]: gauge-covariant spatial derivatives are combined with a gamma-matrix within a fermion bilinear so that the operator is of general form

$$\bar{\psi}\Gamma \overleftrightarrow{D}_i \overleftrightarrow{D}_j \ldots \psi \qquad (7)$$

- The naming scheme of gamma matrix is given in the Table 2, and we use the following notation for our operators:

$$\mathcal{O}^{J,M} = \left(\Gamma \times D^{[N]}_{J_D}\right)^J \qquad (8)$$

	a_0	π	π_2	b_0	ρ	ρ_2	a_1	b_1
Γ	1	γ_5	$\gamma_0\gamma_5$	γ_0	γ_i	$\gamma_0\gamma_i$	$\gamma_5\gamma_i$	$\gamma_0\gamma_5\gamma_i$

Table 2. Gamma matrix naming scheme

[8] J. Dudek, R. Edwards, M. Peardon, D. Richards, C. Thomas, Phys. Rev. Lett. 103 (2009) 262001.

[9] J. Dudek, R. Edwards, M. Peardon, D. Richards, C. Thomas, Phys. Rev. D82 (2010) 034508.

Lattice irreps

- We include operators built from all possible combinations of gamma matrices up to three derivatives and then subduced into lattice irreps:

J	$\Lambda(\mathrm{dim})$
0	$A_1(1)$
1	$T_1(3)$
2	$T_2(3) \oplus E(2)$
3	$T_1(3) \oplus T_2(3) \oplus A_2(1)$
4	$A_1(1) \oplus T_1(3) \oplus T_2 3 \oplus E(2)$

Table 3. Continuum spins subduced into lattice irreps.

- Formula for construction operators that transform in a definite lattice irrep and row from the continuum operator $\mathcal{O}^{J,M}$:

$$\mathcal{O}^{[J]}_{\Lambda,\lambda} = \sum_M S^{\Lambda,\lambda}_{J,M} \mathcal{O}^{J,M} \qquad (9)$$

[8] J. Dudek, R. Edwards, M. Peardon, D. Richards, C. Thomas, Phys. Rev. Lett. 103 (2009) 262001.

[9] J. Dudek, R. Edwards, M. Peardon, D. Richards, C. Thomas, Phys. Rev. D82 (2010) 034508.

“Ideal” operator

- Different interpolators one might use in the variational approach are just the basis one offers to the system. The relative weight of these basis elements come out of the variational procedure:

once generalized eigenvalue problem

$$C_{ij}(t)v_j^{(n)} = \lambda(t)^{(n)} C_{ij}(t_0)v_j^{(n)} \qquad (4)$$

is solved, one can define new interpolators $\Omega^{(n)}$ as a linear combination of the original interpolators:

$$\Omega^{(n)} = \sum_{i=1}^{r} v_i^{(n)*} \mathcal{O}_i \qquad (10)$$

➡ the variational method determines which linear combination of the basis interpolators best describe a physical state (an optimal operator).

Pion decay constant

- There are successful applications of the variational method, the distillation technique and described operator construction to the calculations on the anisotropic lattices (for both excited mesons [8, 9, 10] and baryons [2, 11]).

- One of the interesting challenges is the evaluation of matrix elements for excited states. Within this project, we apply all mentioned above techniques to study the properties of spectrum of excitations of a pion. In particular, we are interested in calculation of the decay constants for pion excitations.

[10] J. Dudek, R. Edwards, B. Joo, M. Peardon, D. Richards, C. Thomas, Phys. Rev. D83 (2011) 111502.

[11] J. Dudek, R. Edwards, Phys. Rev. D85 (2012) 054016.

Pion decay constant

- For a pion at rest, one can calculate the decay constant from the matrix element of the local axial vector current:

$$\langle 0|A_\mu^L(0)|\pi\rangle = f_\pi m_\pi \qquad (11)$$

with $A_\mu^L = \bar{\psi}\gamma_\mu\gamma_5\psi$ and $\pi = \bar{\psi}\gamma_5\psi$.

