The strange degrees of freedom in QCD at high temperature

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Abstract

We use up to fourth order cumulants of net strangeness fluctuations and their correlations with net baryon number fluctuations to extract information

- on the strange meson and baryon contribution to the low temperature hadron resonance gas,
- on the dissolution of strange hadronic states in the crossover region of the QCD transition,
- on the quasi-particle nature of strange quark contributions to the high temperature quark-gluon plasma phase.

$T \lesssim T_c$

$T_c \lesssim T \lesssim 1.3T_c$

$T \gtrsim 1.3T_c$

$T_c :=$ chiral crossover temperature

Outline

1) Introduction

- Definitions of cumulants and correlations
- Motivations to study fluctuations of conserved charges

2) The lattice setup and results

- The HISQ action
- 4^{th} order fluctuations and correlations

3) A closer look to strangeness

- Strangeness in the HRG model
- Disentangling different strangeness sectors

4) Summary

based on BNL-BI: arXiv:1304.7220

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Generalized Susceptibilites / Cumulants

Expansion of the pressure:

$$\frac{p}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk,0}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

$X = B, Q, S$: conserved charges

Lattice

$$\chi_{n,0}^X = \frac{1}{VT} \frac{\partial^n \ln Z}{\partial(\mu_X/T)^n}\bigg|_{\mu_X=0}$$

generalized susceptibilities

$\Rightarrow$ only at $\mu_X = 0$!

Experiment

$$\begin{aligned} VT^3 \chi_2^X &= \left\langle (\delta N_X)^2 \right\rangle \\ VT^3 \chi_4^X &= \left\langle (\delta N_X)^4 \right\rangle - 3 \left\langle (\delta N_X)^2 \right\rangle^2 \\ VT^3 \chi_6^X &= \left\langle (\delta N_X)^6 \right\rangle \\ &\quad -15 \left\langle (\delta N_X)^4 \right\rangle \left\langle (\delta N_X)^2 \right\rangle \\ &\quad +30 \left\langle (\delta N_X)^2 \right\rangle^3 \end{aligned}$$

cumulants of net-charge fluctuations

$$\delta N_X \equiv N_X - \langle N_X \rangle$$

$\Rightarrow$ only at freeze-out $(\mu_f(\sqrt{s}), T_f(\sqrt{s})$)!

ratios of cumulants are volume independent!

Motivations

1) Discover a critical point (if exists)

- Analyze Taylor series/Pade resummations of various susceptiblities: find region of large fluctuations
- Analyze the radius of convergence directly

2) Analyze freeze-out conditions

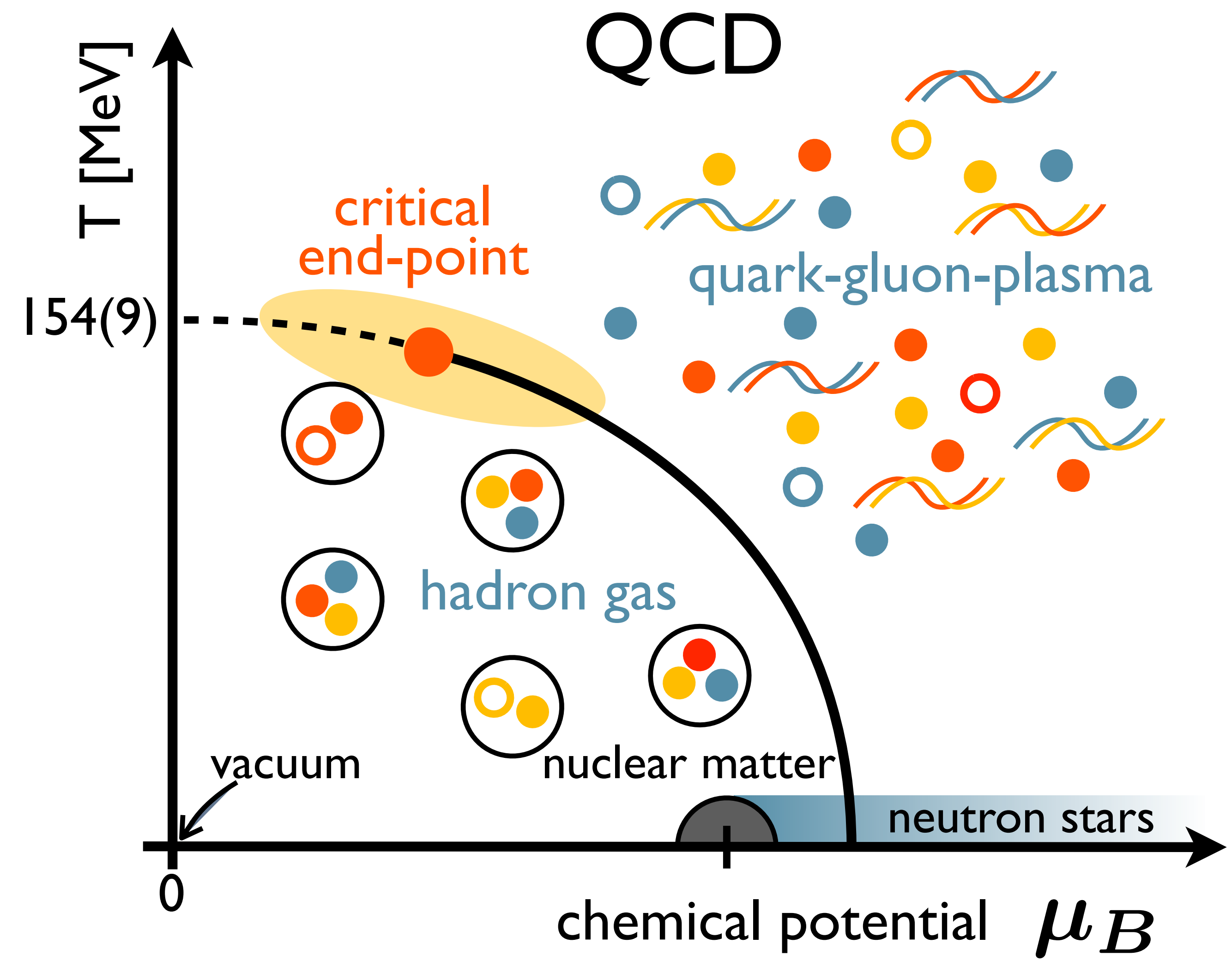
- Match various cumulant ratios of measured electric charge fluctuations to (lattice) QCD results: determine freeze-out parameters.

