Nucleon generalized form factors with twisted mass fermions

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with

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Outline

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   - Nucleon mass
   - Setting the scale

2. High precision study of nucleon observables
   - Excited states contributions
   - Evaluation of disconnected quark loop contributions
   - Axial charge $g_A$
   - Momentum fraction $\langle x \rangle$
   - Scalar charge

3. Results
   - Axial charge
   - Momentum fraction and Nucleon spin
   - Scalar charge

4. Conclusions
Wilson twisted mass lattice QCD

- \( N_f = 2 \): \( \psi = \begin{pmatrix} u \\ d \end{pmatrix} \)

  Change of variables: \( \psi = \frac{1}{\sqrt{2}} [1 + i \tau^3 \gamma_5] \chi \quad \bar{\psi} = \bar{\chi} \frac{1}{\sqrt{2}} [1 + i \tau^3 \gamma_5] \)

  \( \Rightarrow \) mass term: \( \bar{\psi} m \psi = \bar{\chi} i \gamma_5 \tau^3 m \chi \)

  \[ S = S_g + a^4 \sum_x \bar{\chi}(x) \left[ \frac{1}{2} \gamma_\mu (\nabla_\mu + \nabla^*_\mu) - \frac{ar}{2} \nabla_\mu \nabla^*_\mu + m_{\text{crit}} + i \gamma_5 \tau^3 \mu \right] \chi(x) \]

- \( N_f = 2 + 1 + 1 \)

  \( S_h = \sum_x \bar{\chi}_h(x) \left[ D_W + m_{(0,h)} + i \gamma_5 \tau^1 \mu_\sigma + \tau^3 \mu_\delta \right] \chi_h(x) \)

  \( S_g \): Iwasaki action

- \( N_f = 2 \) twisted mass plus clover and Iwasaki action \( \rightarrow \) a good formulation for simulations at the physical point, talk by B. Kostrzewa, Monday, session 2G

  \( \rightarrow \) preliminary results at physical point

Wilson tmQCD at maximal twist

- Automatic \( O(a) \) improvement for a class of observables

- No operator improvement needed, renormalization simplified \( \rightarrow \) important for nucleon structure
Cut-off effects small for these lattice spacings

Fit for $m_\pi < 375$ MeV

New result at near physical pion gives complete agreement with experimental value
Setting the scale

- For baryon observables use nucleon mass at physical limit
- Extrapolate using lowest HBχPT result: $m_N = m_0^N - 4c_1m_\pi^2 - \frac{3g_A^2}{16\pi f_\pi^2}m_\pi^3$
- Estimate systematic error due to chiral extrapolation: i) using next order in HBχPT that includes explicit $\Delta$-degrees of freedom; ii) varying the pion mass range for the fit; iii) allowing the coefficient of the $m_\pi^3$ to be a fit parameter
- Simultaneous fits to $\beta = 1.9$, $\beta = 1.95$ and $\beta = 2.1$ results
- Allows a cross-check with the determination using $f_\pi$

\[
\begin{array}{|c|c|c|}
\hline
\beta & a \ (fm) & O(p^3) \\
\hline
1.90 & 0.0936(13)(32) & \\
1.95 & 0.0823(11)(35) & \\
2.10 & 0.0646(7)(25) & \text{Preliminary} \\
2.10 & 0.0971(26)(3) & \\
\hline
\end{array}
\]

- $\sigma$-term from $m_N$ using $O(p^3)$ and $m_\pi \lesssim 300$ MeV: $\sigma_{\pi N} = 58(8)(7)$ MeV
- Using the nucleon mass we find $r_0 \sim 0.495(5)$ fm in the continuum limit
High precision study of nucleon observables

Dedicated high statistics study

Choose one ensemble to perform a high statistics analysis for:

- excited state contributions
  - nucleon axial charge $g_A$ - weak, S. Dinter, C.A., M. Constantinou, V. Drach, K. Jansen and D. Renner, arXiv: 1108.1076
  - momentum fraction $\langle x \rangle_{u-d}$ - intermediate
  - scalar charge (equivalently $\sigma$-terms) - severe, talk by V. Drach, Tuesday, session 4B

