Moments of structure functions for $N_f = 2$ near the physical point.

S. Collins
University of Regensburg

In collaboration with G. Bali, B. Gläßle, M. Göckeler, J. Najjar, R. Rödel, A. Schäfer, R. Schiel, W. Söldner and A. Sternbeck

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Outline

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- Excited state analysis
- g_A
- $\langle x \rangle_{u-d}$
- g_T and g_S
- Outlook

Introduction

Nucleon structure investigated using matrix elements of the form $\langle N, p' | \bar{q}\Gamma q | N, p\rangle$: $q = p' - p$, (generalised) form factors $\to$ A. Sternbeck.

$p' = p = 0$: lowest moments of pol. and unpol. structure functions.

$$2\int_0^1 \mathrm{d}x\, x\, F_1(x, Q^2) \quad = \quad c_{11}^{(u)}\langle x\rangle_u(\mu) + c_{11}^{(d)}\langle x\rangle_d(\mu) + c_{11}^{(s)}\langle x\rangle_s(\mu)$$

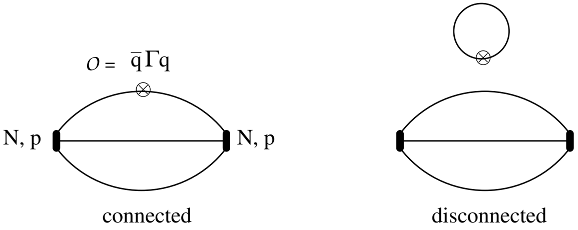
$$\langle x\rangle_u(\mu) = \sum_{q=u,d,s} Z_q(\mu)\langle x\rangle_q^{latt} \quad \langle x\rangle_q^{latt} \quad \propto \langle N, p|\bar{q}[\gamma_4 \overleftrightarrow{D}_4 - \frac{1}{3}\gamma\cdot\overleftrightarrow{\mathbf{D}}]q|N, p\rangle^{latt}$$

$$2\int_0^1 \mathrm{d}x \mathbb{1}\, g_1(x, Q^2) \quad = \quad \frac{1}{2}[e_{10}^{(u)}\langle \mathbb{1}\rangle_{\Delta u}(\mu) + e_{10}^{(d)}\langle \mathbb{1}\rangle_{\Delta d}(\mu) + e_{10}^{(s)}\langle \mathbb{1}\rangle_{\Delta s}(\mu)]$$

$$\langle \mathbb{1}\rangle_{\Delta u}(\mu) = \sum_{q=u,d,s} Z_q'(\mu)\langle \mathbb{1}\rangle_{\Delta q}^{latt}, \quad \langle \mathbb{1}\rangle_{\Delta q}^{latt} \quad \propto \langle N, p, s|\bar{q}\gamma_i\gamma_5 q|N, p, s\rangle^{latt}$$

Also non-Standard Model couplings $\bar{q}q \to g_S$ and $\bar{q}\sigma_{\mu\nu}q \to g_T$.

Evaluate $\langle N, p|\bar{q}\Gamma q|N, p\rangle$



$q = u/d$: both connected and disconnected terms.

$q = s$: only the disconnected term.

Mixing under renormalisation possible.

Disconnected results for $\Gamma = \mathbb{1}$, talk by A. Sternbeck.

Here use isovector combinations $\langle N, p|\bar{u}\Gamma u - \bar{d}\Gamma d|N, p\rangle \rightarrow$
$g_A = \Delta u - \Delta d$, $\langle x\rangle_{u-d}$, g_S^{u-d}, g_T^{u-d}.

Simulation details

$N_f = 2$, NP-improved clover fermions, Regensburg+QCDSF configs.

β	Vol	#	a fm	m_π MeV	Lm_π	t_{sink}
5.20	32×64	823×2	0.08	280	3.7	13
5.29	24×48	1124×2	0.07	430	3.7	15
	32×64	2027×2 (1)		294	3.4	7,9,11,13,15,17
	40×64	2028×2		289	4.2	15
	48×64	3400×2		157	2.7	15
	64×64	940×3		150	3.5	15
5.40	32×64	1170×2	0.06	491	4.8	17
	48×64	2178×2		260	3.8	17

This analysis using sequential propagators: using stochastic estimates see talk J. Najjar on Thursday.

Gauge invariant Wuppertal smearing - optimised for two-point function.

Systematics: in general - finite a, finite V, non-physical m_q, excited states.

