Fine structure of the confining string in an analytically solvable 3D model

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July 29, 2013

Outline

1. Main features of the the $U(1)$ lattice gauge model in 2+1 dimensions.
   - Analytical predictions.
   - Exact dual transformation.
2. Effective string theory predictions
   - Squared string width behaviour at finite temperature.
   - Interquark potential corrections.
3. Numerical Results and Conclusions

The $U(1)$ Lattice gauge theory

The partition function

$$Z = \prod_{x,\mu} \int_{-\pi}^{\pi} \mathrm{d}\vartheta_{x,\mu}\, e^{-\beta \sum_{\mathrm{pl.}} (1 - \cos \vartheta_{x,\mu\nu})}$$

Using discrete forms notation

$$Z = \prod_{c_1} \int_{-\pi}^{\pi} \mathrm{d}(\vartheta)\, e^{-\beta \sum_{c_2} (1 - \cos \mathrm{d}\vartheta)}$$

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In the weak coupling approximation

$$Z = Z_{\text{sw}} Z_{\text{top}} = Z_{\text{sw}} \sum_{\{q\}} e^{-2\pi^2 \beta (q, \Delta^{-1} q)}$$

- Z_{top} describes a Coulomb gas of magnetic monopoles.
- Dual superconductor: electric charges are confined!

- Confinement persists for every value of the coupling constant[1]
- For $\beta \gg 1$ and $q = \pm 1$

$$\sigma \geq \frac{c_\sigma}{\sqrt{2\pi^2\beta}} e^{-\pi^2 \beta v(0)} , \; m_D = c_0 \sqrt{8\pi^2\beta} e^{-\pi^2\beta v(0)}$$

with $v(0) = 0.2527$.

- Since

$$\frac{m_D}{\sqrt{\sigma}} = \frac{c_0}{\sqrt{c_\sigma}} 2\pi (2\pi\beta)^{3/4} e^{-\pi^2 v(0)\beta/2}$$

we can tune the importance of glueball effects by changing β!

[1] (Polyakov, 1976),(Göpfert, 1981)

Expand each plaquette factor in Fourier series[2]

$$e^{-\beta(1-\cos \mathrm{d}\vartheta)} = \sum_{k=-\infty}^{\infty} e^{-\beta} \mathrm{I}_{|k|}(\beta) e^{\imath k \, \mathrm{d}\vartheta}$$

- $\mathrm{I}_{|k|}(\beta)$ the modified Bessel function of order $|k|$.
- Performing the integrals on ϑ in Z yields a constraint for k on each plaquette
$$\delta k = 0$$
- The constraint can be automatically solved by the dual 0-chain ${}^\star l$ such that
$${}^\star k = \mathrm{d}{}^\star l$$

The transformation is exact.

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[2] (Savit, 1977)

We obtain a globally $\mathbb{Z}$ symmetric spin model

$$Z = e^{-\beta N_l} \sum_{\{{}^\star l=-\infty\}}^{\{\infty\}} \prod_{{}^\star c_1} \mathrm{I}_{|\,\mathrm{d}^\star l|}(\beta)$$

- Easier and more efficient to simulate than the original model.
- Sources at a distance R easily included in the partition function

$$Z_R = e^{-\beta N_l} \sum_{\{{}^\star l=-\infty\}}^{\{\infty\}} \prod_{{}^\star c_1} \mathrm{I}_{|\,\mathrm{d}^\star l+{}^\star n|}(\beta)$$

Effective string theory

1 - Effective string action

$$G(R) = \langle P(x) P^\dagger(x+R) \rangle = e^{-S_{\text{eff}}} = e^{-F(R,L)}$$

