Progress in Gauge-Higgs Unification on the Lattice (II)

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Definition: action

Orbifold action

Anisotropic Wilson plaquette action for gauge group $SU(2)$ on a $T \times L^3 \times (N_5 + 1)$ lattice

$$S_W^{\text{orb.}} = \frac{\beta}{2} \sum_n \Big[\frac{1}{\gamma} \sum_{\mu<\nu} w \, \mathrm{tr}\, \{1 - P_{\mu,\nu}(n)\} + \gamma \sum_\mu \mathrm{tr}\, \{1 - P_{\mu,5}(n)\} \Big]$$

$$w = \begin{cases} \frac{1}{2} & \text{boundary plaquette} \\ 1 & \text{in all other cases.} \end{cases} , \qquad \pi R = N_5 a_5 \, ,$$

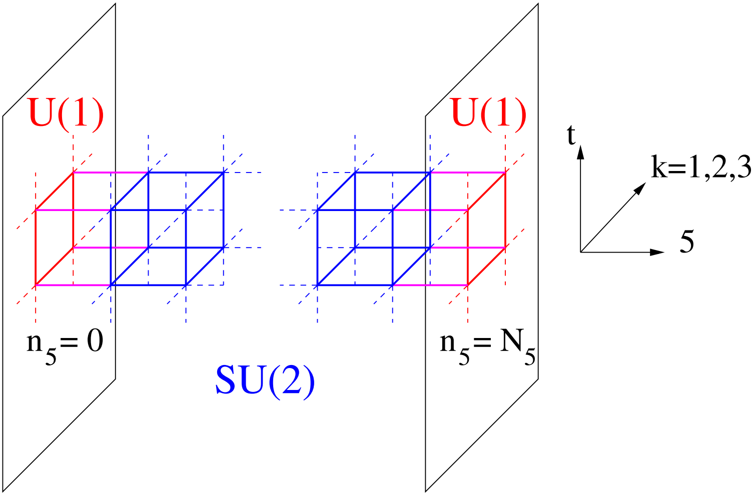
$$\beta_4 = \beta/\gamma \, , \quad \beta_5 = \beta\,\gamma \, , \quad \gamma = a_4/a_5 \text{ (classical level)}$$

Orbifold space: extra dimension is an interval $n_5 \in [0, N_5]$ with Dirichlet boundary conditions

$$U_\mu(n) = g\, U_\mu(n)\, g^{-1} \Rightarrow U_\mu(n) = \mathrm{e}^{i\phi(n)\sigma^3} \in U(1)$$

at $n_5 = 0$ and $n_5 = N_5$ with $g = -i\sigma^3$ [Irges and FK, 2005]

Definition: fields



Definition: symmetries

Local symmetries

Gauge invariance: $SU(2)$ in the bulk, $U(1)$ on the boundaries

Global symmetries

$$S_L \otimes S_R \otimes U(1)_L \otimes U(1)_R \otimes F \otimes C$$

Stick symmetries $S_{L(R)}$ [Ishiyama, Murata, So and Takenaga, 2010]

$$S_L: \quad U_5(n_5{=}0) \to g_s^{-1} U_5(n_5{=}0)\,, \quad U_\nu(n_5{=}0) \to g_s^{-1} U_\nu(n_5{=}0) g_s$$

with $g_s = \mathrm{e}^{i\theta}(-i\sigma^2)$
$U(1)_{L(R)}$ transform only links on the left (right) boundary

$$U(1)_L: \quad U_\nu(n_5{=}0) \to \mathrm{e}^{i\alpha\sigma^3} U_\nu(n_5{=}0)$$

F is a flip with respect to $n_5 = N_5/2$ and C is a global sign flip

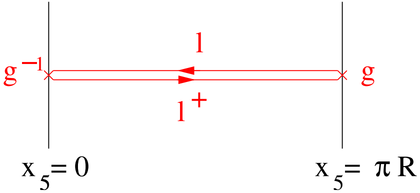
Definition: Higgs operators

Polyakov line P on $S^1/\mathbb{Z}_2$

Orbifolded Polyakov loop

$$P = lgl^\dagger g^{-1}$$

where $g = -i\sigma^3$



Lattice Higgs operators:

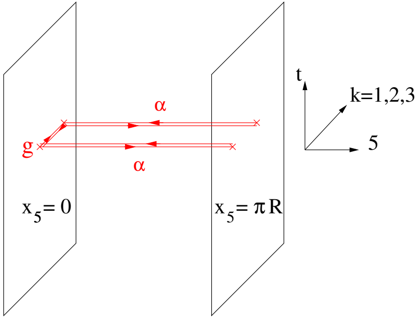
- $\mathrm{tr}\,\{P\}$: $\not{S}_L,\ \not{S}_R,\ U(1)_L,\ U(1)_R,\ F$
- $\mathrm{tr}\,\{\Phi\Phi^\dagger\}$: $S_L,\ S_R,\ U(1)_L,\ U(1)_R,\ F$
  with $\Phi = 1/(4N_5)\,[P - P^\dagger, g] = \phi^1\sigma^1 + \phi^2\sigma^2$

