Analytical Relation between the Polyakov Loop and Dirac Eigenvalues in Temporally Odd-Number Lattice QCD

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Abstract:

Using temporally odd-number lattices, we analytically derive a relation between the Polyakov loop $\langle L_P \rangle$ and Dirac eigenvalues λ_n in QCD.
For the temporally odd-number lattice with an odd-number N_t,
the Polyakov loop $\langle L_P \rangle$ is expressed with the Dirac eigenvalues λ_n :

$$\boxed{\langle L_P \rangle = \mathrm{const} \sum_n \lambda_n^{N_t-1} \langle n|U_4|n\rangle.}$$

From this relation, the contribution of the low-lying Dirac modes to the Polyakov loop is found to be negligibly small in this sum. On the other hand, the low-lying Dirac modes are essential for chiral symmetry breaking (CSB). Then, this relation indicates no direct (one-to-one) correspondence between confinement and CSB in QCD, as was shown in our previous studies.

References:

[1] S.Gongyo, T.Iritani, H.S., Phys. Rev. D86 (2012) 034510,“Gauge-Invariant Formalism with Dirac-mode Expansion for Confinement and Chiral Symmetry Breaking”.

[2] T. Iritani, H.S., arXiv:1305.4049[hep-lat], “Polyakov Loop in terms of Dirac Eigenmodes: Relation between Confinement and Chiral Symmetry Breaking”.

Lattice 2013 July 29, 2013, Mainz

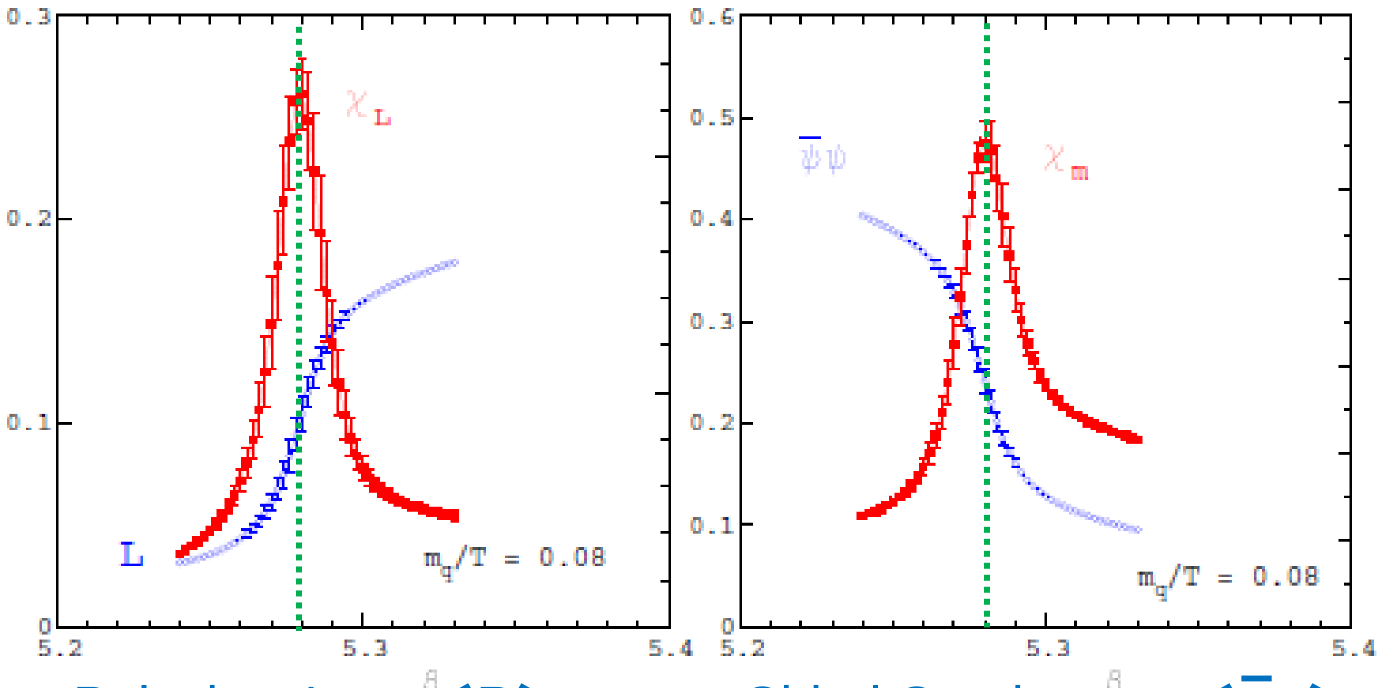
Introduction : Confinement and Chiral Symmetry Breaking

Color Confinement and Chiral Symmetry Breaking (CSB)
are Two of most important phenomena of
Nonperturbative QCD

The relation between
Confinement and CSB is not yet known
directly from QCD.

Correlation between Confinement and CSB is suggested by
Simultaneous Phase Transition of
Deconfinement and Chiral Restoration.

Lattice QCD results at finite temperature F. Karsch, Lect. Notes Phys. (2002)



Polyakov Loop <**P**> Chiral Condensate <$\bar{q}q$>

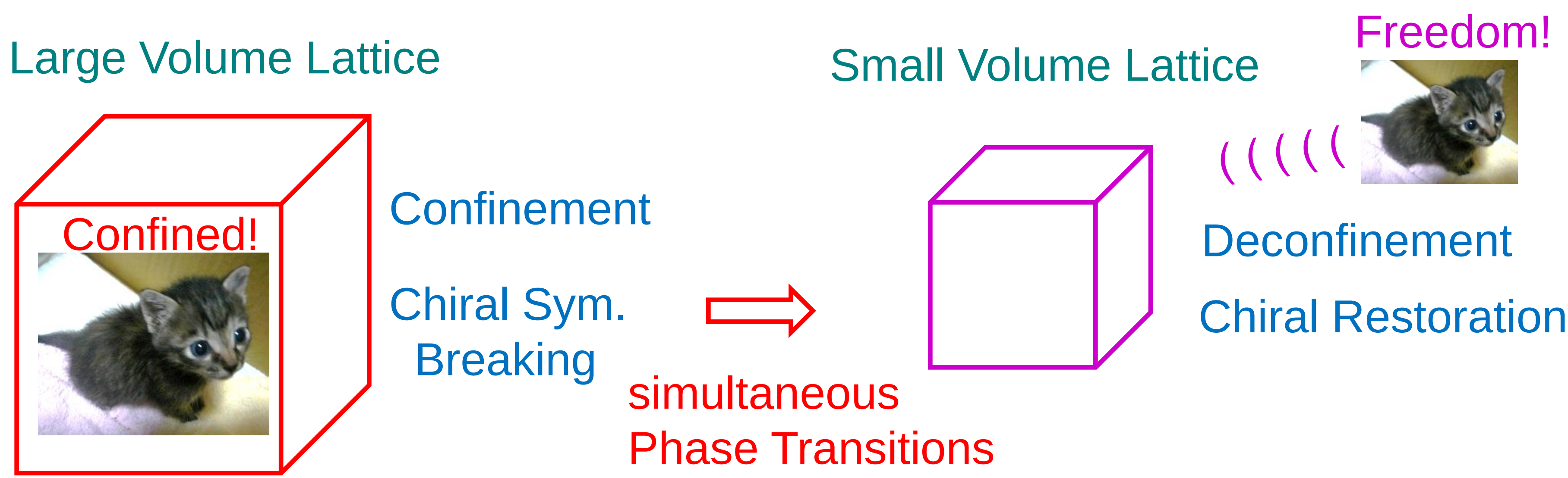
Color Confinement Chiral Symmetry Breaking