Excited state analysis

$$C_{2pt}(t_{sink}) = \sum_{\vec{x}} T^{\alpha\beta} \langle \mathcal{N}_\alpha(\vec{x}, t_{sink}) \overline{\mathcal{N}}_\beta(\vec{0}, 0) \rangle = |Z_0|^2 e^{-m_0 t_{sink}} + |Z_1|^2 e^{-m_1 t_{sink}} + \ldots$$
$$= A_0 e^{-m_0 t_{sink}} [1 + e^{-\Delta m t_{sink}} + \ldots]$$

$$C_{3pt}(t_{sink}, t_{ins}) = \sum_{\vec{x},\vec{y}} T^{\alpha\beta} \langle \mathcal{N}_\alpha(\vec{x}, t_{sink}) O(\vec{y}, t_{ins}) \overline{\mathcal{N}}_\beta(\vec{0}, 0) \rangle$$
$$= |Z_0|^2 \langle N_0|O|N_0\rangle e^{-m_0 t_{sink}} + Z_1 Z_0^* \langle N_1|O|N_0\rangle e^{-m_0 t_{ins}} e^{-m_1(t_{sink}-t_{ins})}$$
$$+ Z_0 Z_1^* \langle N_0|O|N_1\rangle e^{-m_1 t_{ins}} e^{-m_0(t_{sink}-t_{ins})} + |Z_1|^2 \langle N_1|O|N_1\rangle e^{-m_1 t_{sink}} + \ldots$$
$$= A_0 e^{-m_0 t_{sink}} \left(B_0 + B_1 [e^{-\Delta m(t_{sink}-t_{ins})} + e^{-\Delta m t_{ins}}] + B_3 e^{-\Delta m t_{sink}} \right) + \ldots$$

where $B_0 = \langle N_0|O|N_0\rangle$, $B_1 \propto \langle N_1|O|N_0\rangle$, $B_2 \propto \langle N_1|O|N_1\rangle$,

Fit C_{2pt} and C_{3pt} simultaneously for different t_{sink} and compare with constant fit to

$$\frac{C_{3pt}(t_{sink}, t_{ins})}{C_{2pt}(t_{sink})} = B_0 + \ldots$$

A lot of work on excited state contamination recently. For example,

Dinter et al. (2011) arXiv:1108.1076 - multiple t_{sink}.

Owen et al. (2012) arXiv:1212.4668 - variational approach.

Capitani et al. (2012) arXiv:1205.0180 - summation method.

Green et al. (2012) arXiv:1209.1687 - summation method.

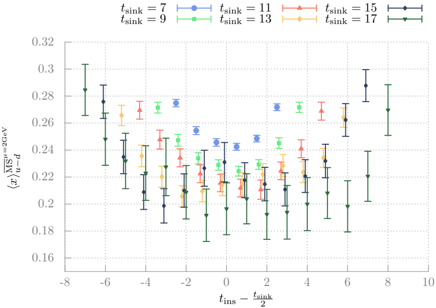
Bhattacharya et al. (2013) arXiv:1306.5435 - fits to multiple t_{sink}.

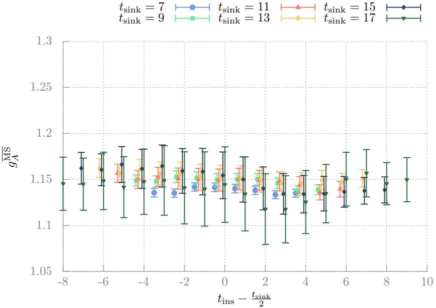
Non-perturbative renormalisation: Constantinou et al. arXiv:1303.6776, Göckeler et al. (QCDSF/UKQCD) arXiv:1003.5756.

All results preliminary! Work in progress.

$\beta = 5.29$, $m_\pi = 290$ MeV, $V = 32 \times 64$ lattice, $t_{sink} = 7, 9, 11, 13, 15, 17$.

Example x_{u-d} and g_A: $C_{3pt}(t_{ins}, t_{sink})/C_{2pt}(t_{sink})$ + additional factors.

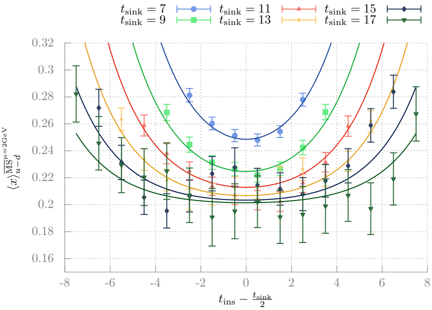


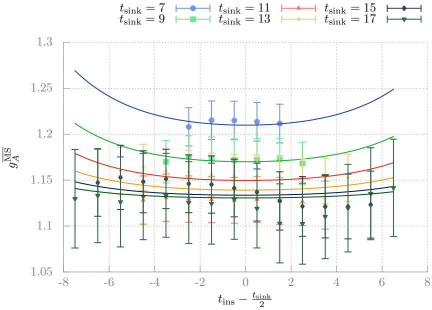


Fit compared to data: $C_{3pt}(t_{ins}, t_{sink})/(A_0 e^{-m_0 t_{sink}})$ compared to

$$F = B_0 + B_1[e^{-\Delta m(t_{sink}-t_{ins})} + e^{-\Delta m t_{ins}}] + B_3 e^{-\Delta m t_{sink}}$$

