Multi-boson spectrum of the SU(2)-Higgs model

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Introduction

- The confinement region contains glueballs and QCD-like bound states of the scalar fields.
- The Higgs region contains the Higgs boson and three degenerate W bosons, which agrees with perturbation theory.
- We want to study the Higgs region using lattice simulations with parameters tuned to match the standard model.

Introduction

- We want to study the entire $I(J^P)$ spectrum (for $I = 0, 1$) on the lattice.
- Our parameters (β,κ,λ) are tuned to match experimental data: $m_H = 125$ GeV, $m_W = 80.4$ GeV, $\frac{g^2}{4\pi} \approx \frac{\alpha}{\sin^2\theta_W} \approx 0.04$
- Our lattice study found more than a dozen energy levels.

Particles versus Fields

- The SU(2)-Higgs Lagrangian contains gauge dependent scalar $\phi(x)$ and gauge $U_\mu(x)$ fields.
- Physical particle states are found from gauge-invariant operators that are composites of the fields.
- Higgs boson $0(0^+)$ couples to $\mathrm{Tr}\left(\phi^\dagger(x)\phi(x)\right)$
- W boson $1(1^-)$ couples to $\mathrm{Tr}\left(-i\sigma^a\phi^\dagger(x)U_\mu(x)\phi(x+\hat{\mu})\right)$
- In principle, particle states couple to any operator with the correct quantum numbers.

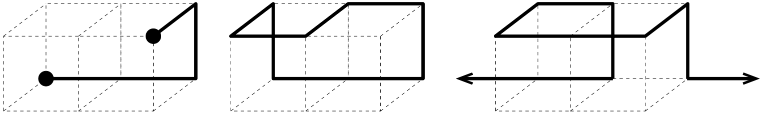
Lattice Irreducible Representations, Λ

Λ	J							
	0	1	2	3	4	5	6	...
A_1	1	0	0	0	1	0	1	...
A_2	0	0	0	1	0	0	1	...
E	0	0	1	0	1	1	1	...
T_1	0	1	0	1	1	2	1	...
T_2	0	0	1	1	1	1	2	...

- Angular momentum on the lattice corresponds to the irreps Λ of the octahedral group of rotations.
- Higgs, $\mathrm{Tr}\left(\phi^\dagger(x)\phi(x)\right)$: $I(\Lambda^P) = 0(A_1^+)$
- W, $\mathrm{Tr}\left(-i\sigma^a\phi^\dagger(x)U_\mu(x)\phi(x+\hat{\mu})\right)$: $I(\Lambda^P) = 1(T_1^-)$

Gauge-Invariant Operators

- Gauge invariant link, Wilson loop and Polyakov loop operators with zero momentum were used to extract the spectrum.
- Intricate operator shapes were used to access all $I(\Lambda^P)$ quantum numbers.



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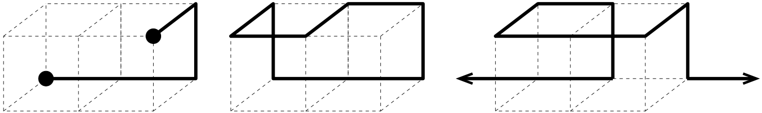


Operator Smearing

- Stout link and scalar field smearing were used to improve the operators, and generate a basis for a variational analysis.
- Number of stout link and scalar smearing iterations: 0, 5, 10, 25, 50, 100, 150, and 200.

Variational Analysis

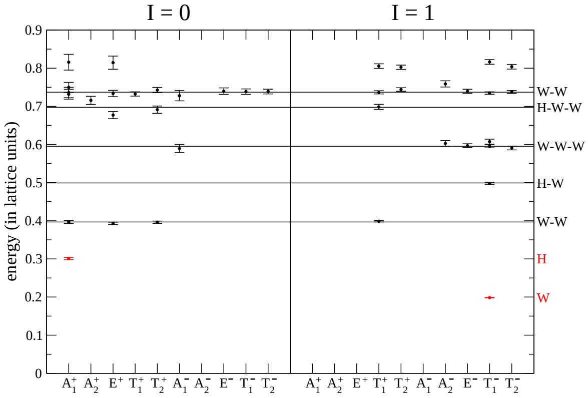
- The low-lying energy spectrum is extracted from correlation matrices of vacuum subtracted gauge-invariant (Hermitian) operators.