- disconnected contributions for all nucleon observables

$$N_f = 2 + 1 + 1$$ twisted mass, $a = 0.082$ fm, $m_\pi = 373$ MeV
Form ratio by dividing the three-point correlator by an appropriate combination of two-point functions:

- Plateau method:

$$R(t_s, t_{\text{ins}}, t_0) \xrightarrow{(t_{\text{ins}} - t_0) \Delta \gg 1} \mathcal{M} [1 + \ldots e^{-\Delta(p)(t_{\text{ins}} - t_0)} + \ldots e^{-\Delta(p')(t_s - t_{\text{ins}})}]$$

- $\mathcal{M}$ the desired matrix element
- $t_s, t_{\text{ins}}, t_0$ the sink, insertion and source time-slices
- $\Delta(p)$ the energy gap with the first excited state
Extracting nucleon matrix elements

Form ratio by dividing the three-point correlator by an appropriate combination of two-point functions:

- Plateau method:

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  R(t_s, t_{\text{ins}}, t_0) \xrightarrow{\frac{(t_{\text{ins}}-t_0)\Delta \gg 1}{(t_s-t_{\text{ins}})\Delta \gg 1}} \mathcal{M}[1 + \ldots e^{-\Delta(p)(t_{\text{ins}}-t_0)} + \ldots e^{-\Delta(p')(t_s-t_{\text{ins}})}]
  \]

  ▶ \(\mathcal{M}\) the desired matrix element
  ▶ \(t_s, t_{\text{ins}}, t_0\) the sink, insertion and source time-slices
  ▶ \(\Delta(p)\) the energy gap with the first excited state

- Summation method: Summing over \(t_{\text{ins}}\):

  \[
  \sum_{t_{\text{ins}}=t_0}^{t_s} R(t_s, t_{\text{ins}}, t_0) = \text{Const.} + \mathcal{M}[(t_s - t_0) + \mathcal{O}(e^{-\Delta(p)(t_s-t_0)}) + \mathcal{O}(e^{-\Delta(p')(t_s-t_0)})].
  \]

  ▶ Excited state contributions are suppressed by exponentials decaying with \(t_s - t_0\), rather than \(t_s - t_{\text{ins}}\) and/or \(t_{\text{ins}} - t_0\)
  ▶ Also works if one does not include \(t_0\) and \(t_s\) in the sum \(\rightarrow\) used for the results shown here
  ▶ However, one needs to fit the slope rather than to a constant or take differences and then fit to a constant

Quark loop contributions

Notoriously difficult

- \( L(x) = Tr \left[ \Gamma G(x; x) \right] \) \( \rightarrow \) need all-to-all propagator
- Large gauge noise \( \rightarrow \) large statistics

- Use stochastic noise \( \eta \) \( \rightarrow \) solve \( Dv_r = \eta_r, \; r = 1, \ldots, N_r \rightarrow D^{-1} = \lim_{N_r \to \infty} \frac{1}{N_r} \sum_{j=1}^{N_r} |s_j \rangle \langle \eta_j| \)
- Reduce noise by increasing statistics
  \( \implies \) Take advantage of graphics cards (GPUs) \( \rightarrow \) CUDA programming language

Special Session: Coding Efforts, Friday, 10G

GPU Strong Scaling

Quark loop contributions

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Reduce noise by increasing statistics

\[ \implies \text{Take advantage of graphics cards (GPUs) } \rightarrow\text{CUDA programming language} \]

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For scalar charge (and \(\sigma\)-terms) a good signal is obtained with \(N_r = 24\) in combination of using advantages of the twisted mass formulation

For other disconnected contributions one has large stochastic noise \(\rightarrow\) Reduce noise by increasing statistics at low cost \(\rightarrow\) use low precision inversions and correct bias (truncated solver method (TSM)),

\[ G. \text{Bali, S. Collins and A. Schäfer, PoSLat2007, 141; G. Bali et al., arXiv:1111.1600} \]

\[
D^{-1} = \lim_{N_{HP} \to \infty} \frac{1}{N_{HP}} \sum_{j=1}^{N_{HP}} \left[ |s_j\rangle \langle \eta_j|_{HP} - |s_j\rangle \langle \eta_j|_{LP} \right] + \frac{1}{N_{LP}} \sum_{j=N_{HP}+1}^{N_{HP}+N_{LP}} |s_j\rangle \langle \eta_j|_{LP}
\]

with \(D|s_j\rangle = |\eta_j\rangle\)