Fig. 2. Deconfinement and chiral symmetry restoration in 2-flavour QCD: Shown is $\langle L \rangle$ (left), which is the order parameter for deconfinement in the pure gauge limit ($m_q \to \infty$), and $\langle \bar{\psi}\psi \rangle$ (right), which is the order parameter for chiral symmetry breaking in the chiral limit ($m_q \to 0$). Also shown are the corresponding susceptibilities as a function of the coupling $\beta = 6/g^2$.

More on correlation between Confinement and Chiral Sym Breaking

Also, similar Coincidence between Deconfinement and Chiral Restoration is found in Finite-Size lattice QCD.

In fact, Simultaneous Phase Transitions occur according to the Box Size.



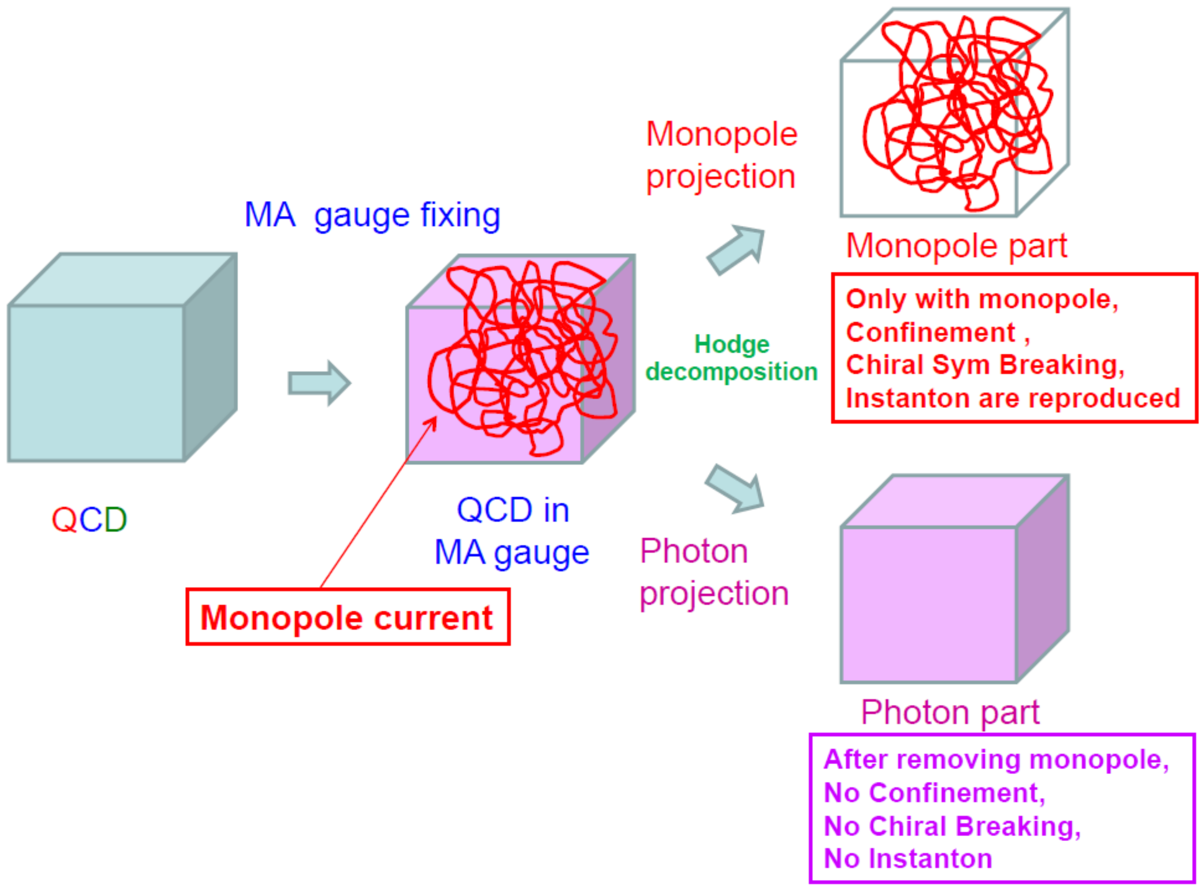
Of course, Finite-Temperature Phase transition is also a kind of Finite-Size effect of Euclidean Lattice in temporal direction.

More on correlation between Confinement and Chiral Sym Breaking

The close relation between Confinement and CSB has been indicated in terms of Monopoles appearing in Maximally Abelian Gauge in QCD.

By removing the Monopoles from the QCD vacuum, the confinement property and CSB are simultaneously lost.