$$C_{ij}(t) = \langle \mathcal{O}_i(t)\mathcal{O}_j(0)\rangle = \sum_n \langle 0|\,\mathcal{O}_i\,|n\rangle\,\langle n|\,\mathcal{O}_j\,|0\rangle \exp\left(-E_n t\right)$$
$$= \sum_n a_i^n a_j^n \exp\left(-E_n t\right)$$

- The variational method iteratively projects out the lightest energies from the correlation matrix $C_{ij}(t)$.

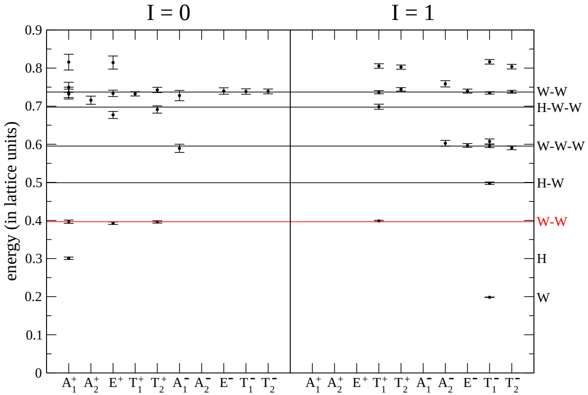
$$C_n(t) = z_n^i C_{ij}(t) z_n^j = A_n \exp\left(-E_n t\right)$$

Spectrum with a Physical Higgs Mass



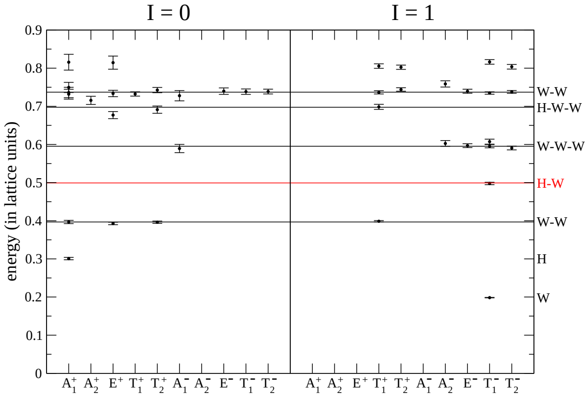
- Lattice parameters: $20^3 \times 40$, $\beta = 8$ (physical gauge coupling), $\kappa = 0.131$, $\lambda = 0.0033 \Rightarrow m_H = 122 \pm 1$ GeV, $m_W = 80.4 \pm 0.2$ GeV

Spectrum with a Physical Higgs Mass



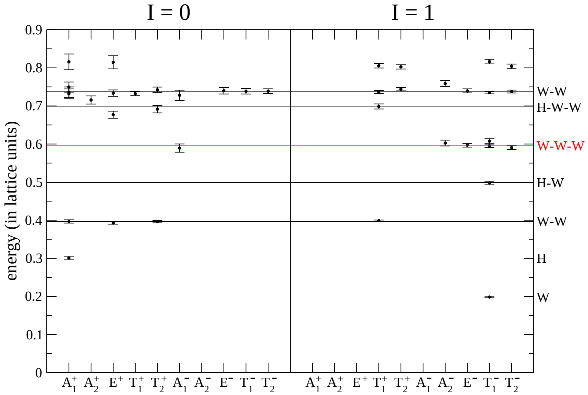
- Expected quantum numbers for two stationary W bosons: $0(0^+)$, $0(2^+)$ and $1(1^+) \Longrightarrow 0(A_1^+)$, $0(E^+)$, $0(T_2^+)$ and $1(T_1^+)$