Definition: Gauge-boson operators

Vector Polyakov line Z_k on $S^1/\mathbb{Z}_2$

$$Z_k = g\, U_k\, \alpha\, U_k^{-1} \alpha$$

with $\alpha = \Phi/\sqrt{-\det\,\Phi}$



cf. 4d SU(2) Higgs model [Montvay, 1985, 1986], but here Φ transforms like a $U(1)$ field strength with charge 2

- $\mathrm{tr}\,\{Z_k\}$: $\not{S}_L,\ S_R,\ \not{U}(1)_L,\ U(1)_R,\ \not{F}$

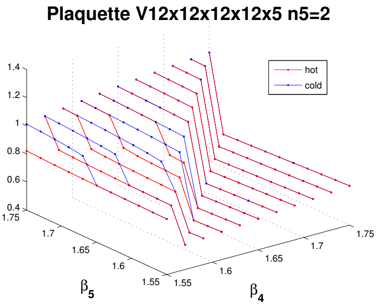
Isotropic

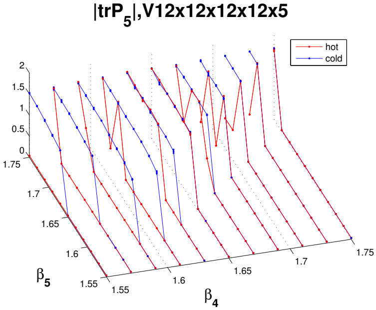
$\gamma = 1$, $12^4 \times 5$

Observables: 4d plaquette at $n_5 = 2$, $|\mathrm{tr}\{P\}|$ and $(\mathrm{tr}\{Z_k\})^2$

red: hot start, blue: cold start (4000 measurements)

Bulk SU(2) phase transition around $\beta = 1.65$ (like with periodic boundary conditions)

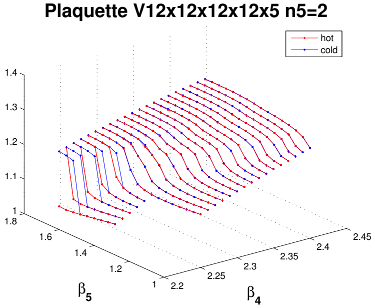


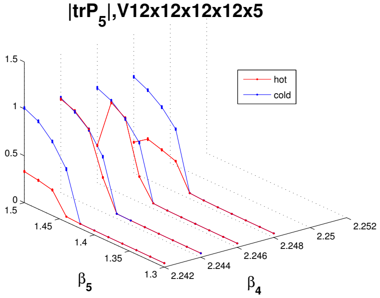


Anisotropic

$12^4 \times 5$

The bulk hysteresis has an end-point at $\gamma < 1$ (cf. mean-field calculation, talk by K. Yoneyama)

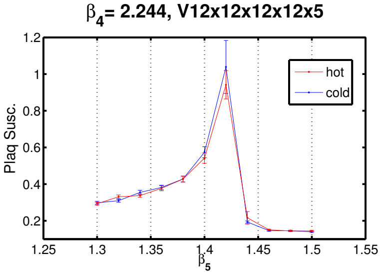


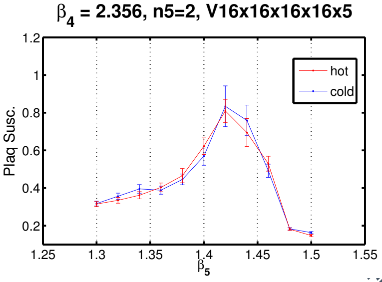


Anisotropic

Nature of the end-point transition

The maximum of the plaquette susceptibility is approximately constant at the end-point transition on $12^4 \times 5$ and $16^4 \times 5$ (its location changes with L) $\Rightarrow$ cross-over





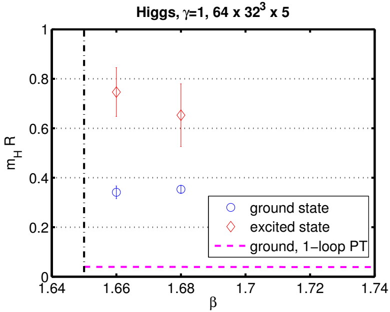
Higgs and Z-boson masses

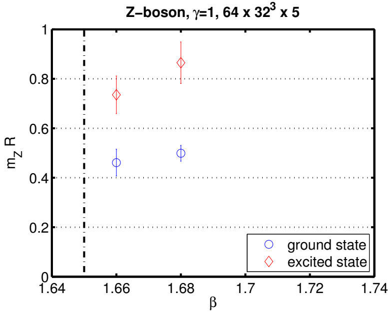
Isotropic

Lattices $64 \times 32^3 \times 5$ at $\gamma = 1$

$m_Z \neq 0$ does not decrease with L (Higgs mechanism!) and $m_Z \gtrsim m_H$

We see excited states for the Higgs and the Z-boson





Conclusions and outlook

Conclusions

- Non-perturbative Gauge-Higgs Unification on orbifold in pure $SU(2)$ gauge theory
- Isotropic : 1st order phase transition, $m_Z \neq 0$ but $\rho_{HZ} = m_H/m_Z \lesssim 1$
- Anisotropic, $\gamma < 1$: end-point of the 1st order transition is most likely a cross-over

Outlook

- Behavior of the masses as the cross-over at $\gamma < 1$ is approached $\longrightarrow$ effective theory?
- Dimensional reduction and its effects on the static potential