Symanzik Flow on HISQ Ensembles

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Outline

Wilson/Symanzik Flow

- Wilson flow is a smoothing of the original gauge fields U towards stationary points of the Wilson action S. [Lüscher, JHEP 1008 (2010) 071]
- Successive links $V(t)$ are updated in flowtime according to the diffusion equation,

$$\frac{d}{dt}V(t)_{i,\mu} = -V_{i,\mu}\frac{\partial S(V)}{\partial V_{i,\mu}}, \quad V(t)_{i,\mu}(0) = U_{i,\mu} \quad \left[\frac{dA_\mu}{dt} = D_\nu F_{\nu\mu}\right]$$

- Cuts out high momenta noise, thereby suppressing statistical fluctuations and discretization effects at minimal computational cost
- Used the Symanzik improved action ($\approx 2x$ cost) to further reduce discretization errors.

Scale Setting

- The scale can be extracted through the flowtime $t[a^2]$.
- Define an improved, dimensionless quantity through the energy density $\langle E(t) \rangle$. [BMW (S. Borsanyi et al.), JHEP 1209 (2012) 010]

$$W(t) = t\frac{d}{dt}\left(t^2\langle E(t)\rangle\right)$$

- In the continuum, the energy density $\langle E(t) \rangle$ is finite (at least to one loop order) when expressed in terms of renormalized quantities. [Lüscher, JHEP 1008 (2010) 071]
- Empirically, the combination $t^2\langle E(t) \rangle$ varies linearly with t for large flow times.
- The $w_0[a]$ scale is defined from the cutoff at 0.3.

$$w_0 = \sqrt{t_c}\,,\ W(t_c) = 0.3$$

- The value of the cutoff is chosen to minimize discretization and finite volume effects.

Ensembles with $m_s \approx m_s^{physical}$

a(fm)	m_l/m_s	nx^3nt	N_{run}	w_0/a (stat) [%]
0.15	1/5	$16^3 48$	1021	1.1221 (06) [0.06%]
0.15	1/10	$24^3 48$	1000	1.1381 (04) [0.04%]
0.15	1/27	$32^3 48$	999	1.1468 (03) [0.03%]
0.12	1/5	$24^3 64$	1040	1.3835 (07) [0.05%]
0.12	1/10	$24^3 64$	1020	1.4020 (10) [0.07%]
0.12	1/10	$32^3 64$	999	1.4047 (06) [0.05%]
0.12	1/10	$40^3 64$	1001	1.4041 (04) [0.03%]
0.12	1/27	$48^3 64$	34	1.4168 (10) [0.07%]
0.09	1/5	$32^3 96$	102	1.8957 (16) [0.08%]
0.09	1/10	$48^3 96$	151	1.9296 (09) [0.05%]
0.09	1/27	$64^3 96$	53	1.9473 (11) [0.06%]
0.06	1/5	$48^3 144$	127	2.8956 (26) [0.09%]
0.06	1/10	$64^3 144$	46	2.9486 (31) [0.11%]
0.06	1/27	$96^3 192$	49	3.0119 (18) [0.06%]

Non-Physical Strange Mass Ensembles

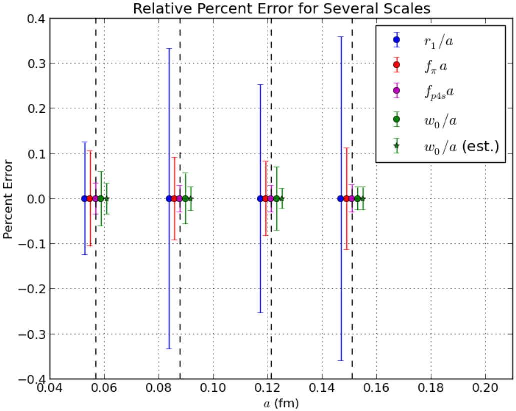
- m'_s and m_l are the sea quark masses
- m_s is the physical strange quark mass

a(fm)	m_l/m_s	m'_s/m_s	nx^3nt	N_{run}	w_0/a (stat) [%]
0.12	0.10	0.10	$32^3 64$	102	1.4833 (13) [0.09%]
0.12	0.10	0.25	$32^3 64$	204	1.4676 (11) [0.07%]
0.12	0.10	0.45	$32^3 64$	205	1.4470 (11) [0.08%]
0.12	0.10	0.60	$32^3 64$	107	1.4351 (20) [0.14%]
0.12	0.175	0.45	$32^3 64$	134	1.4349 (13) [0.09%]
0.12	0.20	0.60	$24^3 64$	255	1.4170 (10) [0.07%]
0.12	0.25	0.25	$24^3 64$	255	1.4336 (16) [0.11%]

- Most ensembles have ≈ 1000 configurations, so N_{run} can still be increased considerably to improve statistics.