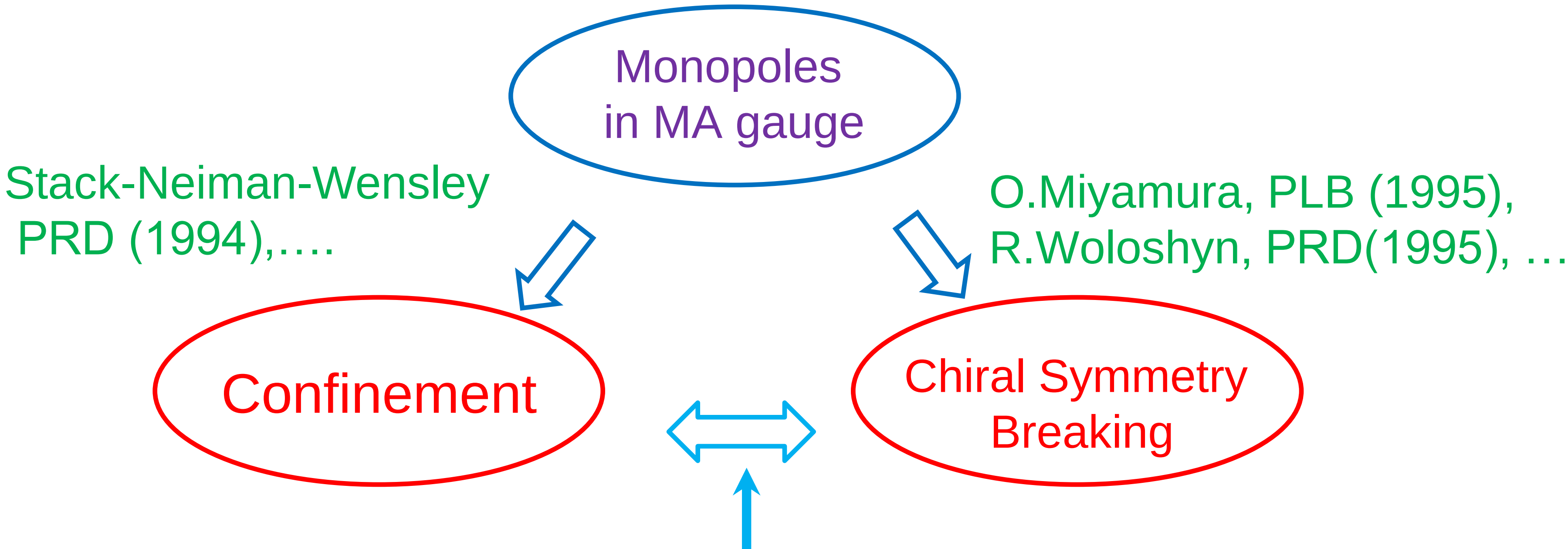
[e.g. Dual GL theory: H.S. et al, NPB (1995),
LQCD : O.Miyamura, PLB (1995), R.Woloshyn, PRD(1995).]



O. Miyamura

Relation between Confinement and Chiral Symmetry Breaking

The lattice QCD studies indicate an important role of monopoles to both Confinement and CSB, and these two nonperturbative phenomena seem to be related through the monopole.



We would like to know the relation between Confinement and CSB in more direct manner.

So, we investigate Confinement in terms the Dirac eigenmode of QCD, because Low-lying Dirac modes are essential for CSB.

Banks-Casher Relation

$$\boxed{\Sigma \equiv \left|\langle \bar{q}q \rangle\right| = \lim_{m\to 0}\lim_{V\to\infty} \pi\,\rho(0)}$$

$$\rho(\lambda) = \frac{1}{V}\left\langle \sum_n \delta(\lambda - \lambda_n) \right\rangle$$ **: QCD Dirac operator eigenvalue density**

$$\hat{\not{D}}|n\rangle = i\lambda_n |n\rangle$$

Zero-eigenvalue density ρ(0) of QCD Dirac operator $\hat{\not{D}}$ gives Chiral Condensate.

⇒ The essential modes for Chiral Sym Breaking are Low-lying Dirac modes.

※ **The non-zero spectrum is symmetric due to** $\{\gamma_5, \hat{\not{D}}\} = 0$

$$\because \hat{\not{D}}\psi_n = \lambda_n \psi_n \to \hat{\not{D}}(\gamma_5\psi_n) = -\lambda_n(\gamma_5\psi_n)$$

Eigen-mode of Dirac operator in Lattice QCD

$$\hat{\not{D}}_{xy} = \frac{1}{2a}\sum_{\mu=1}^{4}\gamma^{\mu}[U_{\mu}(x)\delta_{y,x+\hat{\mu}} - U_{-\mu}(x)\delta_{y,x-\hat{\mu}}]$$:Lattice Dirac operator

$$U_{-\mu}(x) \equiv U_{\mu}^{\dagger}(x-\hat{\mu})$$

$$\hat{\not{D}}|n\rangle = i\lambda_n|n\rangle$$:Dirac eigen-value, Dirac eigen-state

$$\lambda_n \in \mathbf{R}$$

$$\sum_y \hat{\not{D}}_{xy}\psi_n(y) = i\lambda_n\psi_n(x)$$:Dirac eigen-function $\psi_n(x)$

Explicit form of eigen-value equation in lattice QCD

$$\frac{1}{2a}\sum_{\mu=1}^{4}\gamma^{\mu}[U_{\mu}(x)\psi_n(x+\hat{\mu}) - U_{-\mu}(x)\psi_n(x-\hat{\mu})] = i\lambda_n\psi_n(x)$$

Gauge trans. property:
$$\begin{cases} U_{\mu}(x) \to V(x)U_{\mu}(x)V^{\dagger}(x+\hat{\mu}) \\ \psi_n(x) \to V(x)\psi_n(x) \end{cases}$$

same as quark field

apart from an irrelevant phase factor

$$\langle m|n\rangle = \int d^4x\,\psi_m^{\dagger}(x)\psi_n(x) = \delta_{mn}$$:normalization

We introduce

Link-variable operator $\boxed{\hat{U}_{\pm\mu}}$ defined by the matrix element of

$$\boxed{\langle x|\hat{U}_{\pm\mu}|y\rangle = U_{\pm\mu}(x)\delta_{x\pm\hat{\mu},y}}$$

$$U_{-\mu}(x) \equiv U_{\mu}^{\dagger}(x-\hat{\mu})$$

Using link-variable operator, many notations are quite simplified:

$$\boxed{\hat{D}_{\mu} = \frac{1}{2a}(\hat{U}_{\mu} - \hat{U}_{-\mu})}$$

:covariant derivative operator

$$\boxed{\hat{\not{D}} = \frac{1}{2a}\sum_{\mu=1}^{4}\gamma^{\mu}(\hat{U}_{\mu} - \hat{U}_{-\mu})}$$

:Lattice Dirac operator

$$\boxed{\hat{\not{D}}|n\rangle = i\lambda_n|n\rangle}$$

:Dirac eigenvalue, Dirac eigenstate

$$\lambda_n \in \mathbf{R}$$

Previous study: Dirac-mode expansion and projection

S.Gongyo, T.Iritani, H.S., PRD86 (2012) 034510.

$$\sum_n |n\rangle\langle n| = 1$$:completeness of the Dirac-mode basis

$$\boxed{\hat{U}_{\pm\mu} \equiv \sum_m \sum_n |m\rangle\langle m|\hat{U}_{\pm\mu}|n\rangle\langle n|}$$ Dirac-mode expansion

We define Projection operator which restricts the Dirac-mode space.

Projection operator $$\boxed{\hat{P} \equiv \sum_{n\in A} |n\rangle\langle n|}$$ $$\hat{P}^2 = \hat{P} \qquad \hat{P}^+ = \hat{P}$$

In this projection, the Dirac-mode sum is done within a subset *A*.