- Using reconfit_svd package developed by Hadron Spectrum collaboration, we determine f_π from the fits to the smeared-local correlators $\langle 0|A_\mu^L(t)\Omega^S(0)|0\rangle$ constructed using the optimal operator at the source.

To obtain a physical decay constant from the lattice value, we calculate [12]

$$f_\pi = \xi^{-3/2} a_t^{-1} f_\pi^{latt} \qquad (12)$$

[12] J. Dudek, R. Edwards, D. Richards, Phys. Rev. D73 (2006) 074507.

Pion decay constant

- The lattice calculations have already achieved some progress in this direction [13]:

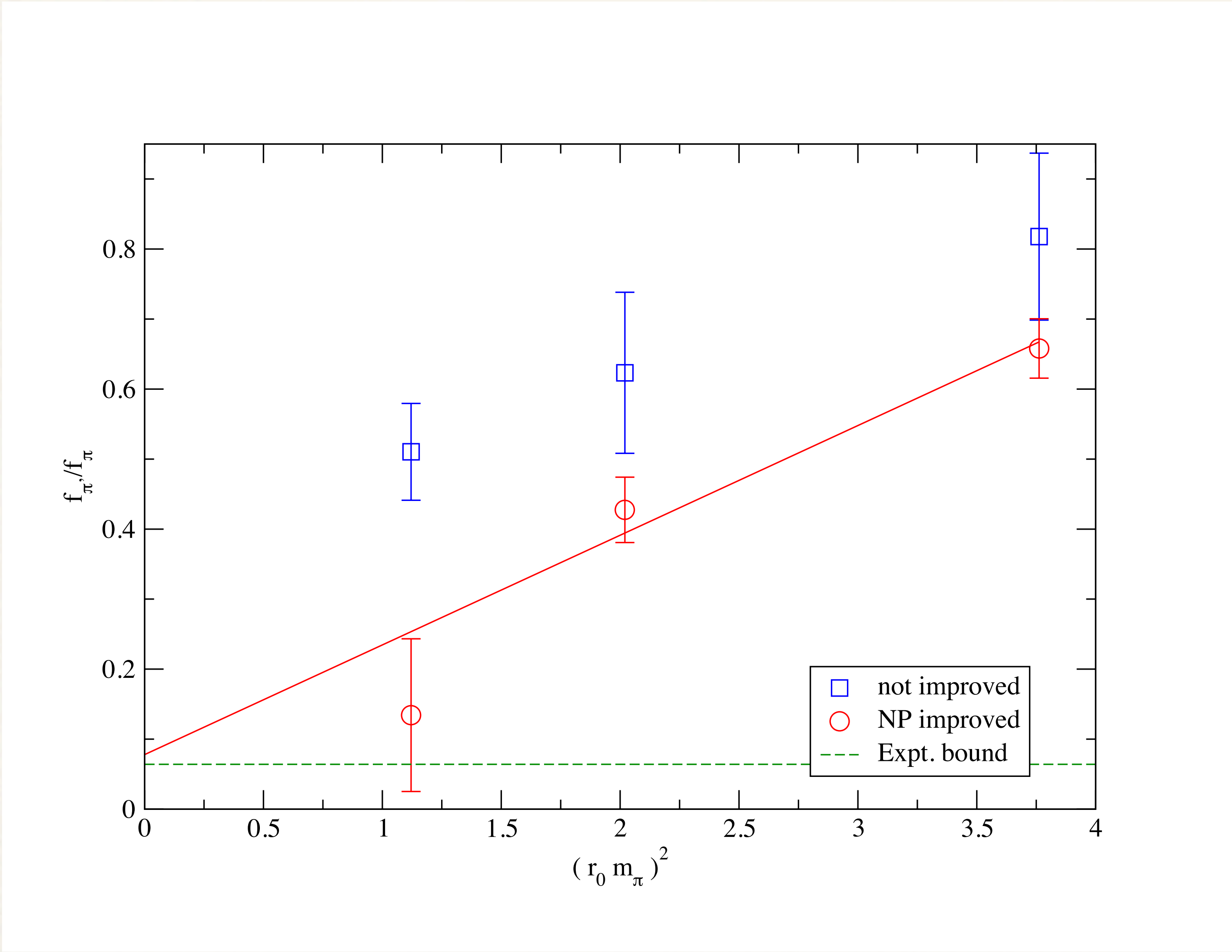

Fig. 3. Ratio of the decay constant of the first excited to ground state light pseudo-scalar meson as a function of the pion mass squared [12].