$\longrightarrow$ see talk by M. Wagner

3) Identify the relevant degrees of freedom (this talk)

- Compare (lattice) QCD fluctuations to various hadronic/quasiparticle models:

Expected phase diagram of QCD:



Does deconfinement take place above the chiral crossover temperature?

Motivations

Does deconfinement take place above the chiral crossover temperature?

The chiral crossover line:

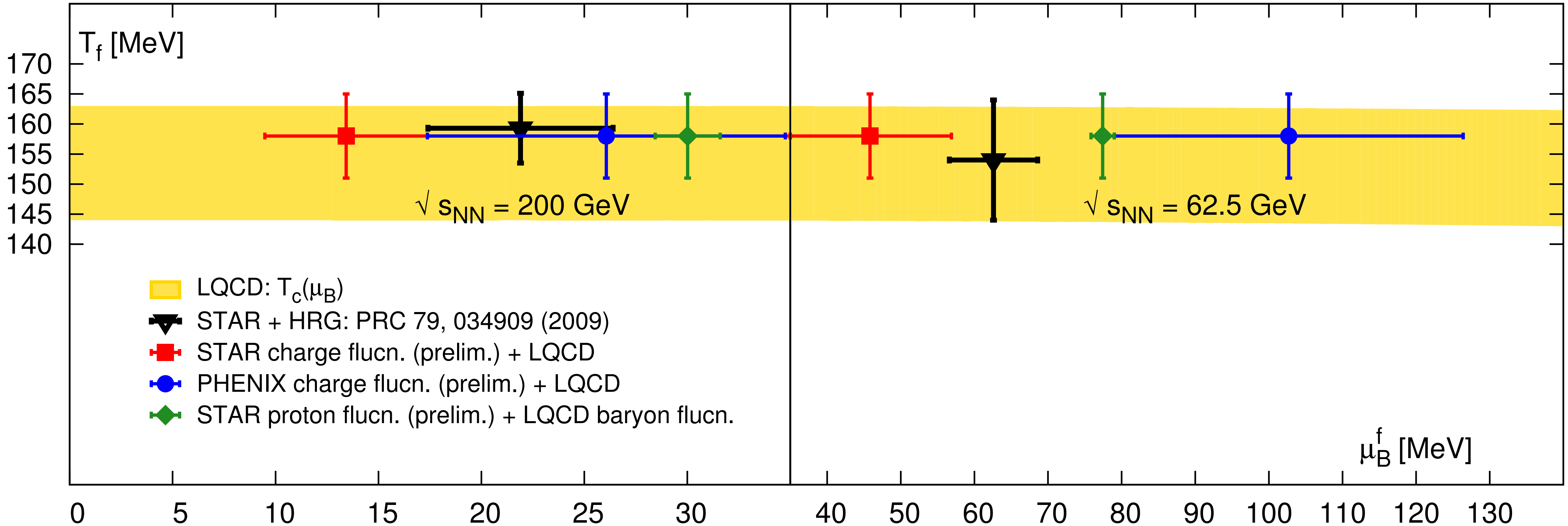
$$T_c = 154(9)\ \mathrm{MeV}$$

HotQCD, PRD 85 (2012) 054503

$$T_c(\mu_B) = T_c(0)\left[1 - 0.0066(7)\mu_B^2\right]$$

BNL-BI, PRD 83 (2011) 014504

Karsch, CPOD 2013, arXiv:1307.3978



$\Rightarrow$ freeze-out points are in agreement with the chiral crossover line

$\Rightarrow$ apparent discrepancies among the freeze-out points that need to be resolved

The Lattice Setup

Action: highly improved staggered quarks (HISQ)

Lattice size: $24^3 \times 6,\ 32^3 \times 8,\ 48^3 \times 12$

Mass: $m_q = m_s/20 \longrightarrow m_\pi \approx 160\ \mathrm{MeV}$

Statistics: O(10^3) - O(10^4)

Observables: traces of combinations of M and $M' = \partial M/\partial\mu$

$$\frac{\partial \ln Z}{\partial \mu} = \frac{1}{Z}\int \mathcal{D}U\ \mathrm{Tr}\left[M^{-1}M'\right]\ e^{\mathrm{Tr}\ \ln M} e^{-\beta S_G}$$

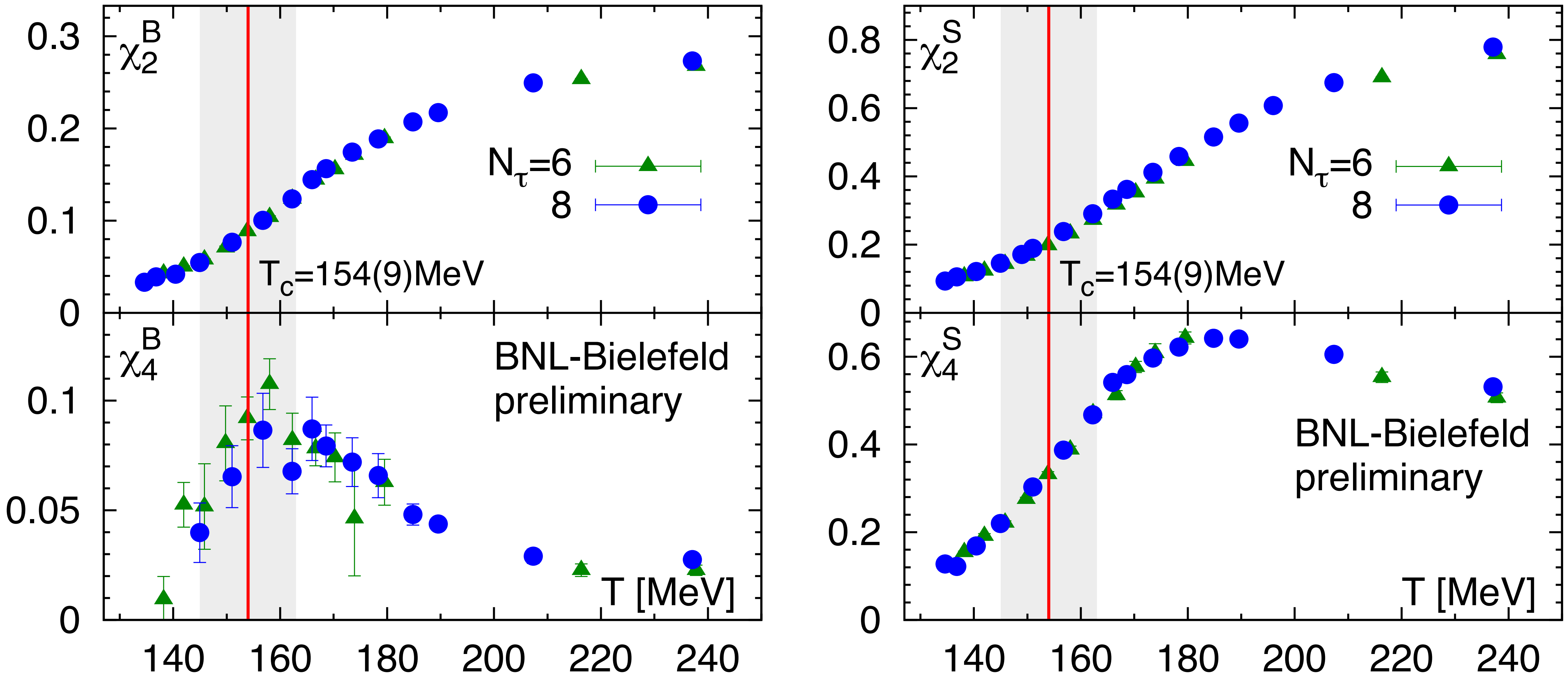
$$= \left\langle \mathrm{Tr}\left[M^{-1}M'\right]\right\rangle$$