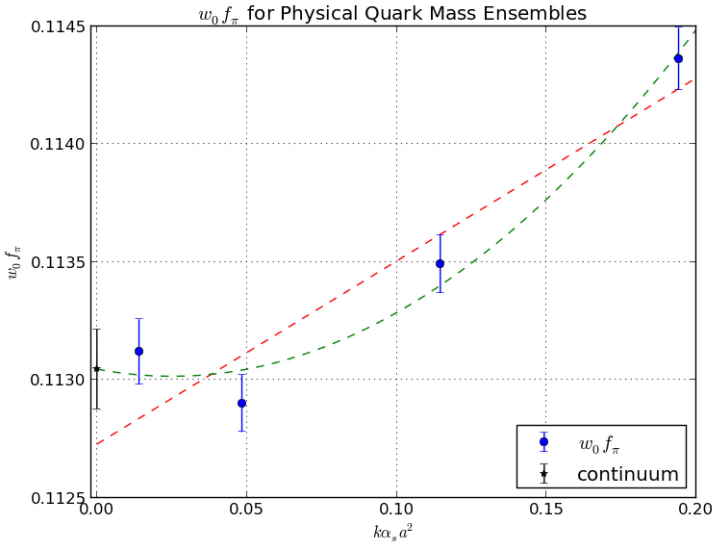
Error Comparison to Other Scales

Physical Quark Mass Ensembles



- 'est' stands for estimate for full ensemble run using conservative estimates of the autocorrelation length
- dashed vertical lines denote the lattice spacing for each ensemble; all scales are at these lattice spacings but data points are separated horizontally to make the comparison easier

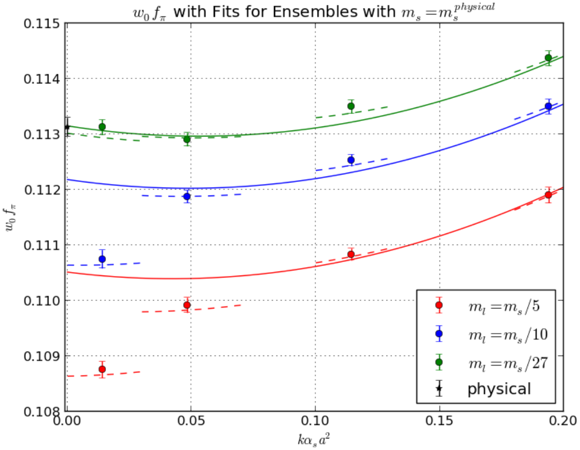
Naive Continuum Extrapolation



Fit Forms

- Including quark mass dependence allows us to include ensembles with $m_s \neq m_s^{physical}$ and correct for fine-tuning errors.
- Using M_π^2 and $2M_K^2 - M_\pi^2$ as proxies for m_l and m_s, included up to cubic powers in the quark mass.
- To extrapolate to the continuum, included $k\alpha_s a^2$ and higher orders of a^2, up to a^6 (k is a constant).
- Due to the large range of m_s covered by the full set of ensembles, some fits drop various ensembles with low values of m_s.

Continuum, Physical Quark Mass Extrapolation

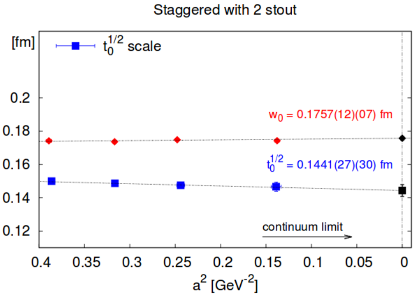


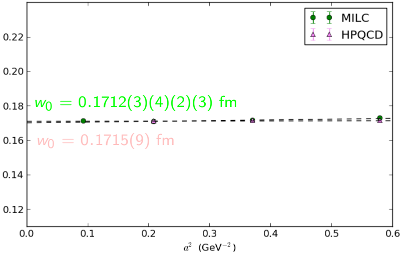
- Only $m_s = m_s^{physical}$ ensembles are plotted, but fit includes all $m_s \leq m_s^{physical}$ ensembles
- Dotted lines are for actual masses run; solid lines are for re-tuned masses per legend