[13] C. McNeile, C. Michael, Phys. Lett.. B 642 (2006) 244.

Pion decay constant

- The energy of two static color sources $V(r)$ separated by distance r serves as a useful reference scale for spectrum calculations (as in Fig. 3). This is most usefully described by Sommer parameter r_0, defined as:

$$-r^2 \frac{\partial V(r)}{\partial r}|_{r=r_0} = 1.65 \qquad (13)$$

- For our ensembles [4] , $a_t m_\pi = 0.1483(2)$ and $r_0/a_s = 3.214(10)$ so, taking anisotropy into account,

$$(r_0 m_\pi)^2 \approx 2.783$$

and we can compare our result with previous calculations (see Fig. 4).

Pion decay constant: first results

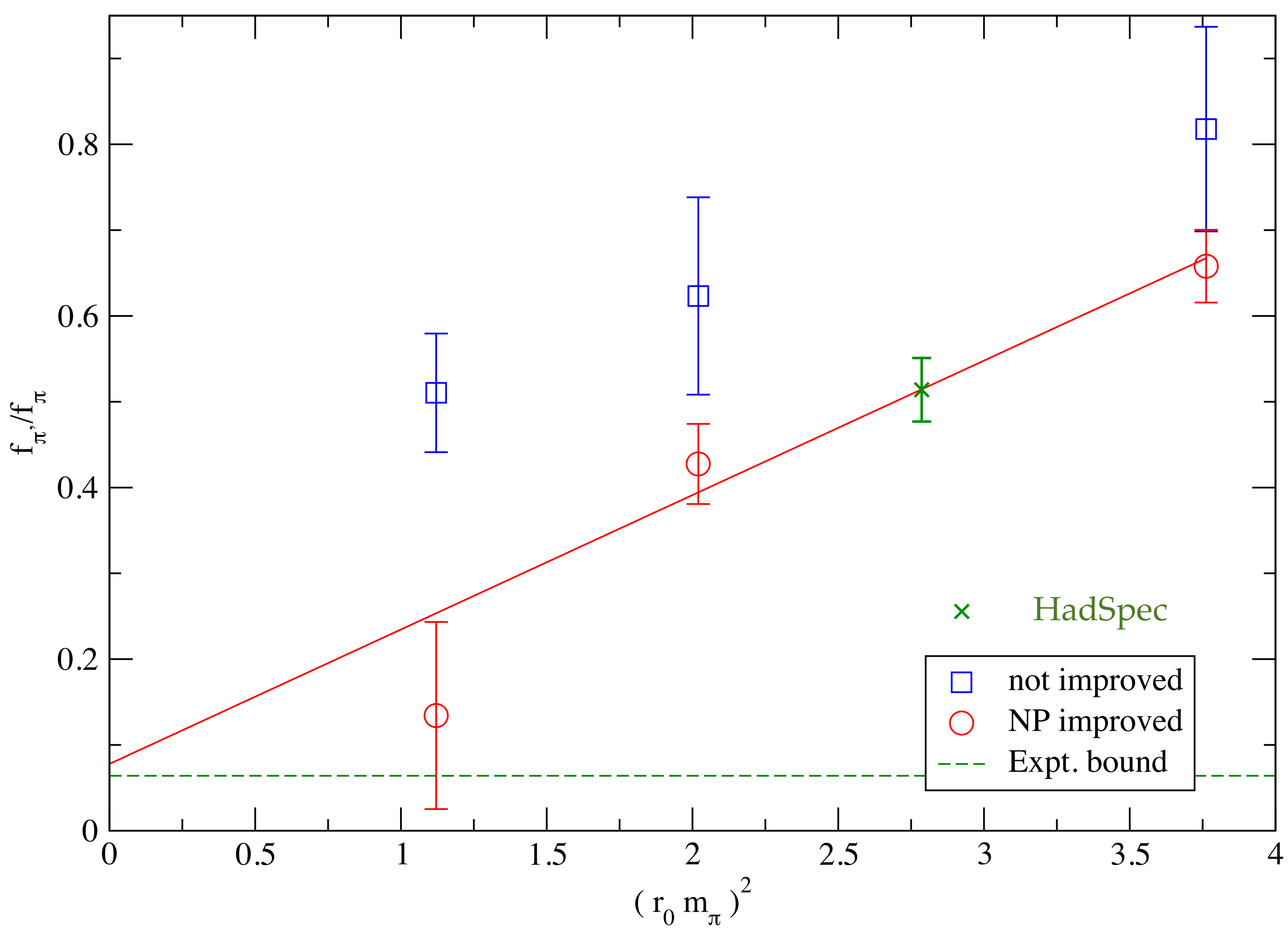

Fig. 4. Ratio of the decay constant of the first excited to ground state light pseudo-scalar meson as a function of the pion mass squared [12] with HadSpec collaboration result.