$$\frac{\partial^2 \ln Z}{\partial \mu^2} = \left\langle \mathrm{Tr}\left[M^{-1}M''\right]\right\rangle - \left\langle \mathrm{Tr}\left[M^{-1}M'M^{-1}\right]\right\rangle + \left\langle \mathrm{Tr}\left[M^{-1}M'\right]^2\right\rangle$$

Method: stochastic estimators with $N = 1500$ random vectors

$$\mathrm{Tr}\left[Q\right] \approx \frac{1}{N}\sum_{i=1}^{N} \eta_i^\dagger Q \eta_i \qquad \text{with} \qquad \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \eta_{i,x}^\dagger \eta_{i,y} = \delta_{x,y}$$

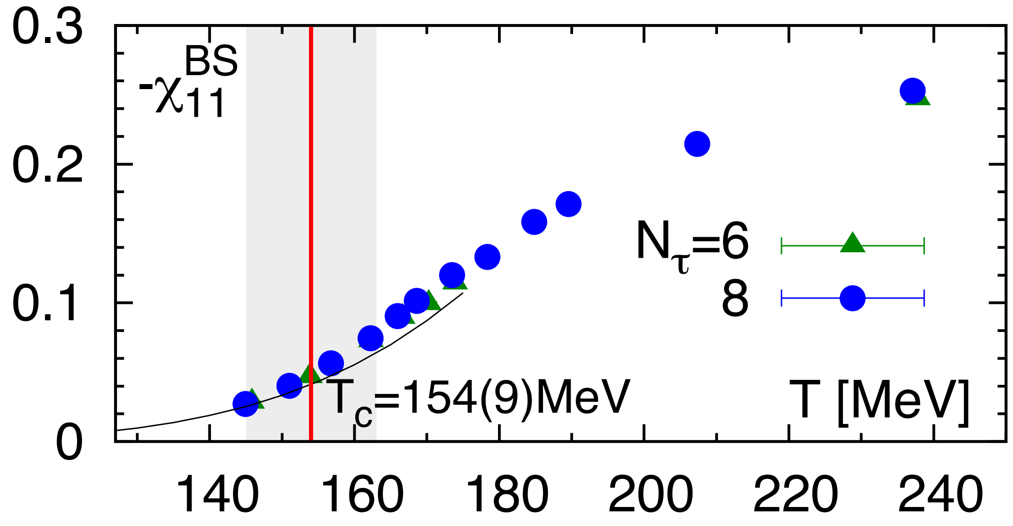
Generalized Susceptibilites / Cumulants



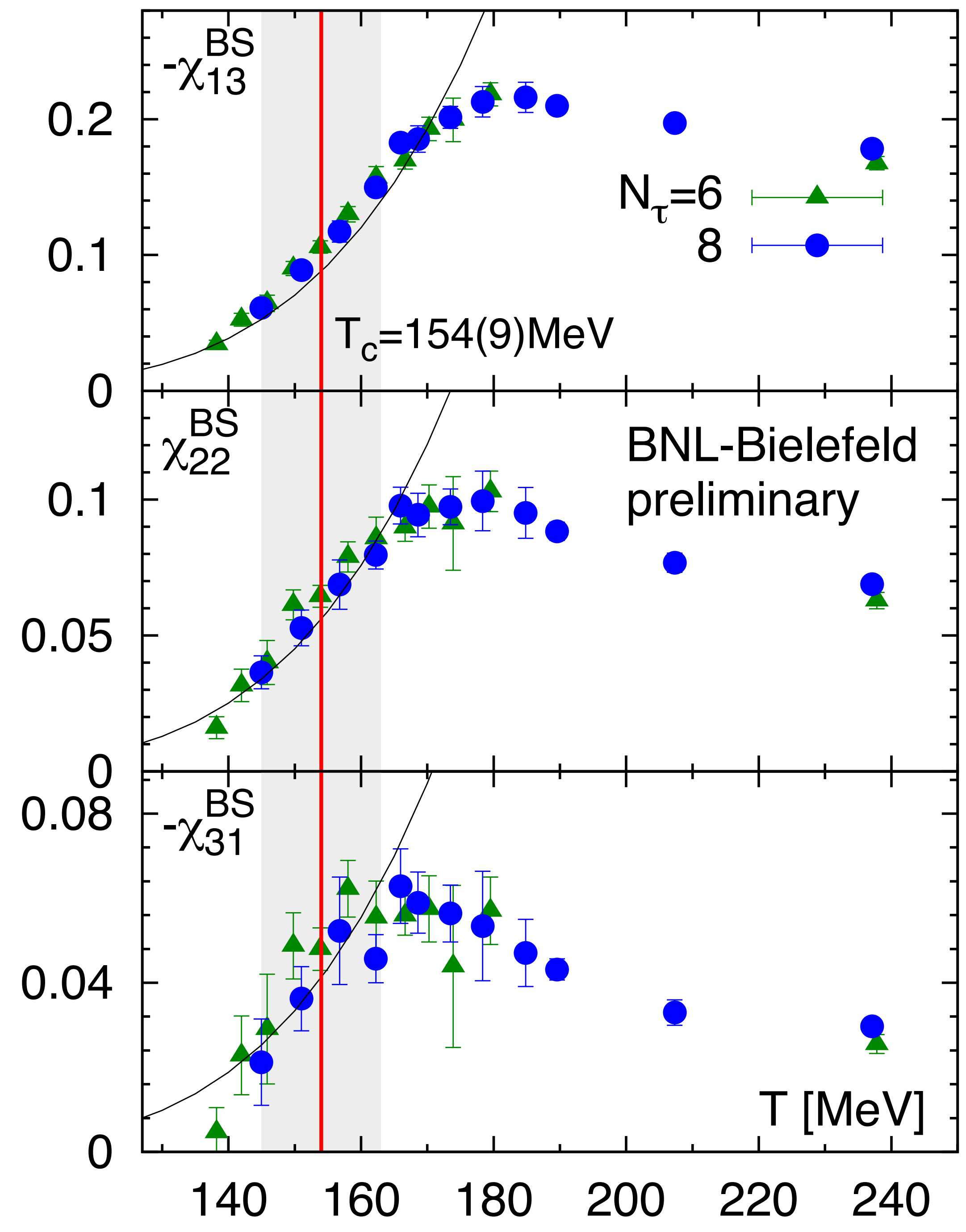
$\Rightarrow$ structure consistent with O(4) critical behavior at $\mu_B = 0,\ m = 0$