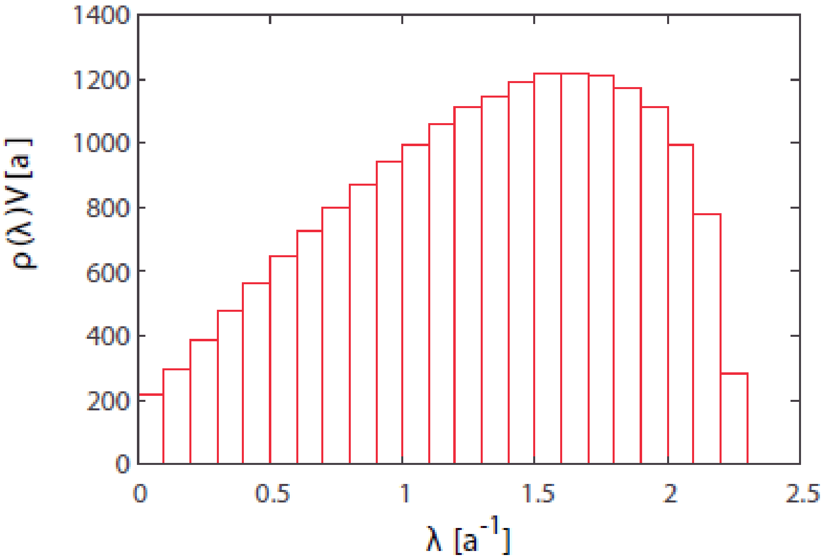
e.g. IR-cut $$\sum_{n\in A} = \sum_{|n|>N_{\mathrm{IR}}}$$

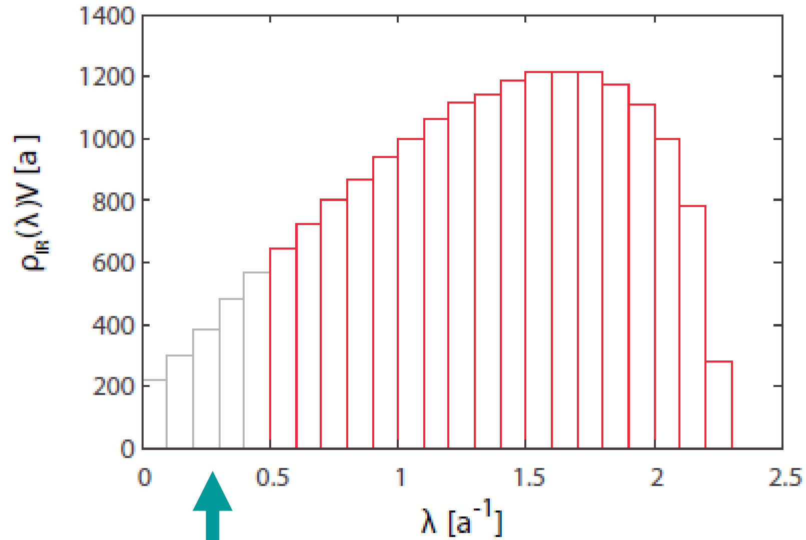
⇒ Projected Link-variable operator

$$\boxed{\hat{U}^P_{\pm\mu} \equiv \hat{P}\hat{U}_{\pm\mu}\hat{P} = \sum_{m\in A}\sum_{n\in A} |m\rangle\langle m|\hat{U}_{\pm\mu}|n\rangle\langle n|}$$

Previous study: Eigen-value distribution of QCD Dirac operator

β=5.6 (a=0.25fm), 6^4





We Remove the contribution of Low-lying Dirac modes.

$$\langle \bar{q}q \rangle_{IR} \propto \sum_{\lambda_n \geq \Lambda_{IR}} \frac{2m}{\lambda_n^2 + m^2}$$

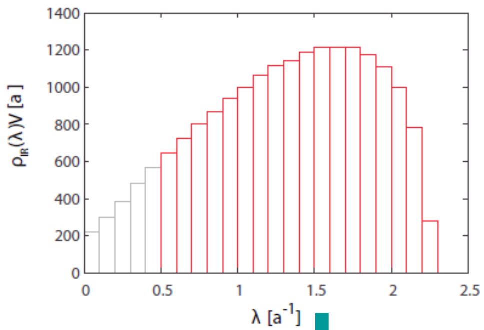
$$\boxed{\frac{\langle \bar{q}q \rangle_{IR}}{\langle \bar{q}q \rangle} \approx 0.02}$$

for m_q~5 MeV

Chiral Condensate is largely reduced (only 2% remains) after removing the low-lying Dirac modes.

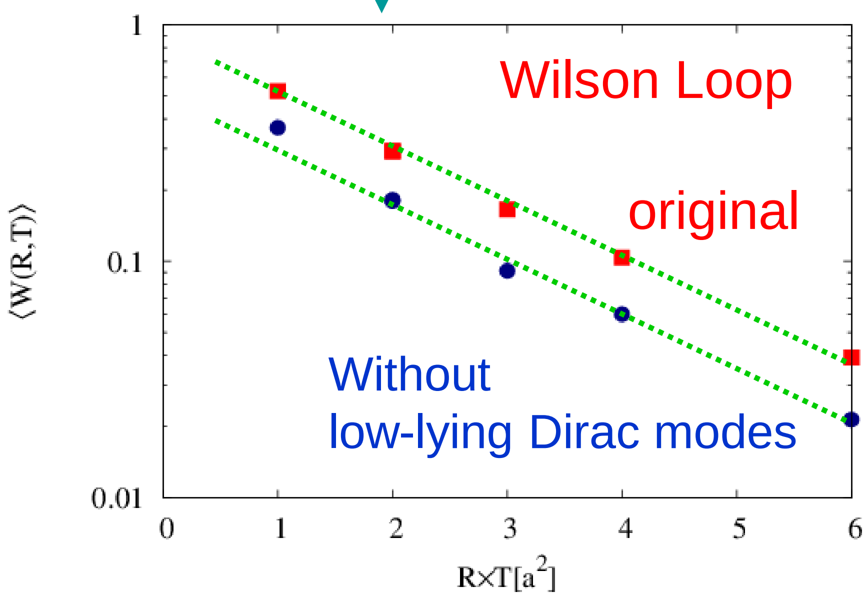
(cf. Banks-Casher relation)