Pion decay constant: first results

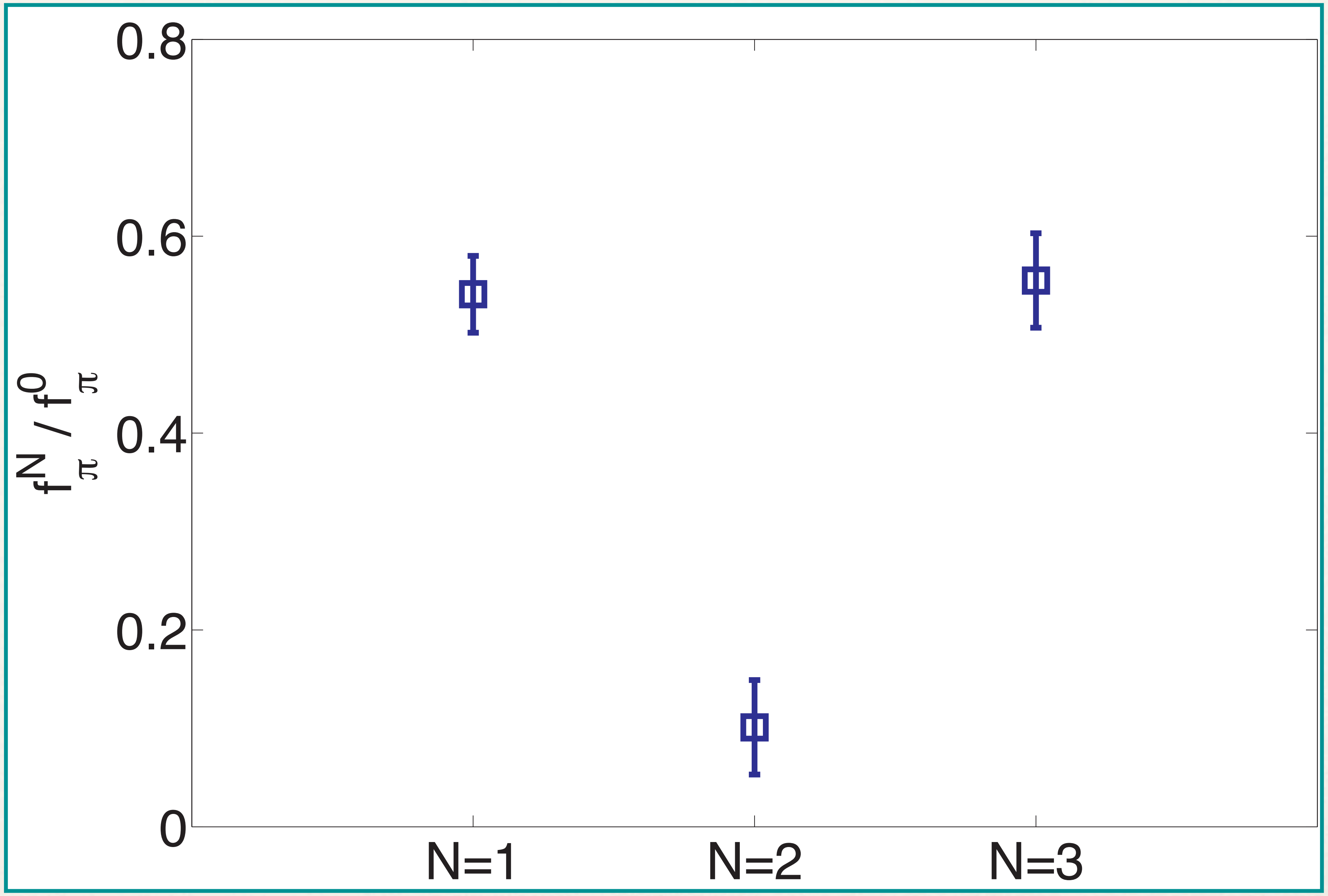

Fig. 5. First results for the ratios of the decay constants for the first 3 excited states.

Pion decay constant: first results

Table 4. Relative weights of the A_1^{-+} basis operators obtained from the variational method.

operator	ground state	1^{st} excited	2^{nd} excited	3^{rd} excited
$(a_0 \times D^{[3]}_{J_{1,3}=1,J=0})^{J=0}$	0.0605631	-0.104103	-0.13088	-0.0256733
$(a_1 \times D^{[3]}_{J_{1,3}=1,J=1})^{J=0}$	0.100849	-0.549734	-0.13767	0.147843
$(b_1 \times D^{[1]}_{J=1})^{J=0}$	0.0992313	-0.923783	-0.253904	1.05823
$(b_1 \times D^{[3]}_{J_{1,3}=0,J=1})^{J=0}$	-0.0735182	0.0189848	0.216605	0.326917
$(b_1 \times D^{[3]}_{J_{1,3}=2,J=2})^{J=0}$	0.111069	-0.443801	-0.348528	-0.137232
$(b_1 \times D^{[3]}_{J_{1,3}=2,J=3})^{J=4}$	-0.0013408	0.0275318	-0.00983878	-0.018632
$(\pi_2 \times D^{[0]}_{J=0})^{J=0}$	0.129139	-0.993586	-0.00597513	0.823323
$(\pi_2 \times D^{[2]}_{J=0})^{J=0}$	0.0587748	-0.338092	-0.0125717	0.319818
$(\pi \times D^{[0]}_{J=0})^{J=0}$	-0.0734256	0.120269	-0.237979	0.288355
$(\pi \times D^{[2]}_{J=0})^{J=0}$	-0.0026432	0.0916815	-0.0851646	0.187708