BS Correlations

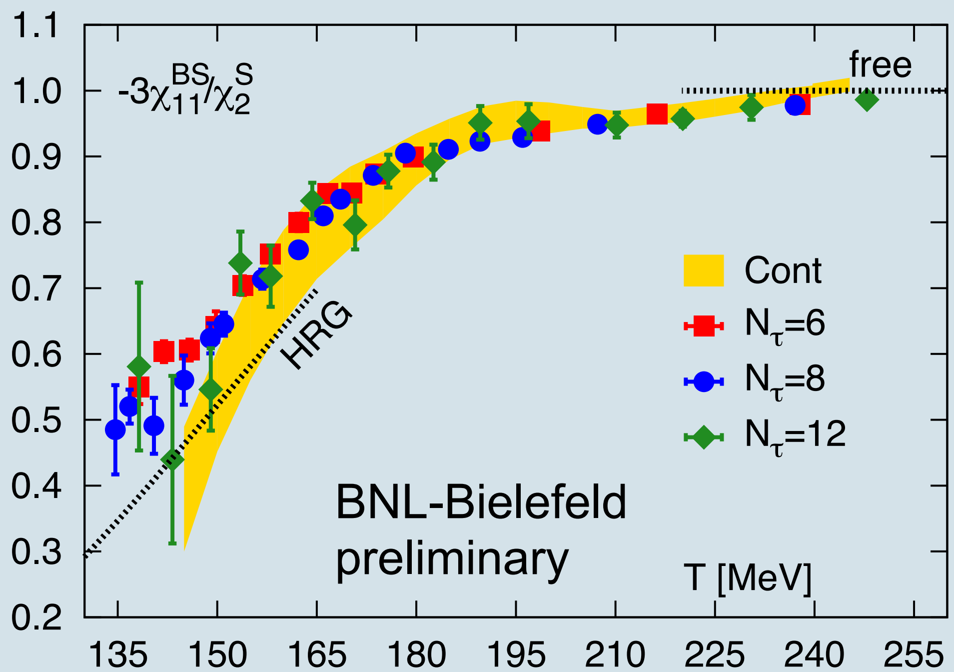
2^{nd} order



4^{th} order



C_{BS} [Koch et al., PRL 95 (2005) 182301]



Strangeness within the HRG

HRG:

$$\frac{p^{HRG}}{T^4} = \frac{1}{VT^3} \sum_{i \in \, mesons} \ln \mathcal{Z}^M_{m_i}(T,V,\mu_S) + \frac{1}{VT^3} \sum_{i \in \, baryons} \ln \mathcal{Z}^B_{m_i}(T,V,\mu_B,\mu_S)$$

with

$$\ln \mathcal{Z}^{M/B}_{m_i} = \frac{VT^3}{\pi^2} d_i \left(\frac{m_i}{T}\right)^2 \sum_{k=1}^{\infty} \frac{(\pm 1)^{k+1}}{k^2} K_2(km_i/T) \cosh\left(k(B_i\hat{\mu}_B + S_i\hat{\mu}_S)\right)$$

here we have set $\mu_Q \equiv 0$

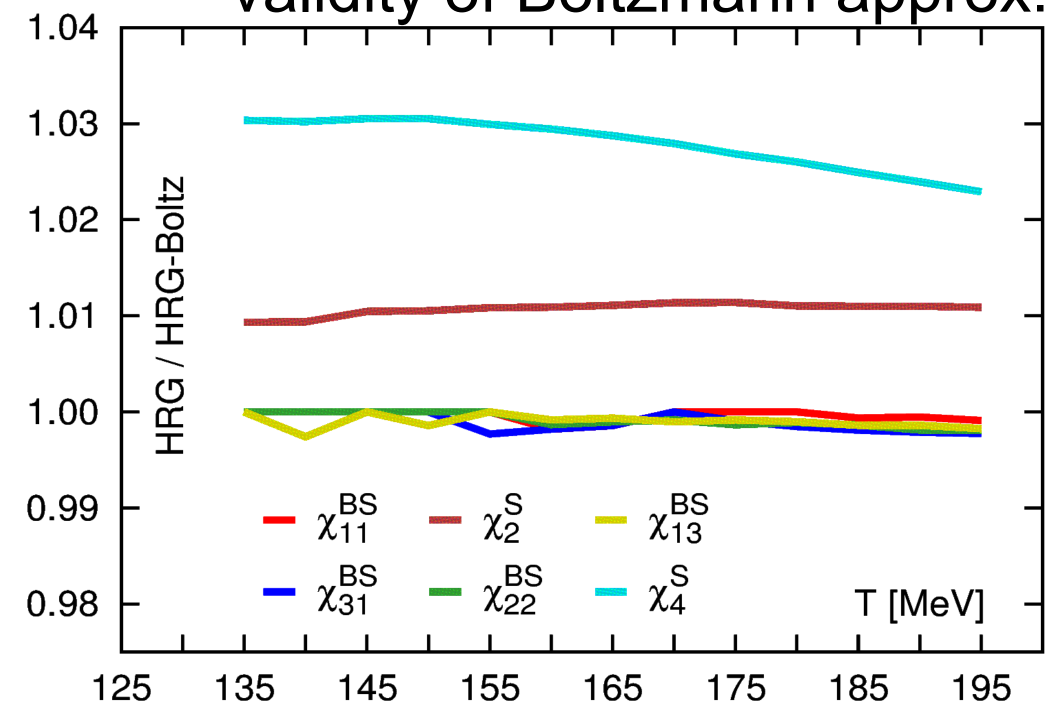
- For baryons and strange particles we can assume Boltzmann approximation:

$$\ln \mathcal{Z}^{M/B}_{m_i} \quad \propto \quad f(m_i,T)\cosh(B_i\hat{\mu}_B + S_i\hat{\mu}_S)$$

$\Rightarrow$ μ_B, μ_S-dependence factorizes

$\Rightarrow$ pressure obtains contributions from different hadronic sectors, defined by all possible (B_i, S_i)-combinations

validity of Boltzmann approx.