Need to tune in addition to the high precision noise vectors \(N_{HP}\) and number of low precision vectors \(N_{LP}\)

Since the LP sources don’t require an accurate inversion, we can take advantage of the half precision algorithms for GPUs - use the QUDA library

To compute the isoscalar disconnected contribution to \(g_A\), we use \(N_{HP} = 24\) and \(N_{LP} \geq 500\)
Axial charge $g_A$

Axial-vector FFs: $A^3_\mu = \bar{\psi} \gamma_\mu \gamma_5 \frac{\tau^3}{2} \psi(x) \implies \frac{1}{2} \bar{u}_N(\vec{p}') \left[ \gamma_\mu \gamma_5 G_A(q^2) + \frac{q^{\mu} \gamma_5}{2m} G_P(q^2) \right] u_N(\vec{p})|_{q^2=0}$

$\to$ yields $G_A(0) \equiv g_A$: i) well known experimentally, & ii) no quark loop contributions
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$N_f = 2 + 1 + 1$ twisted mass, $a = 0.082$ fm, $m_\pi = 373$ MeV, 1200 statistics

Use of incremental eigCG algorithm, A.

Stathopoulos and K. Orginos, arXiv:0707.0131

- One sequential inversion for each $t_s$,
  obtain results for all operator insertions
- $\sim 3 \times$ cheaper

Consistent results between summation and plateau methods

No detectable excited states contamination, agrees with high precision study, S. Dinter et al., arXiv:1108.1076 and C. Alexandrou et al., arXiv:1112.2931

- Same plateau for multiple $t_s$'s
- No curvature in summed ratio, consistent results for various fit-ranges
Disconnected $N_f = 2 = 1 + 1$ TMF, $a = 0.082$ fm, $32^3 \times 64$, $m_\pi = 373$ MeV, 200000 statistics (on 4700 confs)

Disconnected isoscalar, agrees with Bali et al. (QCDSF),


- Renormalization currently under study

\[ \text{Perturbatively the difference between isovector and isoscalar at two-loop, shown to be small for the Wilson action, H. Panagoupolous and A. Skouroupathis, arXiv:0811.4264} \]

For now neglect the difference
Momentum fraction

Isovector momentum fraction carried by quarks: Extracted from the nucleon matrix elements of
\[ O^{\mu_1 \mu_2} = \bar{\psi} \gamma^{i} \{ \mu_1 \, i \, \vec{D} \, \mu_2 \} \psi \] at \( q^2 = 0 \) with no disconnected contributions and known experimentally

\[ N_f = 2 + 1 + 1 \] twisted mass, \( a = 0.082 \text{ fm}, \ m_\pi = 373 \text{ MeV}, \ 1200 \text{ statistics} \]

Connected

Noticeable excited state contamination, especially for the iso-scalar

For the plateau method one needs to show convergence by varying the sink-source time separation also requires a number of sequential inversions consistency of plateau and summation method gives confidence in the results
Momentum fraction

Twisted Mass, $a = 0.082$ fm, $32^3 \times 64$, $m_\pi = 373$ MeV, $\sim 200\,000$ statistics (on 4700 confs)

Disconnected

- Can put bound on its value
- Including momentum in the sink/source improves statistical accuracy

Renormalization currently under study
However, difference between isovector and isoscalar at two-loop, shown to be small for the Wilson action, H. Panagopoulos and A. Skouroupathis, arXiv:0811.4264
For now neglect the difference
Scalar charge

The scalar charge $g_s$ and the tensor charge $g_T$ provide constrains for possible scalar and tensor interactions at the TeV scale. For $g_T$, talk by M. Constantinou in this session

- Isovector: $g_s = \langle N|\bar{u}u - \bar{d}d|N\rangle$
- Isoscalar: $g_s^{u+d} = \langle N|\bar{u}u + \bar{d}d|N\rangle \to$ analogous to the calculation of the nucleon sigma-terms, uses advantages for twisted mass fermions, talk by V. Drach, Tuesday, session 4B