$(\rho_2 \times D^{[2]}_{J=1})^{J=0}$	0.171492	-0.278485	-0.479525	-0.767749
$(\rho \times D^{[2]}_{J=1})^{J=0}$	0.142252	-0.378953	-0.225052	-0.520194

Rho decay constant

- Two different definitions of the rho-meson decay constant:

$$\langle 0|V_\mu(0)|\rho\rangle = \epsilon_\mu \frac{m_\rho^2}{f_\rho} \qquad (14)$$

$$\langle 0|V_\mu(0)|\rho\rangle = \epsilon_\mu f_\rho m_\rho \qquad (15)$$

(here $V_\mu = \bar{\psi}\gamma_\mu\psi$ and ϵ_μ is the polarization vector of rho).

We calculate f_ρ from the correlation function $\langle 0|V_k(t)V_k^\dagger(0)|0\rangle$.

- Preliminary result for the ratio of the first to the ground state rho decay constant:

$$\frac{f_\rho^1}{f_\rho^0} = 0.933(27) \qquad (16)$$

Future plans

- increase the number of gauge configurations; use lighter quark masses to enable comparison with QCD-inspired models of the pion;
- compute the leptonic decay constants of the rho meson and its excitations;
- complete calculation of the matching coefficients in order to relate the lattice decay constants and other computed on the lattice parameters to those measured experimentally.