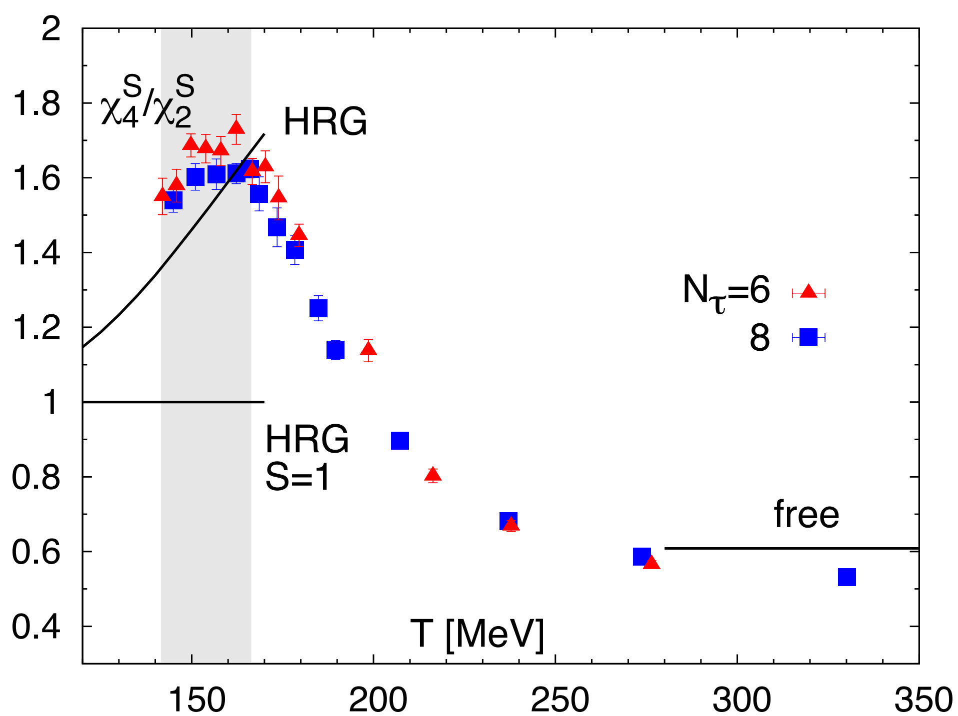
Strangeness within the HRG

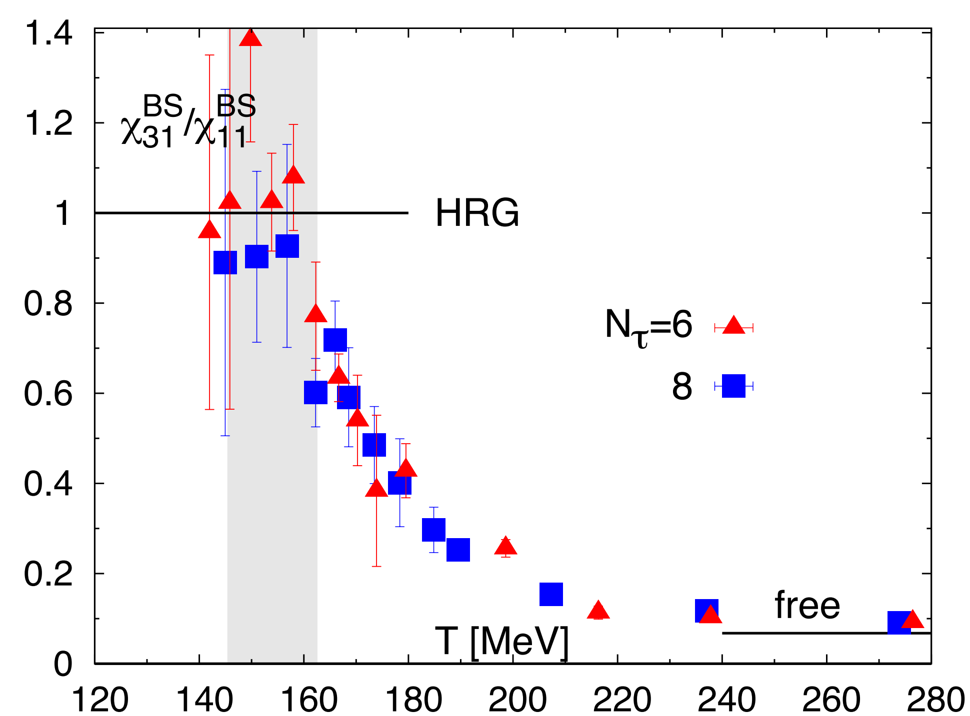
The pressure obtains contributions from different hadronic sectors:

$$
\begin{aligned}
\frac{p^{HRG}}{T^4} = & \left. f_{(0,0)}(T) + f_{(0,1)}(T)\cosh(-\hat{\mu}_S) \right\} \textit{mesons} \\
& \left. \begin{aligned} &+ f_{(1,0)}\cosh(\hat{\mu}_B) + f_{(1,1)}(T)\cosh(\hat{\mu}_B - \mu_S) \\ &+ f_{(1,2)}(T)\cosh(\hat{\mu}_B - 2\hat{\mu}_S) + f_{(1,3)}(T)\cosh(\hat{\mu}_B - 3\hat{\mu}_S) \end{aligned} \right\} \textit{baryons}
\end{aligned}
$$

$\Longrightarrow$ for diagonal fluctuations: $\chi_n^B \equiv \chi_{n+2}^B$, whereas $\chi_n^S \not\equiv \chi_{n+2}^S$ (multi-strange hadrons)

$\Longrightarrow$ for correlations: $\chi_{n,m}^{BS} \not\equiv \chi_{n+2,m}^{BS}$





- at $T \lesssim 160\ \mathrm{MeV}$ we find reasonable agreement with the HRG (within 20%)
- at $T \gtrsim 160\ \mathrm{MeV}$ deviations from HRG become large