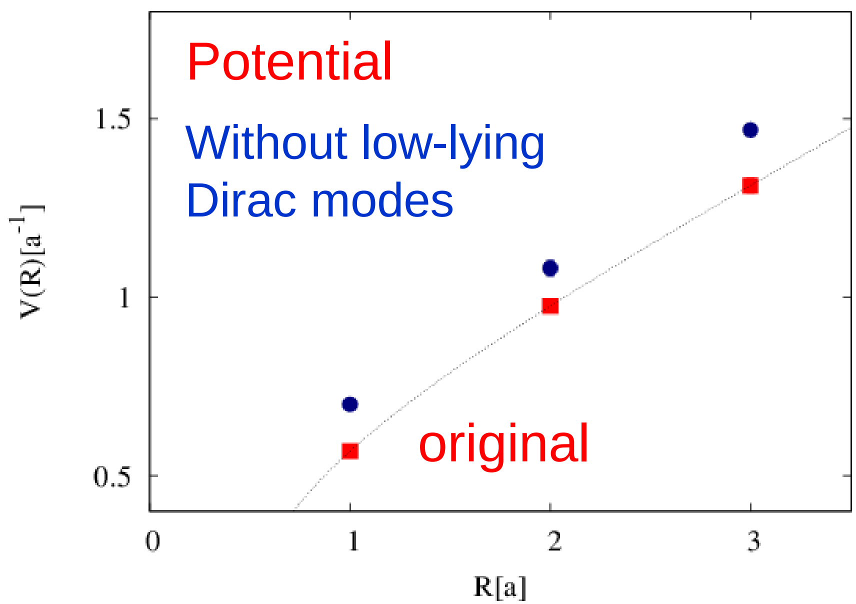
Previous study: Wilson Loop after removing low-lying Dirac modes



S.Gongyo, T.Iritani, H.S., PRD86 (2012) 034510.

Lattice QCD result of
Wilson Loop and Inter-Quark Potential
after removing low-lying Dirac modes





Wilson Loop obeys the Area Law and
the confining force is almost unchanged
even after removing the low-lying Dirac modes,
which are responsible to chiral symmetry breaking.

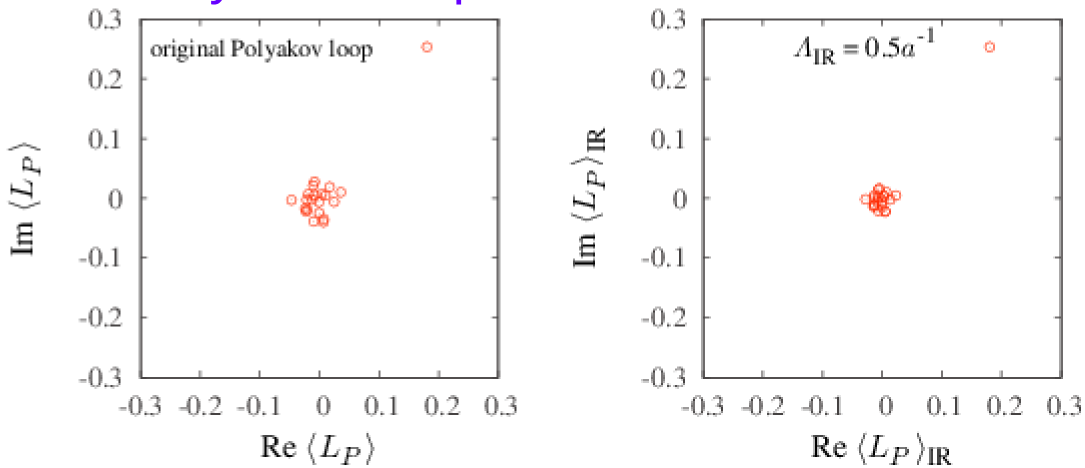
Previous study: *Dirac-mode projected Polyakov Loop*

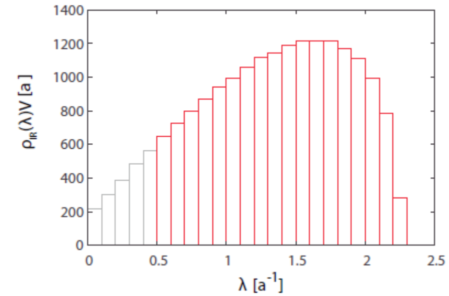
S.Gongyo, T.Iritani, H.S., PRD86 (2012).
T. Iritani, H.S., arXiv:1305.4049[hep-lat],

Dirac-mode projected Polyakov Loop

$$\mathrm{Tr}\,\hat{L}_P^{\,proj} \equiv \mathrm{Tr}(\hat{U}_4^P)^T = \sum_{n_1,n_2,\ldots,n_T \in A} \mathrm{tr}\,\langle n_1|\hat{U}_4|n_2\rangle\langle n_2|\hat{U}_4|n_3\rangle\cdots\langle n_T|\hat{U}_4|n_1\rangle$$

Polyakov Loop | *Without IR-Dirac modes* | on periodic lattice





Dirac spectral density

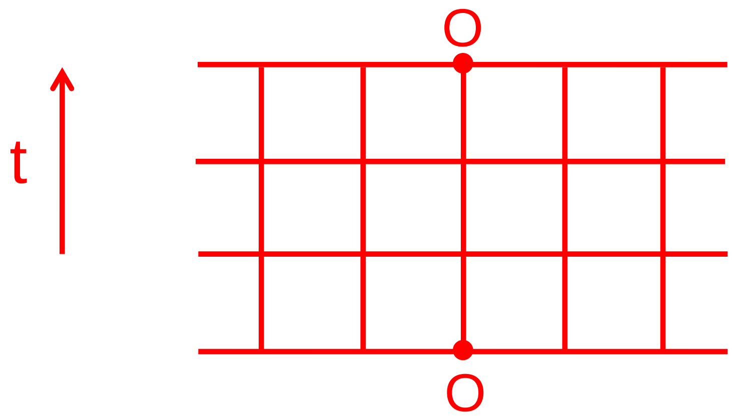
FIG. 6: The scatter plot of the Polyakov loop. The left figure shows the original Polyakov loop $\langle L_P \rangle$. The right figure shows the Polyakov loop $\langle L_P \rangle_{\mathrm{IR}}$ after cutting off the low-lying Dirac modes below the IR-cutoff $\Lambda_{\mathrm{IR}} = 0.5a^{-1}$.

Even after removing the low-lying Dirac modes, Polyakov loop remains to be zero, which means confinement phase and unbroken Z_3-center symmetry.

Main Dish !

Temporally Odd-Number Lattice

In this study, we use a standard square lattice.
But we consider temporally odd-number lattice,
where the temporal length N_t is odd.