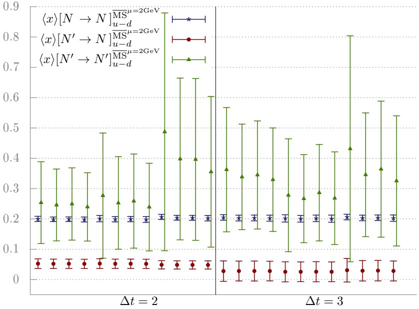
+ additional factors.

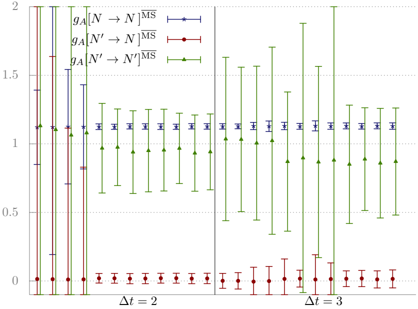




Excited state contribution to g_A cancels in C_{3pt}/C_{2pt}.

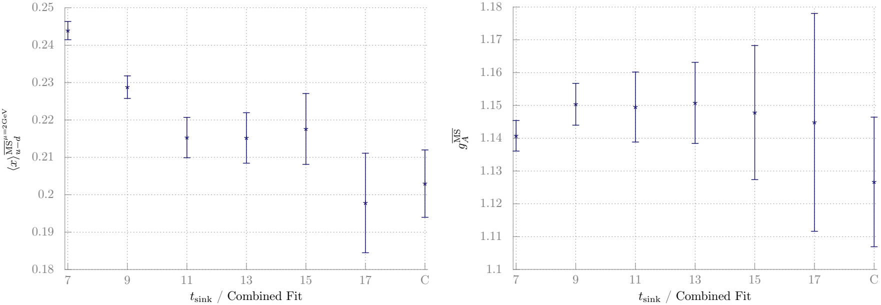
Variation in fitting range: $\Delta t = t_{min} = t_{sink} - t_{max}$ for C_{3pt}.



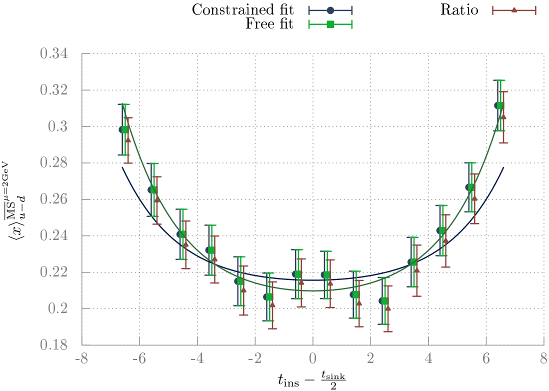


Variation for C_{2pt}, $t_{max} = 13, 17, 20, 23$ and $t_{min} = 2, 3, 4$.

Comparison with constant fits to C_{3pt}/C_{2pt} for different t_{sink}.



Apply to ensembles with single t_{sink}: $\beta = 5.29$, $m_\pi = 157$ MeV, $V = 48^3 \times 64$.

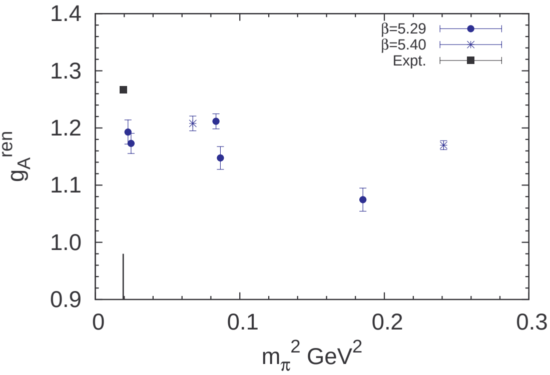


$$C_{3pt}(t_{sink}, t_{ins}) = A_0 e^{-m_0 t_{sink}} \left(B_0 + B_1[e^{-\Delta m(t_{sink}-t_{ins})} + e^{-\Delta m t_{ins}}] + B_3 e^{-\Delta m t_{sink}} \right)$$

Free fit: $B_3 = 0$.