Disentangling different strangeness sectors

The pressure obtains contributions from **4 different strangeness** sectors:

$$\frac{p^{HRG}}{T^4} = f_{(0,0)}(T) + f_{(0,1)}(T)\cosh(-\hat{\mu}_S) \quad \Big\} \textit{ mesons}$$

$$+ f_{(1,0)}\cosh(\hat{\mu}_B) + f_{(1,1)}(T)\cosh(\hat{\mu}_B - \mu_S) + f_{(1,2)}(T)\cosh(\hat{\mu}_B - 2\hat{\mu}_S) + f_{(1,3)}(T)\cosh(\hat{\mu}_B - 3\hat{\mu}_S) \quad \Big\} \textit{ baryons}$$

$$\Rightarrow \left(\frac{p^{HRG}}{T^4}\right)_{S\neq 0} = M_1 + B_1 + B_2 + B_3$$

$\Rightarrow$ diagonal fluctuations and correlations are given as linear combinations of the different strangeness sectors

$$\begin{aligned}
\chi_2^S &= (-1)^2 M_1 + (-1)^2 B_1 + (-2)^2 B_2 + (-3)^2 B_3 \\
\chi_4^S &= (-1)^4 M_1 + (-1)^4 B_1 + (-2)^4 B_2 + (-3)^4 B_3 \\
\chi_{11}^{BS} &= (-1) B_1 + (-2) B_2 + (-3) B_3 \\
\chi_{22}^{BS} &= (-1)^2 B_1 + (-2)^2 B_2 + (-3)^2 B_3 \\
&\vdots
\end{aligned}$$

invert this relation!

Disentangling different strangeness sectors

Idea: separate strangeness sectors by making use of all diagonal strangeness fluctuations and baryon-strangeness correlations up to the 4th order

$$\boldsymbol{x_1\chi_{11}^{BS} + x_2\chi_{31}^{BS} + x_3\chi_2^{S} + x_4\chi_{22}^{BS} + x_5\chi_{13}^{BS} + x_6\chi_4^{S} = y_1M_1 + y_2B_1 + y_3B_2 + y_4B_3}$$

solve: $\boldsymbol{A\vec{x} = \hat{e}_i}$ with

$$\boldsymbol{A} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 \\ -1 & -1 & 1 & 1 & -1 & 1 \\ -2 & -2 & 4 & 4 & -8 & 16 \\ -3 & -3 & 9 & 9 & -27 & 81 \end{pmatrix}$$

defined by powers of strangeness charges

Rank=4

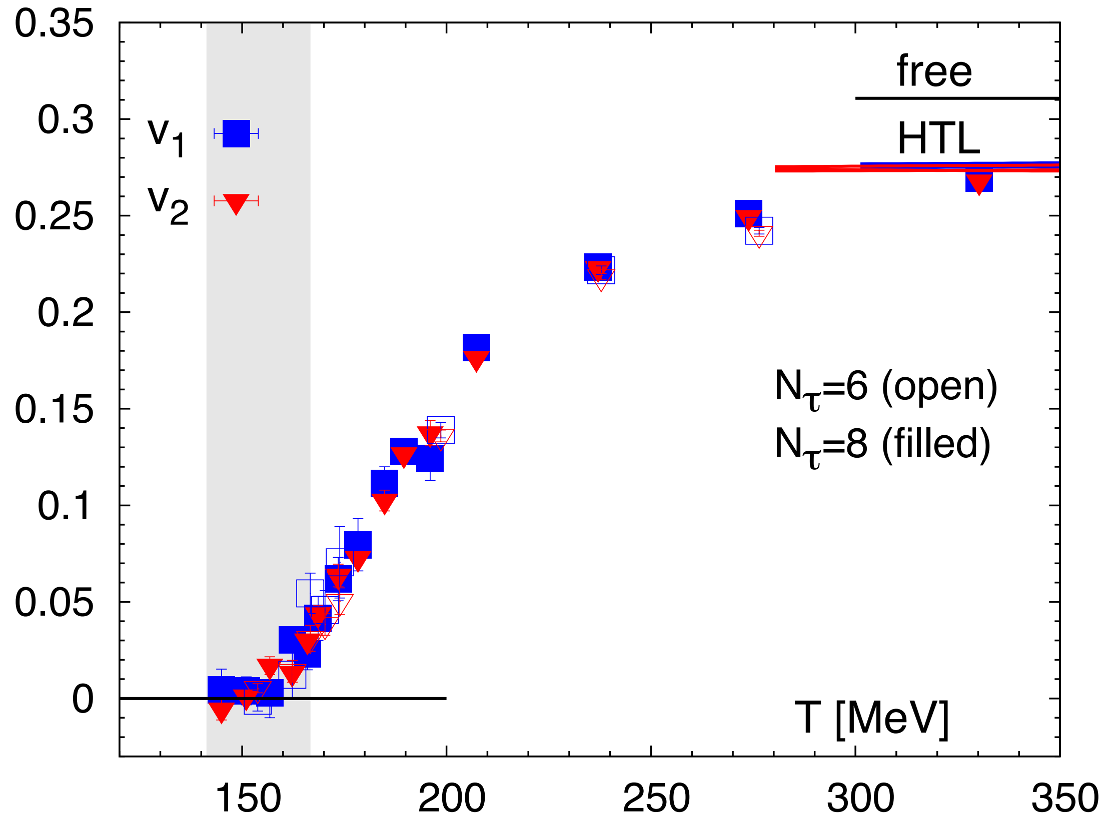
$\Longrightarrow$ dim (Kernel)=2, spanned by $\boldsymbol{v_1, v_2}$