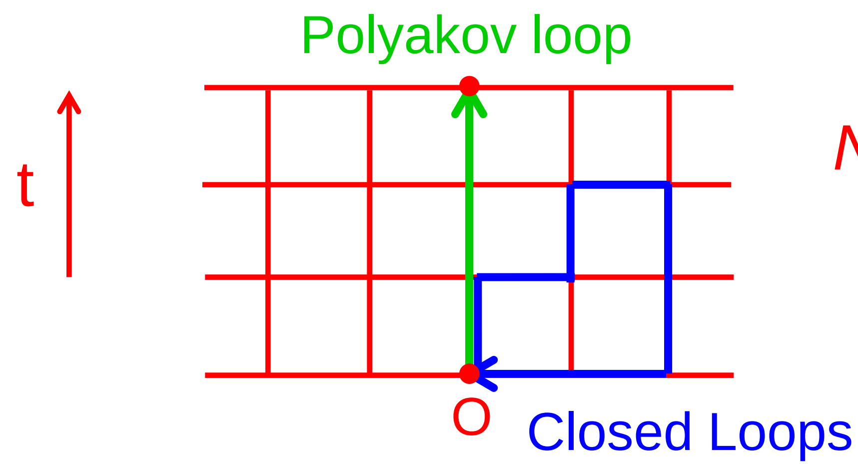
N_t =3 case

NB: in the continuum limit of $a \to 0$, $N_t \to \infty$,
any number of large N_t must give the same result.
Then, it is no problem to use the odd-number lattice.

For the simple notation,
we take the lattice unit a=1 hereafter.

Temporally Odd-Number Lattice

In general, only gauge-invariant quantities such as Closed Loops and the Polyakov loop survive in QCD. (Elitzur's Theorem)



N_t =3 case

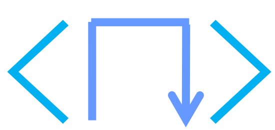
All the non-closed loops are gauge-variant and their expectation values are zero.

e.g.

$$\mathrm{Tr}\hat{U}_4\hat{U}_1\hat{U}_{-4} = \sum_x \mathrm{tr}\{U_4(x)U_1(x+\hat{4})U_4^\dagger(x+\hat{1})\}$$

gauge-variant

$$\propto \left\langle \mathrm{tr}\{U_4(x)U_1(x+\hat{4})U_4^\dagger(x+\hat{1})\}\right\rangle \underline{=0}$$


=0

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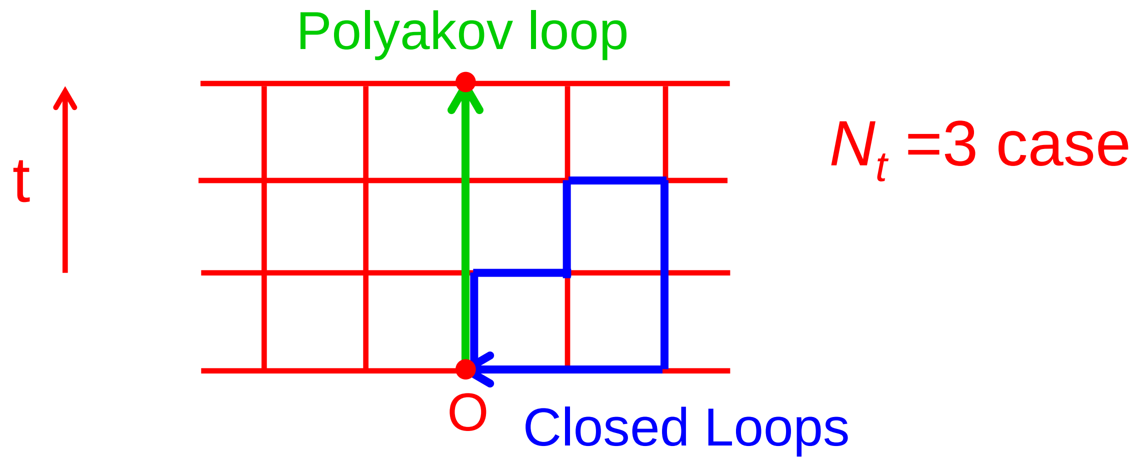
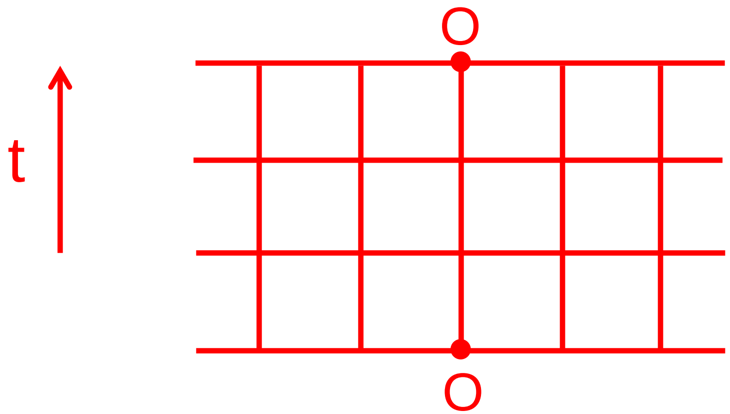


All the non-closed loops are gauge-variant and their expectation values are zero.

NB: any closed loop needs even-number link-variables on the square lattice.

Temporally Odd-Number Lattice



N_t =3 case

On the temporally odd-number lattice,
we consider the functional trace:

$$\boxed{\underline{I \equiv \mathrm{Tr}\hat{U}_4 \hat{\not{D}}^{N_t-1}} = \sum_x \left\langle x \,|\, \mathrm{tr}\hat{U}_4 \hat{\not{D}}^{N_t-1} \,|\, x \right\rangle \equiv \left\langle \mathrm{tr}\hat{U}_4 \hat{\not{D}}^{N_t-1} \right\rangle_{space\text{–}time}}$$

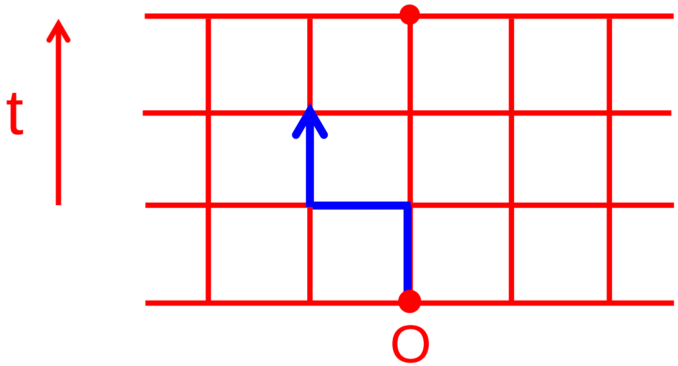
$$\mathrm{Tr} = \sum_x \mathrm{tr_c}\ \mathrm{tr}_\gamma \qquad \mathrm{tr} = \mathrm{tr_c}\mathrm{tr}_\gamma$$