$N_f = 2 + 1 + 1$ twisted mass, $a = 0.082$ fm, $m_\pi = 373$ MeV, 1200 statistics

**Severe contamination from excited states**

Agreement of summation, plateau and two-states fits give confidence to the correctness of the final result
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Connected

- Severe contamination from excited states
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Scalar charge

\[ N_f = 2 + 1 + 1 \] twisted mass, \( a = 0.082 \text{ fm} \), \( m_\pi = 373 \text{ MeV} \), \( \sim 200 \text{ 000 statistics} \)

Disconnected

\[ \sum_{t_{\text{ins}}} R(t_{\text{ins}}) \]

\[ (t_{\text{ins}} - t_s/2)/a \]

\[ t_s/a = 14 \quad t_s/a = 16 \quad t_s/a = 18 \quad t_s/a = 20 \]

SM
Results: I. Axial charge $g_A$

Renormalized using non-perturbative $Z_A$

High statistics analysis at $m_\pi = 373$ MeV reveal no excited states contamination for sink-source separation $\sim 1$ fm

Results using the plateau method and sink-source time separation (1-1.2) fm

Results: I. Axial charge $g_A$

Comparison with other groups

Results obtained using the plateau method with sink-source time separation $\sim (1.0 - 1.2)$ fm

- Results at near physical pion mass are now becoming available $\rightarrow$ need dedicated study at physical point with high statistics and larger volumes
- A number of collaborations are engaging in systematic studies, e.g.
  - $N_f = 2 + 1$ Clover, J. R. Green et al., arXiv:1209.1687
  - $N_f = 2$ Clover, R. Hosley et al., arXiv:1302.2233
  - $N_f = 2$ Clover, S. Capitani et al. arXiv:1205.0180
  - $N_f = 2 + 1$ Clover, B. J. Owen et al., arXiv:1212.4668
  - $N_f = 2 + 1 + 1$ Mixed action (HISQ/Clover), T. Bhattacharya et al., arXiv:1306.5435
  - Also several talks in Lattice 2013 e.g. S. Ohta, M. Lin, Thursday, session 7B

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- Volume effects may not be the full story if we compare the result by QCDSF ($Lm_\pi \sim 2.7$) and LHPC ($Lm_\pi \sim 4.2$) at near physical pion mass

Results: II. Momentum fraction and the nucleon spin

Total quark spin: \[ J^q = \frac{1}{2} (A_{20}^q(0) + B_{20}^q(0)) \]

\( A_{20}(Q^2) \) and \( B_{20}(Q^2) \) extracted from the nucleon matrix elements of \( \mathcal{O}^\mu_1^\mu_2 = \bar{\psi} \gamma^{\mu_1} i \slashed{D}^{\mu_2} \psi \)

For \( Q^2 = 0 \): moments of parton distributions measured in DIS

\[ A_{20}^q(0) \equiv \langle x \rangle_q = \int_0^1 dx \ x [q(x) + \bar{q}(x)] \]

Unpolarized quark distribution: \[ q(x) = q(x)_\downarrow + q(x)_\uparrow \]

Polarized quark distribution: \[ \Delta q(x) = q(x)_\downarrow - q(x)_\uparrow \]

Helicity quark distribution: \[ \delta q = q_T + q_\perp \], talk by M. Constantinou, this session
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Results in the \( \overline{\text{MS}} \) scheme at \( \mu = 2 \) GeV using non-perturbative renormalization, C. Alexandrou et al., arXiv:1006.1920
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We also need \( B_{20}^q(0) \)
Where is the nucleon spin?

Spin sum: \[ \frac{1}{2} = \sum_q \left( \frac{1}{2} \Delta \Sigma^q \right) + J^G \]

Connected contributions

\[ J^u \]

\[ J^d \]

\[ \Delta \Sigma^u + \Delta \Sigma^d \]

\[ L^{u+d} \]

\[ m^2 (\text{GeV}^2) \]

\[ m^2 (\text{GeV}^2) \]

\[ \frac{1}{2} \Delta \Sigma^u + \frac{1}{2} \Delta \Sigma^d \]

\[ L^{u+d} \]

\[ J^u \sim 0.25 \text{ and for d-quark } J^d \sim 0 \]

\[ L^{u+d} \sim 0 \text{ at physical point} \]

\[ \Delta \Sigma^{u+d} \text{ in agreement with experimental value at physical point} \]

\[ \text{The total spin } J^{u+d} \sim 0.25 \text{ Where is the other half?} \]

However, more statistics and checks of systematics are needed for final results at the physical point.
Where is the nucleon spin?

