Determination of the non-degenerate light quark masses from electromagnetic mass splittings in 2+1 flavour lattice QCD+QED

Lattice 2013, Mainz

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I'm working with Tom Blum, Ran Zhou, Taku Izubuchi, Chris Sachrajda.

OUTLINE

INTRODUCTION

BACKGROUND

QED

QCD + QED

CONCLUSION

MOTIVATION

Up until now there has been no QED on the lattice.
All charges = 0 and $m_u = m_d$
If up and down quarks are the same, isospin is conserved.
Not true in nature:

- Masses are not equal (QCD breaking of isospin)
- Charges are not equal (QED breaking of isospin)

$$m_u = 2.3^{+0.7}_{-0.5}\ \mathrm{MeV} \quad q_u = +\frac{2}{3}e \tag{1}$$

$$m_d = 4.8^{+0.7}_{-0.3}\ \mathrm{MeV} \quad q_d = -\frac{1}{3}e \tag{2}$$

(quoted in $\overline{\mathrm{MS}}$ 2GeV) Source: PDG (2012)

The (small) breaking of isospin has observable effects.

MOTIVATION

If isospin were conserved, anything where an up quark can be swapped with a down quark (and vice versa) would be identical.

- Charged-Neutral Pions: $\pi^+(u\bar{d}) \equiv \pi^-(d\bar{u}) \equiv \pi^0 \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$.
  But $m_{\pi^\pm} - m_{\pi^0} = 4.5936(5)\text{MeV}$.

- Protons and neutrons: $p(uud) \equiv n(udd)$.
  But $m_n - m_p = 1.2933322(4)\text{MeV}$.

- Sigma Baryons: $\Sigma^+(uus) \equiv \Sigma^-(dds)$.
  But $m_{\Sigma^-} - m_{\Sigma^+} = 8.08(8)\text{MeV}$.

MOTIVATION

- Expected size of IB effects: $\alpha_{em} \simeq 0.0073$ and $\frac{m_d - m_u}{\Lambda_{QCD}} \simeq 0.01$.
- Taken together: 1% effect.
- We are coming to the stage in lattice calculations where we can resolve 1% effects.
- Everything we calculate on the lattice can be improved by including QED.

Main point of this work is to find physical quark masses: m_u, m_d, m_s

Suggested reading: [arXiv:hep-lat/9602005]
[0708.0484]
[1006.1311]
[1011.4408]

WHAT IS QED?

$$S_{QED}[A] = \frac{1}{4}\int d^4x\ (\partial_\mu A_\nu(x) - \partial_\nu A_\mu(x))^2 \tag{3}$$

This is called the non-compact formulation of QED. The compact formulation would introduce photon self-interactions that dissapear in the continuum.
This is a *free* theory, Gaussian noise $\rightarrow$ can generate configurations quickly.
Once we generate A^{QED}, we exponentiate (compactify) it via:

$$U_\mu = exp(ieQA_\mu^{QED})U_\mu^{QCD} \tag{4}$$

to couple it to QCD.
This is then Quenched QED. Can reuse previously generated ensembles.

QED ON LATTICE

We have a problem with QED on the lattice because the photon is massless (no mass gap). We might expect finite volume effects to be large.
Include terms to account for the finite volume.
Additionally, the zero mode of the photon has to be removed for the theory to be well defined.

$$\int \frac{dk}{k^2} \to \frac{1}{V} \sum \frac{1}{k^2} \tag{5}$$

A single point of the integral has measure zero, so we can change it (take to zero).
Because this is non-compact QED, we need to gauge fix it. We use the Feynman gauge.

FULL DISCUSSION: PORTELLI ET AL. (2011) [ARXIV:1011.4189]

WHAT DO WE EXPECT?

Dashen's theorem states that the QED squared mass difference between the charged pseudoscalar mesons and their neutral partners are equal in the SU(3) chiral limit:

$$\Delta_{QED} M_K^2 - \Delta_{QED} M_\pi^2 = 0 \tag{6}$$

where $\Delta_{QED} M_P^2 = (M_{P^\pm}^2 - M_{P^0}^2)_{m_u = m_d}$.