- At the lowest order (classical) $S_{\text{eff}} = F_{\text{cl}} = \sigma RL + k(L)$.
- Taking into account quantum fluctuations of the string (leading order)

  $$S_{\text{eff}} = \sigma RL + F_{\text{lo}}$$

  with

  $$F_{\text{lo}}(R,L) = (d-2)\log \eta\left(\frac{\imath L}{2R}\right)$$

Effective string theory

2 - Effective string action

- Up to order $(\sigma RL)^{-3}$ Lorentz invariance constraints the shape of next order terms of the effective string action[3].
- At next-to-leading order

$$S_{\text{eff}} = F_{\text{cl}} + F_{\text{lo}} + F_{\text{nlo}}$$

with

$$F_{\text{nlo}} = -\frac{\pi^2 L}{1152\sigma R^3}\left(2E_4\left(\frac{\imath L}{2R}\right) - E_2^2\left(\frac{\imath L}{2R}\right)\right)$$

- After the next to leading order, the boundary[4] term

$$F_b(R, L) = -b_2\frac{\pi^3 L}{60R^4}E_4\left(\frac{\imath L}{2R}\right)$$

with b_2 fittable parameter.

[3](Aharony, 2010)
[4](Aharony, 2010)

Effective string theory

Corrections to the interquark potential

- Measure $Q(R) = F(R+1, L) - F(R, L)$ to test effective string corrections to the interquark potential.
- snake algorithm[5]: great increase in precision!

$$Q(R) = -\log \frac{G(R+1)}{G(R)} = \frac{Z_{R+1}}{Z_R^{L_t-1}} \frac{Z_R^{L_t-1}}{Z_R^{L_t-2}} \cdots \frac{Z_R^1}{Z_R}$$

  where Z_R is the partition function of a system with static charges at a distance R.
- To obtain $Q(R)$ measure L_t local observables in independent simulations.

[5] (deForcrand, 2000), (Panero, 2005)

Effective string theory

String width behaviour

$$\omega^2(R, L) = \frac{\sum_R h^2(R) E_l(R)}{\sum_R E_l(R)}$$

- At the leading order

$$\omega^2 = \frac{1}{2\pi\sigma} \log \frac{L}{L_c} + \frac{R}{4\sigma L} - \frac{e^{-2\pi \frac{R}{L}}}{\sigma\pi} \sim \frac{R}{4\sigma L}, \text{ for } R \gg L$$

- On the lattice, in the presence of two static charges[6]

$$\langle F(x) \rangle_{q\bar{q}} = \frac{\langle \mathrm{d}^\star I \rangle}{\sqrt{\beta}}$$

[6] (Zach, 1997)

- The dual model was simulated on 32^3 and $64^2 x L_t$ lattices with $L_t = 16, 64$, at $\beta = 1.7$, $\beta = 2.2$ and $\beta = 2.75$.
- Site-by-site Metropolis update algotihm, hierarchical lattice update when useful.

Preliminary results

1 - Wilson loops - 32^3 lattice at $\beta = 2.2$

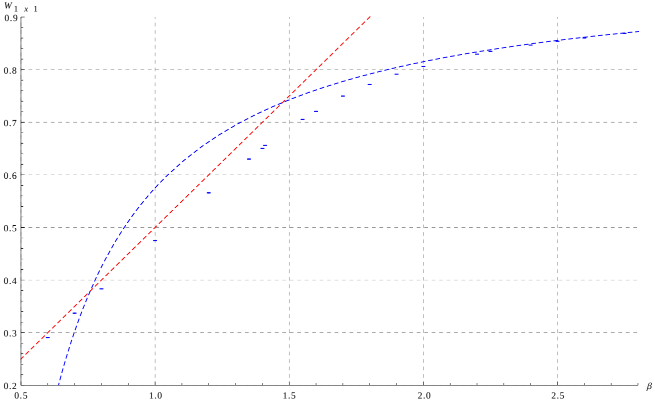
d	$\langle W_{10\times d}\rangle \cdot 10^{-3}$	$\langle W_{10\times d}\rangle \cdot 10^{-3}$ Irbäck, Peterson
2	56.9(1)	57.2(3)
3	23.6(1)	23.9(2)
4	10.60(4)	10.81(17)
5	4.98(2)	5.07(12)
6	2.35(1)	2.41(9)
7	1.129(6)	1.15(7)
8	0.544(3)	0.54(5)
9	0.263(1)	0.25(4)
10	0.128(1)	0.12(3)

Irbäck, Peterson[7] simulated the original model: We are simulating the same system!