Current Results for w_0

- Central fit has $\chi^2/dof = 7.5/10$, $p = 0.68$
- Found 78 different fits with $p > 0.01$; used the standard deviation of the fits' extrapolated values to estimate the systematic uncertainty at $4e^{-4}$ fm
- There is also residual finite volume error in f_π that cannot be corrected for, adding another systematic error of $2e^{-4}$ fm.
- Preliminary Result: $w_0 = 0.1712(3)(4)(2)(3)$ fm
  First is the statistical error, then systematic error from the continuum extrapolation, residual finite volume effects, and experimental value of f_π, respectively.
- As a sanity check, the naive fit through the four physical quark mass ensembles found 0.1711(3)(3)(2) fm. The naive fit is in good agreement with the improved fit.

Comparison (HPQCD, BMW)

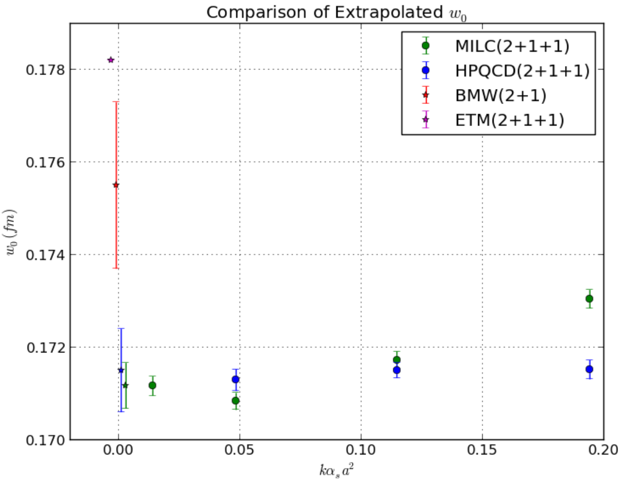




BMW: [BMW (S. Borsanyi et al.), JHEP 1209 (2012) 010]

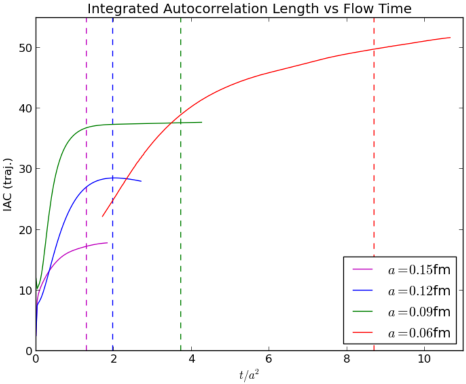
HPQCD: [HPQCD (R. J. Dowdall et al.), arXiv:1303.1670]

Comparison cont...



- ETM and MILC values are preliminary
- ETM did not provide an error estimate for w_0.
- The BMW point is for their final, quoted value on the HEX smeared Wilson ensembles.

Integrated AutoCorrelation Length of $\langle E(t) \rangle$



- Each solid lines corresponds to the ensemble at the ratio $m_l/m_s = 1/10$ and specified a
- Dashed lines correspond to the value of w_0 for each ensemble
- The $a = 0.06$fm ensemble ran with a larger separation between configurations; the low resolution yields noise at low autocorrelation lengths.

Summary / Discussion

- Our preliminary value of

  $$w_0 = 0.1712(3)(4)(2)(3) \text{ fm}$$

  agrees with HPQCD within 1σ, but deviates from BMW by 2.2σ compared to their final, HEX smeared Wilson result $w_0 = 0.1755(18)(04)$ fm
- This deviation may be due to the difference in N_f. However, ETM also used Nf $= 2 + 1 + 1$ ensembles and found a central value even higher than that of BMW. But without an error estimate, the significance of this result is unclear.
- Statistical errors are still being improved.
- We found integrated autocorrelation lengths that are fairly large: up to 55 trajectories on the $a = 0.06$ fm ensemble. These are comparable to but generally smaller than those found for twisted mass ensembles [ETM (A. Deuzeman, U. Wenger), PoS (Lattice 2012) 162].