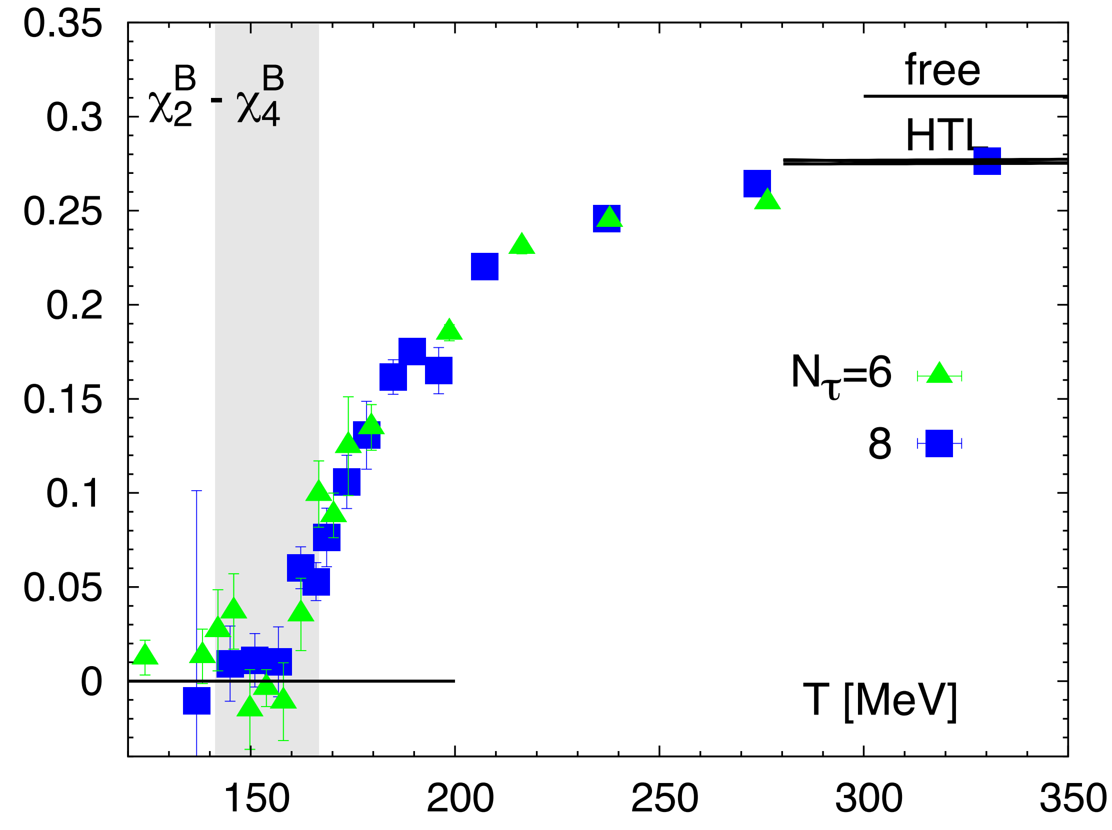
$$v_1 = \chi_{31}^{BS} - \chi_{11}^{BS}$$

$$v_2 = \frac{1}{3}(\chi_2^S - \chi_4^S) - 2\chi_{13}^{BS} - 4\chi_{22}^{BS} - 2\chi_{31}^{BS}$$

must vanish if HRG is a valid description!

Disentangling different strangeness sectors





- strange baryons carry baryon number 1
- partial pressure from strange particles is hadronic

- all baryons carry baryon number 1

$\Rightarrow$ indicator for the validity of the HRG

- at $T \lesssim 160\ \mathrm{MeV}$ we find reasonable agreement with the HRG
- at $T \gtrsim 160\ \mathrm{MeV}$ deviations from HRG become large

Disentangling different strangeness sectors

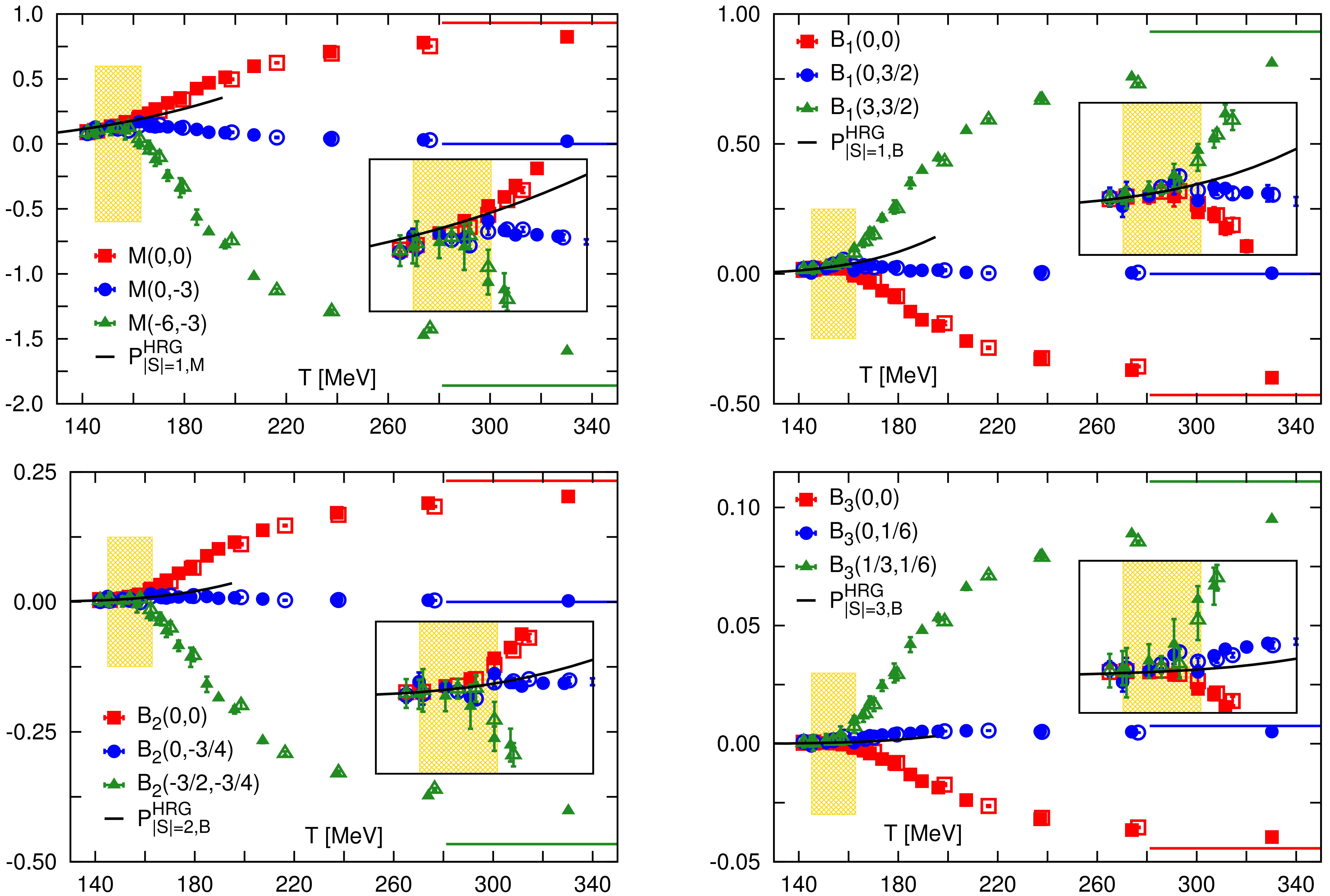
solving the 4 inhomogenous systems $\boldsymbol{A\vec{x} = \hat{e}_i}$