[7] (Irbäck, 1987)

Preliminary results

2 - The plaquette - 64^3 lattice.



$$W_{1x1}^{\text{sc, red}} = \frac{\beta}{2}\,, \quad W_{1x1}^{\text{wc, blue}} = 1 - \frac{1}{3\beta} - \frac{1}{18\beta^2} - \frac{0.0331}{\beta^3} - \frac{0.0028}{\beta^4}$$

Analytical predictions

1 - The string tension

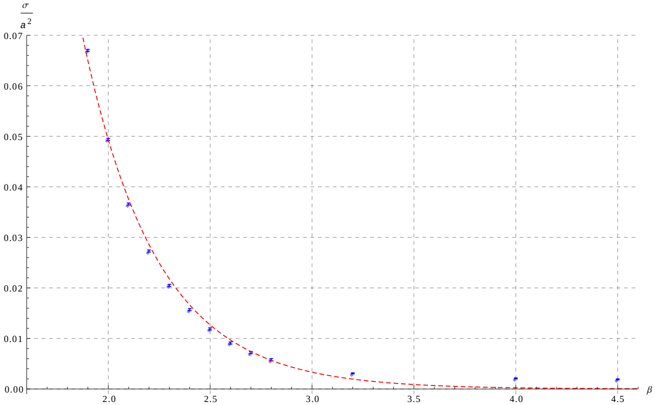


Figure: Fit with $\frac{c_\sigma}{\sqrt{2\pi^2\beta}} e^{-\pi^2\beta v(0)}$, $c_\sigma = 45.4(1)$.

Analytical predictions

2 - The glueball mass

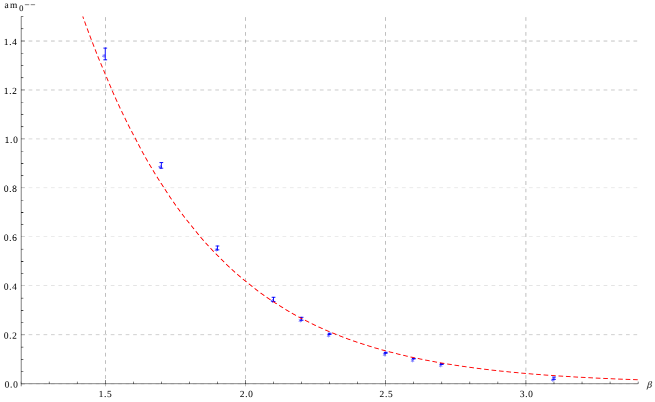


Figure: Fit with $c_g\sqrt{8\pi^2\beta}e^{-\pi^2\beta v(0)}$, $c_g = 4.89(2)$ (in agreement with Loan et al. (2001))

Analytical predictions

3 - The ratio $\frac{m(0^{--})}{\sqrt{\sigma}}$

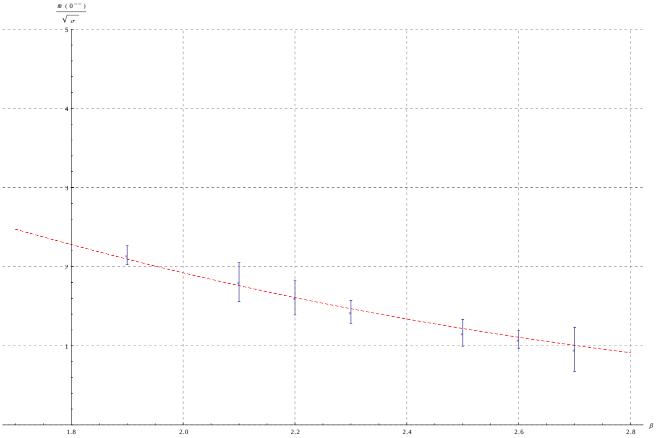


Figure: Fit with $A \cdot 2\pi(2\pi\beta)^{3/4}e^{-\pi^2\frac{\beta}{2}v(0)}$, $A = 0.55(2)$.