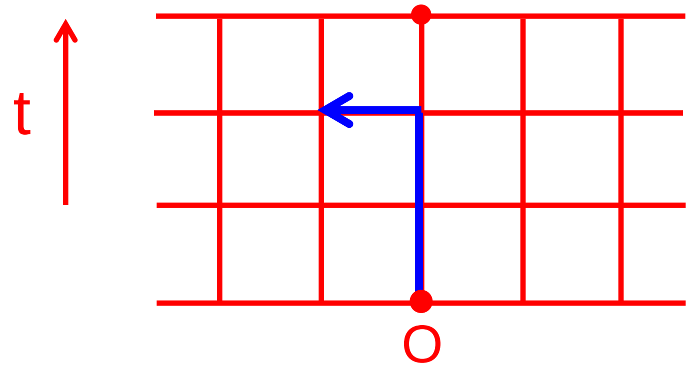
color & spinor

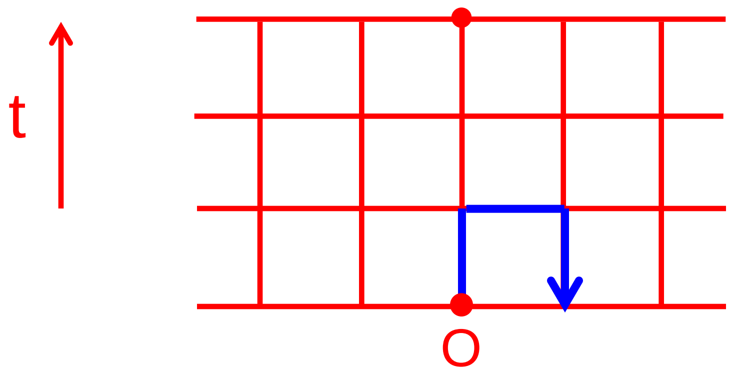
Property on functional trace $I \equiv \mathrm{Tr}\hat{U}_4 \hat{\not{D}}^{N_t - 1} = \left\langle \mathrm{tr}\hat{U}_4 \hat{\not{D}}^{N_t - 1} \right\rangle_{space-time}$

NB: $I \equiv \mathrm{Tr}\hat{U}_4 \hat{\not{D}}^{N_t - 1}$ includes N_t link-variable operators,
since the Dirac operator $\hat{\not{D}} = \frac{1}{2}\sum_{\mu=1}^{4} \gamma^{\mu}(\hat{U}_{\mu} - \hat{U}_{-\mu})$
includes a link-variable operator in each direction $\pm\mu$.

$I \equiv \mathrm{Tr}\hat{U}_4 \hat{\not{D}}^{N_t - 1}$ includes many trajectories on the square lattice.







N_t =3 case

Any closed loop needs
even-number link-variables
on the square lattice.

Property on functional trace $I \equiv \mathrm{Tr}\hat{U}_4 \hat{\not{D}}^{N_t-1} = \left\langle \mathrm{tr}\hat{U}_4 \hat{\not{D}}^{N_t-1} \right\rangle_{space\text{–}time}$

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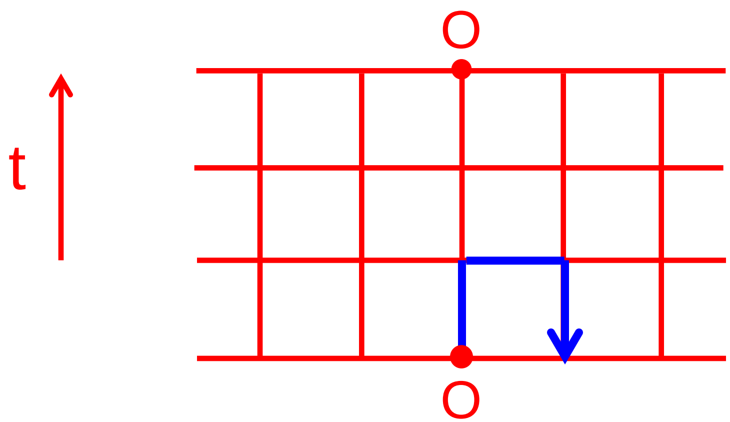
$I \equiv \mathrm{Tr}\hat{U}_4 \hat{\not{D}}^{N_t-1}$ includes many trajectories on the square lattice.

In this functional trace $I \equiv \mathrm{Tr}\hat{U}_4 \hat{\not{D}}^{N_t-1}$,
it is impossible to form a closed loop on the square lattice,
because the total number of the link-variable, N_t, is odd.
Only the exception is the Polyakov loop.



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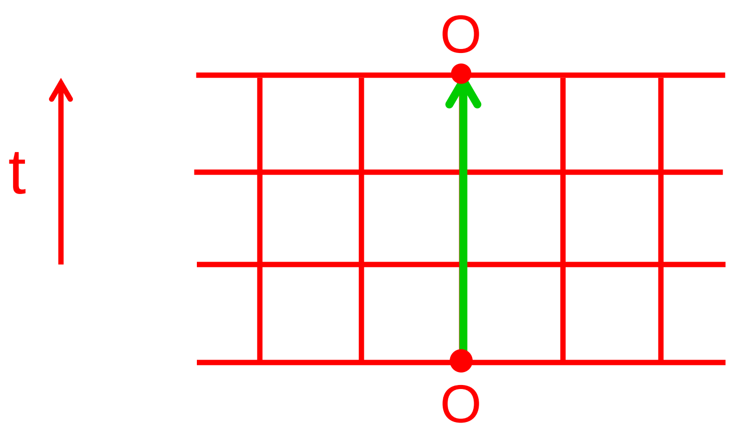

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Only the exception is the Polyakov loop.

Therefore, in this functional trace $I \equiv \mathrm{Tr}\hat{U}_4 \hat{\not{D}}^{N_t - 1}$,
only the Polyakov-loop ingredient can survive:

$$I \equiv \mathrm{Tr}\hat{U}_4 \hat{\not{D}}^{N_t - 1} = \mathrm{Tr}\hat{U}_4 (\gamma_4 \hat{D}_4)^{N_t - 1} = \mathrm{Tr}\hat{U}_4 \hat{D}_4^{N_t - 1}$$

$$\propto \mathrm{Tr}\hat{U}_4 (\hat{U}_4 - \hat{U}_{-4})^{N_t - 1} = \mathrm{Tr}\hat{U}_4^{N_t} = \mathrm{Tr}\,\hat{L}_P = \left\langle \mathrm{tr}\,\hat{L}_P \right\rangle_{space\text{–}time}$$

$$I \equiv \mathrm{Tr}\hat{U}_4 \hat{\not{D}}^{N_t-1}$$

$$= \mathrm{Tr}\hat{U}_4(\gamma_4\hat{D}_4)^{N_t-1}$$ (∵ only gauge-invariant quantities survive)