$\Rightarrow$ the solutions are translations of the kernel

$$
\begin{aligned}
M(c_1,c_2) &= \chi_2^S - \chi_{22}^{BS} + c_1 v_1 + c_2 v_2 \\
B_1(c_1,c_2) &= \frac{1}{2}\left(\chi_4^S - \chi_2^S + 5\chi_{13}^{BS} + 7\chi_{22}^{BS}\right) + c_1 v_1 + c_2 v_2 \\
B_2(c_1,c_2) &= -\frac{1}{4}\left(\chi_4^S - \chi_2^S + 4\chi_{13}^{BS} + 4\chi_{22}^{BS}\right) + c_1 v_1 + c_2 v_2 \\
B_3(c_1,c_2) &= \frac{1}{18}\left(\chi_4^S - \chi_2^S + 3\chi_{13}^{BS} + 3\chi_{22}^{BS}\right) + c_1 v_1 + c_2 v_2
\end{aligned}
$$

Disentangling different strangeness sectors

BNL-BI: arXiv:1304.7220



Disentangling different strangeness sectors

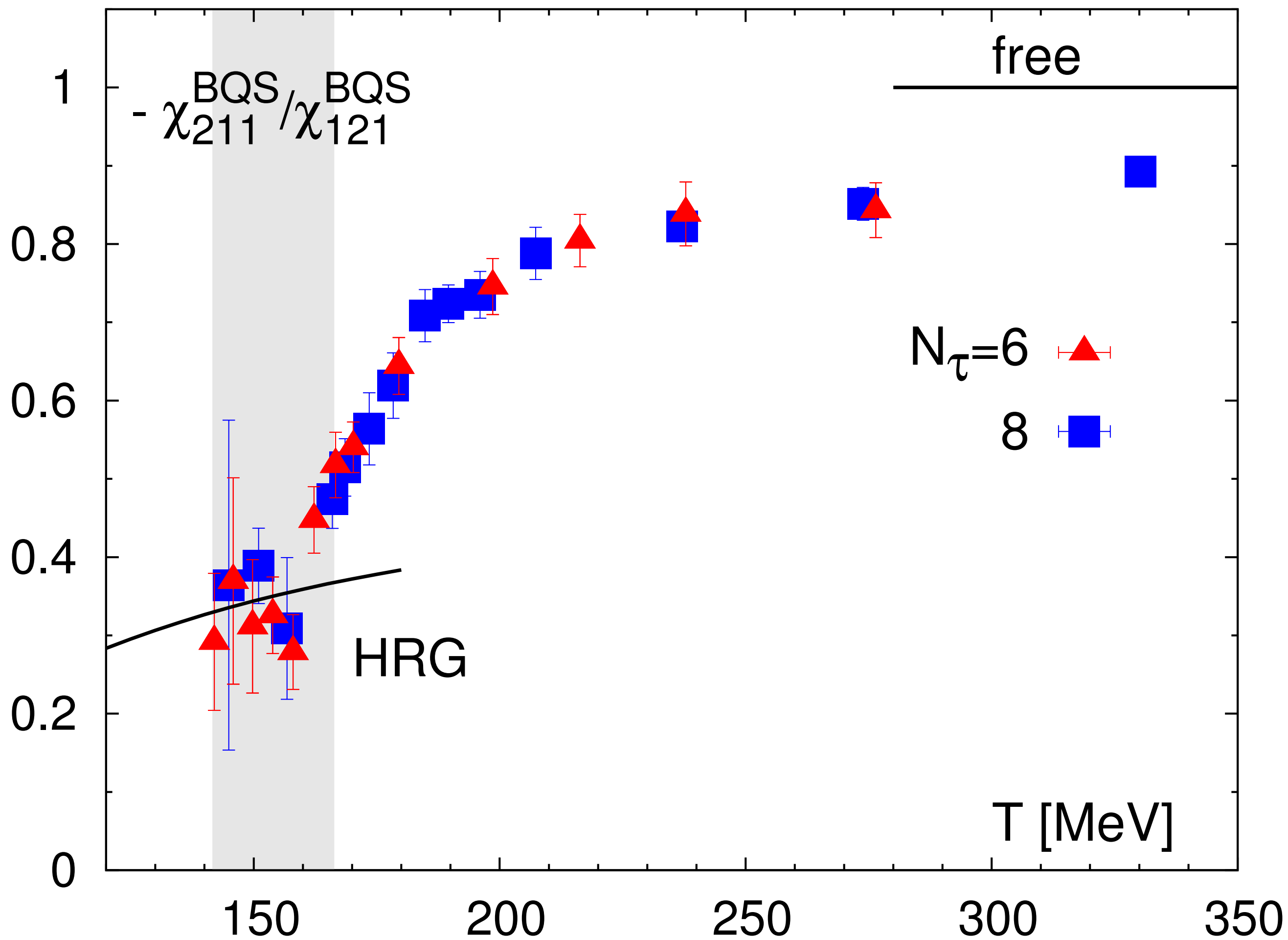
- For $T \lesssim 155$ MeV different strange sectors agree **separately** with the HRG
- For higher temperatures the deviations from HRG set in abruptly and rapidly become large

$\Rightarrow$ modifications of the strange hadron contribution to bulk thermodynamics become apparent in the crossover region and follow the same pattern present also in the light quark sector.

Strangeness in the QGP

- We now probe the quasi-particle picture, i.e. to what extent the susceptibilities involving strangeness contributions can be understood in terms of elementary degrees of freedom that carry quantum numbers

$$S = \pm 1, B = \pm 1/3, Q = \pm 1/3$$



- for a free (uncorrelated) strange **quarks** we have $B^2Q = -BQ^2$

$$\Rightarrow \lim_{T\to\infty}\left(-\chi_{211}^{BQS}/\chi_{121}^{BQS}\right) = 1$$

- for free (uncorrelated) strange **hadrons** we have contributions from two charge sectors.

$$\Rightarrow \left(-\chi_{211}^{BQS}/\chi_{121}^{BQS}\right) < 1$$

Summary

- We have provided evidence that in QCD the strange hadron sector gets modified strongly in the vicinity of the pseudo-critical temperature determined from the light quark chiral susceptibilities.
- Deviations from the HRG model start becoming large in the transition region and follow a pattern similar to that known for the light quark sector.
- There thus is no evidence that deconfinement and the dissolution of hadronic bound states may be shifted to higher temperatures for strange hadrons.

- We also showed that at temperatures larger than $T>1.3T_C$ a simple quasi-particle model may be sufficient to describe properties of mixed strangeness-baryon number susceptibilities.

- Closer to T_C the structure of these susceptibilities becomes more complicated. A feature well-known also from bulk thermodynamic quantities like the pressure.