Effective String theory predictions

1 - The string width - 64^2x16 Lattice, $\beta = 2.2$

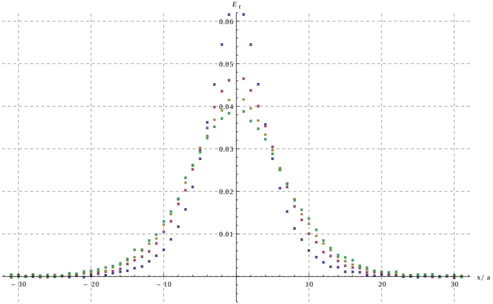


Figure: Measured values of E in the symmetry plane of two sources for various values of intersource distance.

Effective String theory predictions

1 - The string width - $64^2 x 16$ Lattice, $\beta = 2.2$

$$\omega^2 = \frac{1}{2\pi\sigma} \log \frac{L}{L_c} + \frac{R}{4\sigma L} - \frac{e^{-2\pi \frac{R}{L}}}{\sigma\pi}$$

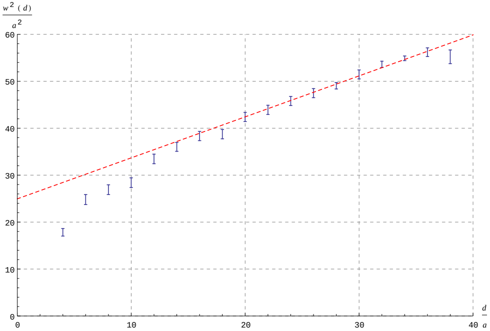
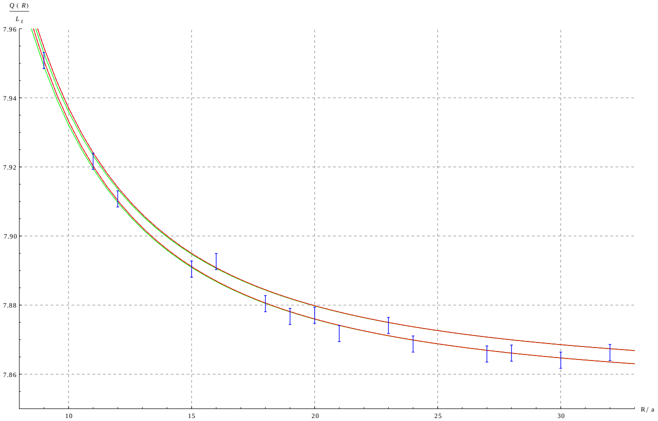


Figure: Measured values of ω^2 fitted with $\omega^2 = a + bd$ for $d \gg 1$. The fit parameters take the values $a = 25(2)$ and $b = 0.87(6)$ in agreement with $\frac{1}{4\sigma L}$.

Effective String theory predictions

2 - $Q(R) = F(R+1, L) - F(R, L)$ at $L = 64$, $\beta = 1.7$

$$\frac{m_D}{\sqrt{\sigma}} \sim 2.5$$



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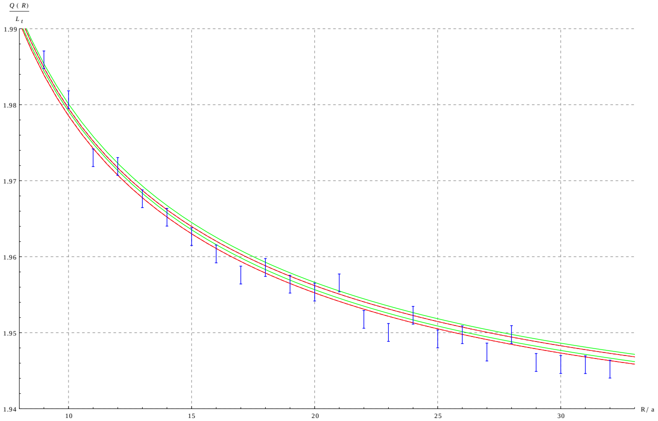