Spin sum: \( \frac{1}{2} = \sum_q \left( \frac{1}{2} \Delta \Sigma^q + L^q \right) + J^G \)

Connected contributions

\( \Delta \Sigma^u, d \) consistent with experimental values

\( L^d \sim -L^u \)

However, more statistics and checks of systematics are needed for final results at the physical point.
Where is the nucleon spin?
Spin sum: \( \frac{1}{2} = \sum_q \left( \frac{1}{2} \Delta \Sigma^q + L^q \right) + J^G \)

For one ensemble at \( m_\pi = 373 \) MeV we have the disconnected contribution → we can check the effect on the observables, \( \mathcal{O}(200000) \) statistics

- Disconnected quark loop contributions non-zero for \( \Delta \Sigma^{u,d,s} \)
- Consistent with zero for \( J^{u,d} \)
- The total spin \( J^{u+d} \sim 0.25 \) \( \Rightarrow \) Where is the other half?
- Quark loop contributions are small
- Contributions from \( J_g ? \) → on-going efforts to compute them, K.-F. Liu et al. (\( \chi \text{QCD} \)), arXiv:1203.6388, talk by C. Wiese, this session
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Connected isoscalar taking sink-source time separation $\sim (1.0 - 1.2)$ fm
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Increasing the sink-source time separation $\sim 1.5$ fm for one TMF ensemble
Isoscalar scalar charge

Besides the connected contribution it has a disconnected part

![Graph showing data points and error bars with legend]

- $N_f=2+1+1$ TMF $a=0.082\,\text{fm}$
- $N_f=2+1+1$ TMF $a=0.084\,\text{fm}$
- $N_f=2$ TMF/Clover $a=0.097$
- $N_f=2+1+1$ HISQ/Clover $a=0.12\,\text{fm}$

$m_\pi^2$ (GeV$^2$) vs $g_s$
Isoscalar scalar charge

Besides the connected contribution it has a disconnected part

\[ N_f=2+1+1 \text{ TMF } a=0.082\text{fm, with disconn. contr.} \]
Simulations at the physical point \( \rightarrow \) that’s where we always wanted to be!
\[ \Rightarrow \text{Physical results on } g_A, \langle x \rangle_{u-d} \text{ etc are now directly accessible} \]
But will need high statistics and careful cross-checks \( \rightarrow \) noise reduction techniques are crucial e.g.
AMA, TSM, smearing etc
Evaluation of quark loop diagrams has become feasible
Predictions for other hadron observables are emerging e.g. axial charge of hyperons and charmed baryons
Confirmation of experimentally known quantities such as \( g_A \) will enable reliable predictions of others \( \rightarrow \)
provide insight into the structure of hadrons and input that is crucial for new physics such as the nucleon \( \sigma \)-terms, \( g_s \) and \( g_T \)
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Conclusions

- Simulations at the physical point → that’s where we always wanted to be!
  → Physical results on $g_A$, $\langle x \rangle_{u-d}$ etc are now directly accessible
  But will need high statistics and careful cross-checks → noise reduction techniques are crucial e.g. AMA, TSM, smearing etc

- Evaluation of quark loop diagrams has become feasible

- Predictions for other hadron observables are emerging e.g. axial charge of hyperons and charmed baryons

- Confirmation of experimentally known quantities such as $g_A$ will enable reliable predictions of others → provide insight into the structure of hadrons and input that is crucial for new physics such as the nucleon $\sigma$-terms, $g_s$ and $g_T$
Thank you for your attention

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Two-state fits

Fitting the ratio to two-states simultaneous for several sink-source separations works for the scalar charge and momentum fraction. As stressed $g_A$ does not pick up contributions from excited states.
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Not useful for predicting the large time dependence