Spectrum with a Physical Higgs Mass



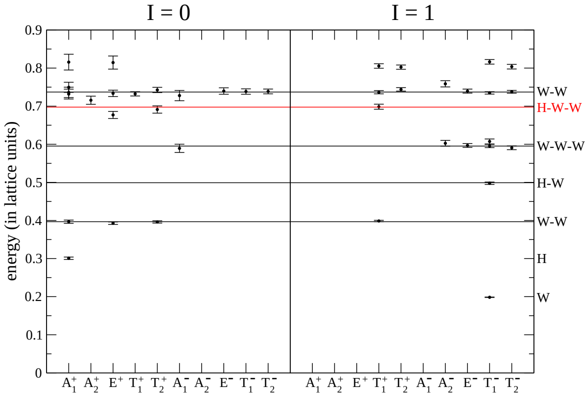
- Expected quantum numbers for stationary Higgs and W boson: Same as a single W boson: $1(1^-) \Longrightarrow 1(T_1^-)$

Spectrum with a Physical Higgs Mass



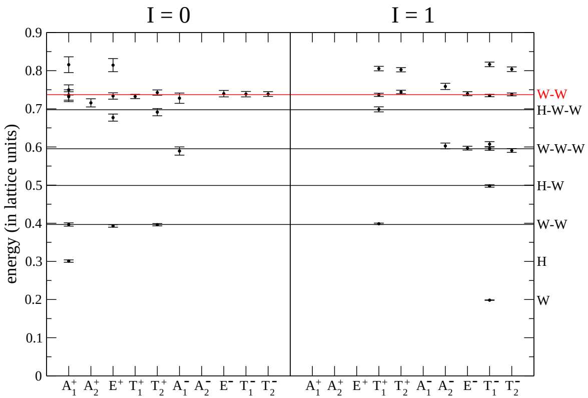
- Expected quantum numbers for three stationary W bosons: $0(0^-)$, $1(1^-)$, $1(2^-)$, $1(3^-)$ $\Longrightarrow$ $0(A_1^-)$, $1(T_1^-)$, $1(E^-)$, $1(T_2^-)$, $1(A_2^-)$

Spectrum with a Physical Higgs Mass



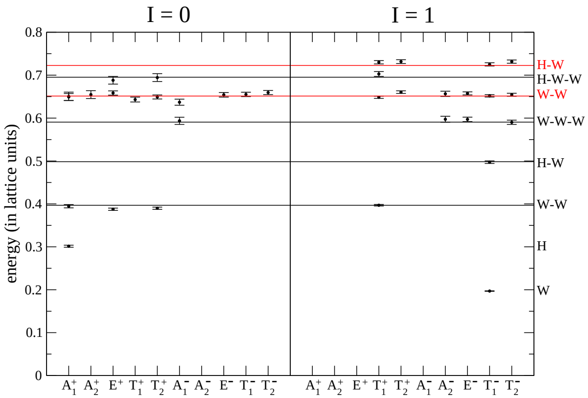
- Expected quantum numbers for two stationary W bosons and a Higgs boson: Same as two W bosons

Spectrum with a Physical Higgs Mass



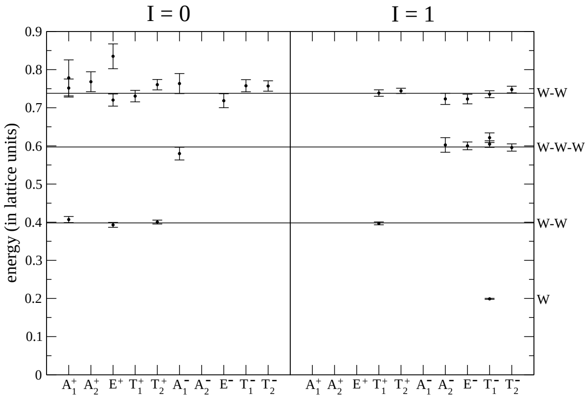
- Expected quantum numbers for two moving W bosons with total momentum zero and the minimal back to back internal momentum: All $I(\Lambda^P)$ channels!