$$= \mathrm{Tr}\hat{U}_4\hat{D}_4^{N_t-1}$$ (∵ $\gamma_4^{N_t-1}=1$, NB: N_t−1 is even)

$$= \frac{1}{2^{N_t-1}}\mathrm{Tr}\hat{U}_4(\hat{U}_4-\hat{U}_{-4})^{N_t-1}$$

$$= \frac{1}{2^{N_t-1}}\mathrm{Tr}\hat{U}_4^{N_t}$$ (∵ only gauge-invariant quantities survive)

$$= \frac{1}{2^{N_t-1}}\mathrm{Tr}\,\hat{L}_P$$

$$= \frac{4}{2^{N_t-1}}\left\langle \mathrm{tr}_c\ \hat{L}_P \right\rangle_{space\text{–}time}$$ ($\because\ \mathrm{tr}_\gamma 1=4$, $\mathrm{Tr}=\sum_{space\text{–}time}\mathrm{tr}_c\ \mathrm{tr}_\gamma$)

Thus, the quantity $I \equiv \mathrm{Tr}\hat{U}_4 \hat{\not{D}}^{N_t-1}$ is proportional to the Polyakov loop $\left\langle \mathrm{tr}_c\ \hat{L}_P \right\rangle_{space\text{–}time}$

Thus, we obtain

$$\boxed{I \equiv \mathrm{Tr}\hat{U}_4 \hat{\not{D}}^{N_t-1} = \frac{4}{2^{N_t-1}} \left\langle \mathrm{tr}_c\ \hat{L}_P \right\rangle_{space-time}}$$

On the other hand,
using the complete set of the Dirac eigen-states $|n\rangle$

$$\boxed{I \equiv \mathrm{Tr}\hat{U}_4 \hat{\not{D}}^{N_t-1} = \sum_n \langle n | \hat{U}_4 \hat{\not{D}}^{N_t-1} | n \rangle = i^{N_t-1} \sum_n \lambda_n{}^{N_t-1} \langle n | \hat{U}_4 | n \rangle}$$

$$\sum_n |n\rangle\langle n| = 1 \qquad\qquad \hat{\not{D}}|n\rangle = i\lambda_n |n\rangle$$

Combining them, we obtain the analytical relation:

$$\boxed{\left\langle \mathrm{tr}_c\ \hat{L}_P \right\rangle_{space-time} = \frac{(2i)^{N_t-1}}{4} \sum_n \lambda_n{}^{N_t-1} \langle n | \hat{U}_4 | n \rangle}$$

$$\boxed{\left\langle \mathrm{tr}_c\, \hat{L}_P \right\rangle_{space-time} = \frac{(2i)^{N_t-1}}{4} \sum_n \lambda_n{}^{N_t-1} \langle n | \hat{U}_4 | n \rangle}$$

Here, the sum of RHS can be expressed with Dirac eigenvalue λ_n ,
Dirac eigenfunction $\psi_n(x)$, and temporal link-variable $U_4(x)$:

$$\sum_n \lambda_n{}^{N_t-1} \langle n | \hat{U}_4 | n \rangle = \sum_n \lambda_n{}^{N_t-1} \sum_x \langle n | x \rangle \langle x | \hat{U}_4 | x+\hat{t} \rangle \langle x+\hat{t} | n \rangle$$
$$= \sum_n \lambda_n{}^{N_t-1} \sum_x \psi_n^\dagger(x) U_4(x) \psi_n(x+\hat{t})$$

Each Dirac-mode contribution specified by n can be individually calculated in actual lattice QCD simulations.

Each term is manifestly Gauge Invariant.

Gauge trans. property: $\begin{cases} U_\mu(x) \to V(x) U_\mu(x) V^\dagger(x+\hat{\mu}) \\ \psi_n(x) \to V(x) \psi_n(x) \end{cases}$

Comment: There is no cancellation between chiral-pair Dirac states,
$|n\rangle$ and $\gamma_5 |n\rangle$, because $N_t - 1$ is even and $(-\lambda_n)^{N_t-1} = \lambda_n^{N_t-1}$

$$\left\langle \mathrm{tr}_c\ \hat{L}_P \right\rangle_{space-time} = \frac{(2i)^{N_t-1}}{4} \sum_n \underline{\lambda_n{}^{N_t-1}} \langle n | \hat{U}_4 | n \rangle$$

As a remarkable fact, because of $\lambda_n{}^{N_t-1}$, the contribution from small λ_n region is negligibly small in this sum.

(in comparison with other terms with large λ_n)

Here, the matrix element $\langle n | \hat{U}_4 | n \rangle$ is generally nonzero.

Comments:

If RHS *were not* a sum *but a product*,
the small λ_n region should have given a large contribution and a critical reduction factor to the Polyakov loop.
However, in the sum, the small λ_n contribution is negligible.

Even in the presence of a possible multiplicative renormalization factor for the Polyakov loop, the small λ_n contribution is negligible in this sum, relatively in comparison with other non-zero terms.

$$\left\langle \mathrm{tr}_c \, \hat{L}_P \right\rangle_{space-time} = \frac{(2i)^{N_t-1}}{4} \sum_n \underline{\lambda_n^{\;N_t-1}} \langle n | \hat{U}_4 | n \rangle$$

Conclusion

From this relation, the contribution of low-lying Dirac modes to the Polyakov loop is negligibly small in this sum, while the low-lying Dirac modes are essential for CSB.

Then, this analytical relation indicates no direct (one-to-one) correspondence between confinement and CSB in QCD.

Comment:

In this study, we have used temporally odd-number lattice.
However, in the continuum limit of $a \to 0$, $N_t \to \infty$,
any number of large N_t must give the same result.
Then, it is no problem to use the odd-number lattice.

Summary and Concluding Remarks

Using the temporally odd-number lattice with an odd-number N_t, we have analytically derived a relation between the Polyakov loop $\langle L_P \rangle$ and Dirac eigenvalues λ_n in QCD:

$$\boxed{\left\langle \mathrm{tr_c}\, \hat{L}_P \right\rangle_{space-time} = \frac{(2i)^{N_t-1}}{4} \sum_n \lambda_n^{N_t-1} \langle n | \hat{U}_4 | n \rangle}$$

From this relation, we have shown that the contribution of low-lying Dirac modes to the Polyakov loop is negligibly small.

On the other hand, the low-lying Dirac modes are essential for CSB.

Then, this relation indicates no direct (one-to-one) correspondence between confinement and CSB in QCD.

In the next talk by T.M.Doi, using actual lattice QCD calculations, we confirm this analytical relation in both confined and deconfined phases, and also show the negligible contribution of low-lying Dirac modes to the Polyakov loop numerically.

Thank you!