DASHEN'S THEOREM

This is violated by terms of order $\mathcal{O}(\alpha_{em} m)$ away from the chiral limit. Using chiral perturbation theory, we can identify these corrections and find the non-degenerate quark masses by matching to experimentally measured mass splittings.
Violations to Dashen's theorem can be parametrised by the FLAG parametrisation:

$$\varepsilon = \frac{\Delta_{QED} M_K^2 - \Delta_{QED} M_\pi^2}{\Delta M_\pi^2} \tag{7}$$

DEFINED IN: G. COLANGELO ET AL. (FLAG). REVIEW OF LATTICE RESULTS EUR. PHYS. J. C 71 P. 1695 (2011)

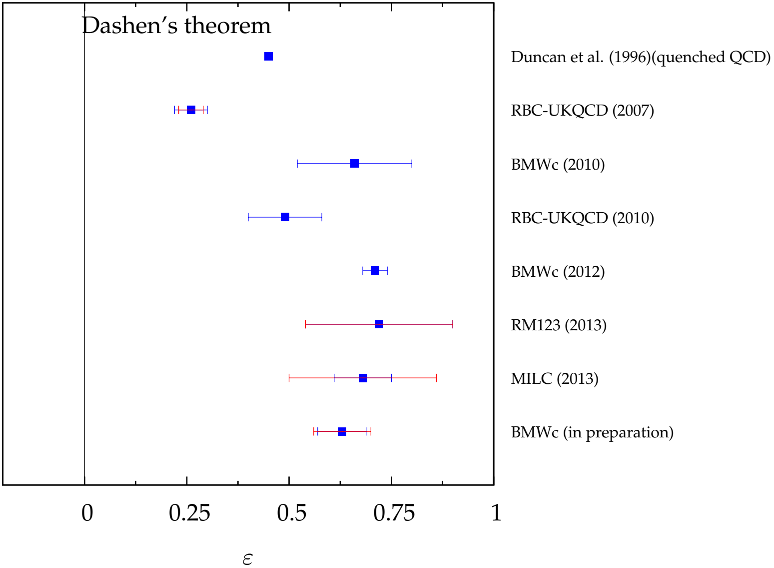

Figure: Summary of lattice calculations of Dashen's theorem violation, ε. Blue error: statistical, Red error: systematic. Source: Portelli (2013) [arXiv:1307.6056]

CHIRAL PERTURBATION THEORY

- Using an effective theory to extrapolate down to physical quark masses.
- Expand the action for small quark masses and derive an effective action.
- Unknown non-perturbative parameters - Low Energy Constants. Must be simulated.

We are using $SU(2)_L \times SU(2)_R$ plus kaon at NLO. $SU(3)$ is poorly convergent for $m \sim m_s$.
Essentially, we have formulae for:

$$\Delta M^2 = f(\text{QCD LECs}, \text{QED LECs}, m_1, m_3, q_1, q_3) \quad (8)$$

$$M^2 = g(\text{QCD LECs}, \text{QED LECs}, m_u, m_d, m_s, q_u, q_d, q_s) \quad (9)$$

Where $\Delta M^2 = M(e \neq 0)^2 - M(e = 0)^2$.

SOURCE: BLUM ET AL. (2010) [ARXIV:1006.1311]

CHIRAL PERTURBATION THEORY FOR PION (NLO)