Spectrum from a Larger ($24^3 \times 48$) Lattice



- Two-W states with minimal internal momentum now appear at 0.65
- Higgs-W states with minimal internal momentum at 0.72

Spectrum with a Heavy Higgs



- Repeat with $20^3 \times 40$, $\beta = 8$, $\kappa = 0.40$, $\lambda = \infty$
- $m_H = 720 \pm 70$ GeV and $m_W = 80.4 \pm 0.6$

Successes for Part 1

- Identified the Higgs-W multi-particle spectrum.
- Two-W, three-W, Higgs-W and Higgs-W-W states with no internal momentum appeared in all of the expected channels.

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Remaining for Part 2

- Find the two-Higgs state, which was missing in Part 1.
- Explain missing irreps for two-particle states with equal and opposite internal momentum.

Solution: Construct "Two"-Particle Operators

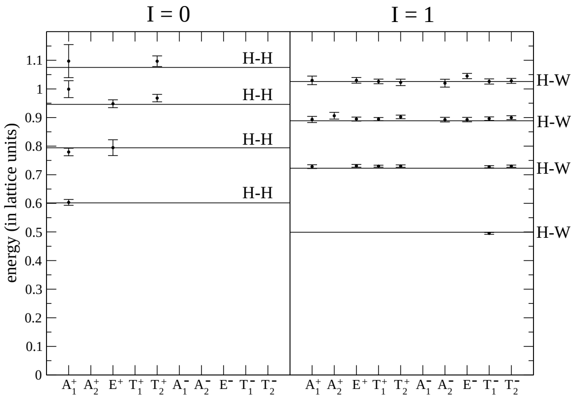
$$H(\vec{p}) = \sum_{\vec{x}} \frac{1}{2} \operatorname{Tr}\left\{\phi^\dagger(x)\phi(x)\right\} \exp\left\{i\vec{p}\cdot\vec{x}\right\}$$

$$W_\mu^a(\vec{p}) = \sum_{\vec{x}} \frac{1}{2} \operatorname{Tr}\left\{-i\sigma^a\phi^\dagger(x)U_\mu(x)\phi(x+\hat{\mu})\right\} \exp\left\{i\vec{p}\cdot\left(\vec{x}+\tfrac{1}{2}\hat{\mu}\right)\right\}$$

- Multiply operators of definite momentum:

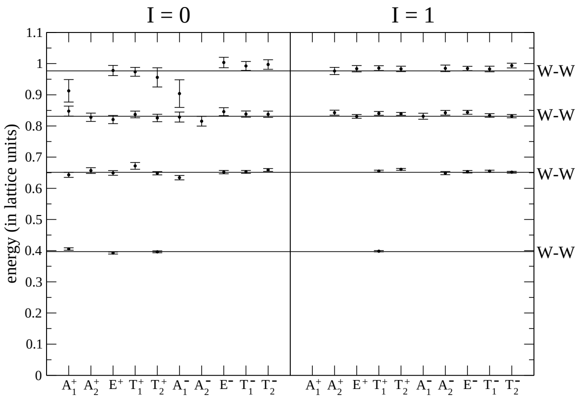
$$\begin{aligned} H(\vec{p})H(-\vec{p}) \quad &, \; I = 0 \\ H(\vec{p})W_\mu^a(-\vec{p}) \quad &, \; I = 1 \\ W_\mu^a(\vec{p})W_\nu^a(-\vec{p}) \; &, \; I = 0 \\ \epsilon^{abc}W_\mu^b(\vec{p})W_\nu^c(-\vec{p}) \; &, \; I = 1 \end{aligned}$$

Higgs-Higgs and Higgs-W Spectrum



- Two-Higgs states are now found.

W-W Spectrum



- The direction of the internal momentum of multi-particle states affects the allowed lattice irreps.

Conclusion

- The entire SU(2)-Higgs energy spectrum has been studied with all parameters tuned to match the standard model.
- Multi-boson spectrum was observed and is consistent with collections of weakly interacting Higgs and W bosons.