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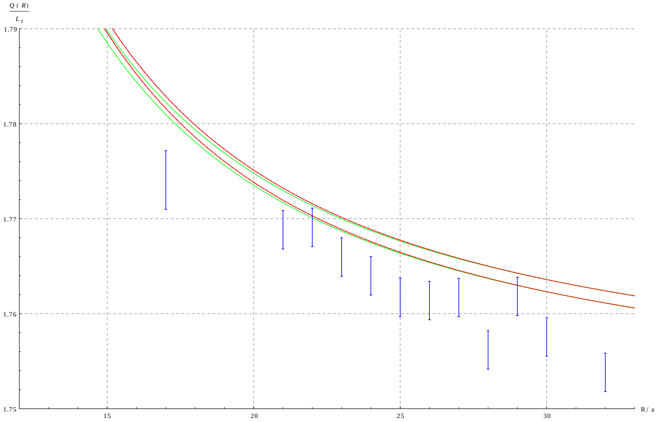

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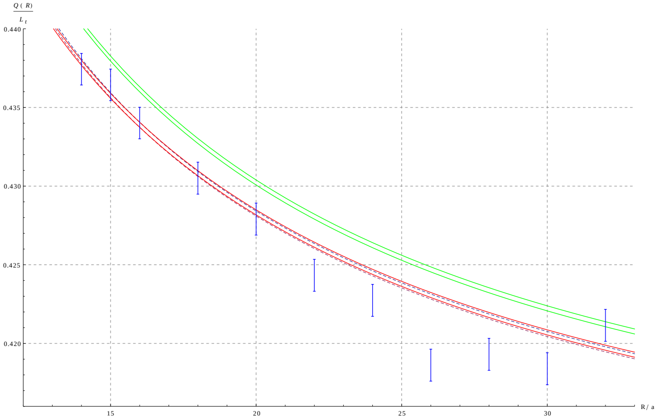

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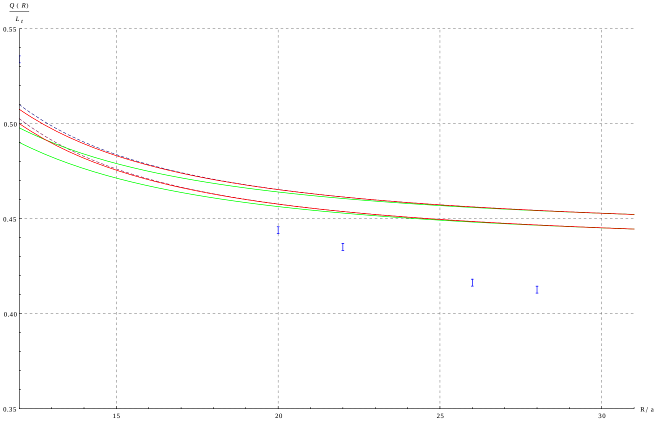

Effective String theory predictions

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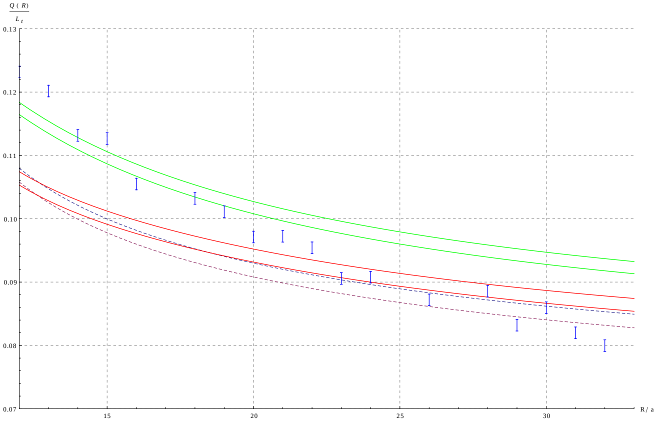

Conclusions

- The behaviour of $Q(R)$ predicted in the framework of effective string theory is confirmed by the data, within errors, at the next-to-leading order for $\beta = 1.7$.
- The deviations from the predicted behaviour seem to grow with β: they are bigger where glueball effects are expected to be important.
- The predicted behaviour of the flux tube width with intercharge distance is confirmed by the data, within errors, at the leading order.