$$
\begin{aligned}
M^2 = \chi_{13} \Bigg\{ & 1 + \frac{24}{F^2}(2L_6 - L_4)\frac{\chi_4 + \chi_5}{3} + \frac{8}{F^2}(2L_8 - L_5)\chi_{13} \\
& + \frac{1}{2}\frac{1}{16\pi^2 F^2}\left(R^{\pi}_{13}\,\chi_\pi \,\log\frac{\chi_\pi}{\mu^2} + R^{1}_{\pi 3}\,\chi_1 \,\log\frac{\chi_1}{\mu^2} + R^{3}_{\pi 1}\,\chi_3 \,\log\frac{\chi_3}{\mu^2}\right)\Bigg\} \\
& + \frac{2Ce^2}{F^2} q_{13}^2 \\
& - 12e^2 Y_1 \bar{q}^2 \chi_{13} + 4e^2 Y_2 q_p^2 \chi_p + 4e^2 Y_3 q_{13}^2 \chi_{13} \\
& - 4e^2 Y_4 q_1 q_3 \chi_{13} + 12e^2 Y_5 q_{13}^2 \frac{\chi_4 + \chi_5}{3} \\
& - e^2 \frac{3}{16\pi^2}\chi_{13} \log\frac{\chi_{13}}{\mu^2} q_{13}^2 + e^2 \frac{1}{4\pi^2}\chi_{13} q_{13}^2 \\
& - e^2 \frac{C}{F^4}\frac{1}{8\pi^2} q_{13}\left(q_{14}\chi_{14}\log\frac{\chi_{14}}{\mu^2} + q_{15}\chi_{15}\log\frac{\chi_{15}}{\mu^2} - q_{34}\chi_{34}\log\frac{\chi_{34}}{\mu^2} - q_{35}\chi_{35}\log\frac{\chi_{35}}{\mu^2}\right) \\
& + e^2 \delta_{m_{\mathrm{res}}}(q_1^2 + q_3^2)\,.
\end{aligned}
$$

QCD LECs: $B, F, L_4, L_5, L_6, L_8, m_{res}, a^{-1}$ $\quad \chi_{ij} = B(m_i + m_j) \quad \mu = 1\,\mathrm{GeV}$

QED LECs: $C, Y_2, Y_3, Y_4, Y_5, \delta_{mres}$

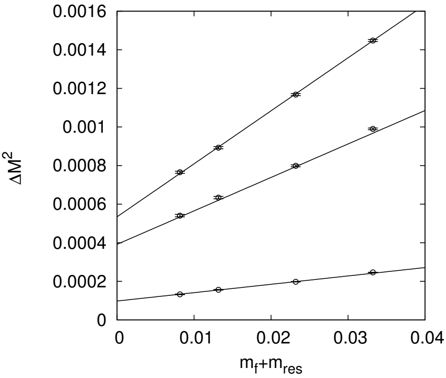

Figure: Meson mass-squared splittings. 24^3 lattice size. Infinite volume linear fit. Data points correspond to $u\bar{d}$, $u\bar{u}$ and $d\bar{d}$ mesons respectively, from top to bottom.

BLUM ET AL. (2010) [ARXIV:1006.1311]

STRATEGY

1. Simulate charged and neutral mesons
2. Find their masses $M(e \neq 0)$ and $M(e = 0)$
3. Compute the mass squared difference $\Delta M^2 = M(e \neq 0)^2 - M(e = 0)^2$
4. Do this for all combinations of masses / charges
5. We now have many values of the mass squared difference as a function of mass / charge of the valence quarks
6. Can fit the QED low energy parameters and get estimates on them
7. Now can extrapolate to physical pion and find non-degenerate masses of quarks

We are using $M_{\pi^+},\ M_{K^0},\ M_{K^+}$ to fix the quark masses. We don't use M_{π^0} - disconnected diagrams.

SIMULATION DETAILS

Action	β	$(am_{ud,sea})$	(am_{val})	$(am_{s,sea})$	L/a	N_{conf}	a (fm)
I	2.13	0.005	0.001, 0.005, 0.01, 0.02, 0.03	0.04	24	195	0.114
		0.01	0.001, 0.01, 0.02, 0.03			180	
		0.02	0.02			360	
		0.03	0.03			360	
I	2.25	0.04	0.04	0.03	32	131	0.086
		0.06	0.06			188	
		0.08	0.08			~ 81	
I+DSDR	1.75	0.001	0.001	0.045	32	48(13)	0.146
		0.0042	0.0042			44(12)	

- RBC-UKQCD Collaboration ensembles. 2+1 flavour Domain Wall Fermions and Iwasaki Gauge Action.
- Iwasaki: $24^3 \times 64 \times 16$, $32^3 \times 64 \times 16$. Iwasaki+DSDR: $32^3 \times 64 \times 32$
- Iwasaki: Lightest unitary pion 293 MeV.
- Iwasaki + DSDR: Lightest unitary pion 170 MeV.
- Varying the charges of each quark in the set $\{-2, -1, 0, 1, 2\} \times 1/3e$

DETAILS: RBC AND UKQCD, R. ARTHUR ET AL., (2012), [ARXIV:1208.4412]

WHAT HAS BEEN DONE ALREADY

I have reproduced the results from the 24^3 paper. The 32^3 results need more analysis.
Initially had only Iwasaki - need more sensitivity to strange. Add Iwasaki+DSDR.
Tried just fitting to π^+ and π^0 to find m_u, m_d. Errors due to neglecting disconnected diagrams too great in π^0.
Tried fitting to K^+, K^0 and π^+ but just set $m_s = 95$ MeV. Seems to work well.

$$\begin{aligned} m_u &= 2.264(82)\ \text{MeV} \\ m_d &= 4.815(40)\ \text{MeV} \\ m_u/m_d &= 0.471(18) \\ m_d - m_u &= 2.551(97)\ \text{MeV} \end{aligned}$$

(quoted in $\overline{\text{MS}}$ 2 GeV). Infinite volume fit.
Current status: Combining I and ID ensembles to give access to strange mass dependence. Adding more configurations.

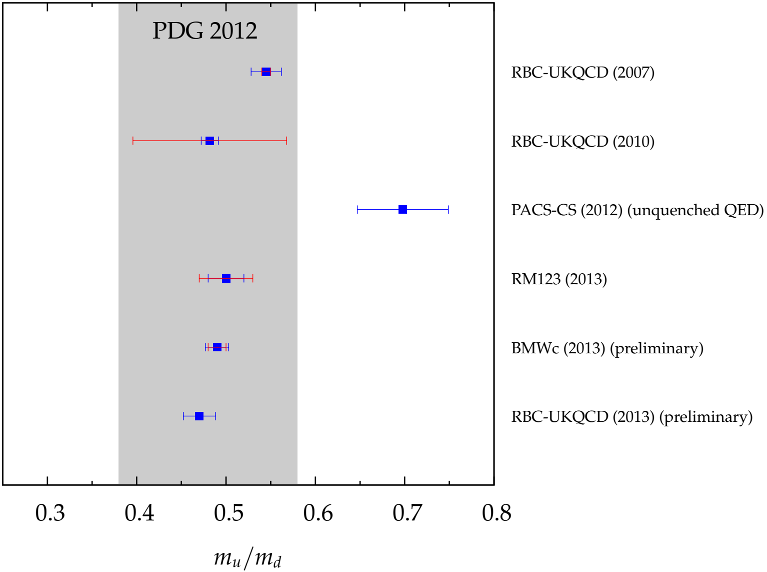

Figure: Summary of lattice calculations of the up and down quark mass ratio.
Source: Portelli (2013) [arXiv:1307.6056]

WHAT HAS TO BE DONE?

We have a number of configurations on 32^3 lattices.

- Iwasaki has been analysed.
- Add in Iwasaki+DSDR.
- Get sensitivity to strange quark mass.
- Increase statistics for Iwasaki+DSDR.
- Add corrections due to finite volume effects.

CONCLUSION

- In lattice calculations we are getting to the stage where we can resolve 1% effects.
- If we consider $\alpha_{em} \simeq 0.0073$ and $\frac{m_d - m_u}{\Lambda_{QCD}} \simeq 0.01$ taken together, they are are of order 1%.
- The strong and EM breaking of isospin must therefore be taken into account if we want to have accurate predictions of hadron masses.
- Additionally, we wish to resolve the difference in mass between the physical up and down quarks.
- 32^3 results have to be analysed.

CONCLUSION

THANK YOU!