Constrained fit: using excited state parameters from multi-t_{sink} fits. $\langle N_1|O|N_0\rangle$ and $\langle N_1|O|N_1\rangle$

g_A

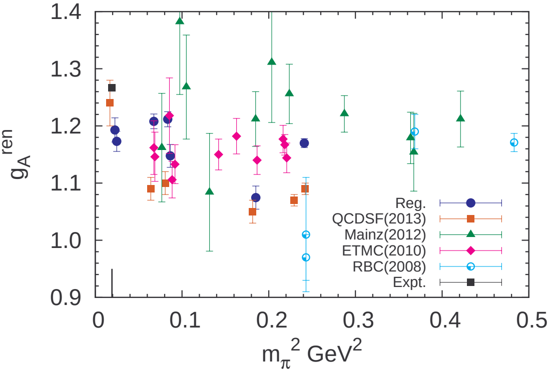


Results from a constant fit to C_{3pt}/C_{2pt}.

Finite volume effects: $m_\pi \sim 290$, $Lm_\pi = 3.4$ and 4.2
$m_\pi \sim 150$, $Lm_\pi = 2.7$ and 3.5

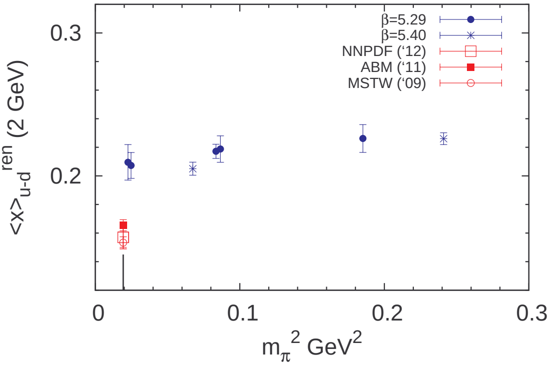
Finite a effects possible.

Comparison of $N_f = 2$ results.



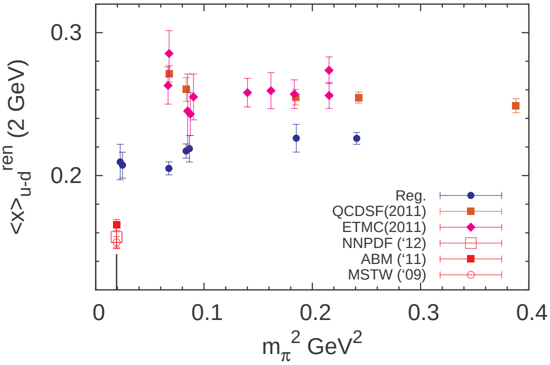
$N_f = 2 + 1$: RBC/UKQCD (2010), QCDSF (2010), HSC (2011), CSSM (2012), LHPC (2012). $N_f = 2 + 1 + 1$: ETMC (2013), PNDME (2013).

$\langle x \rangle_{u-d}$



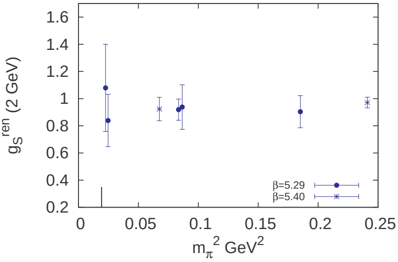
- No significant finite volume effects
- Possible a effect?

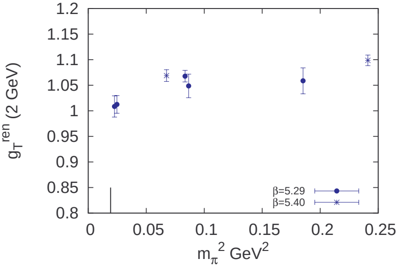
Comparison of NP renormalised $N_f = 2$ results.



$N_f = 2 + 1$: RBC/UKQCD (2010), LHPC (2010), LHPC (2012).
$N_f = 2 + 1 + 1$: ETMC (2013)

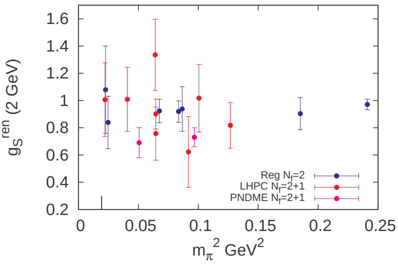
g_S^{u-d} and g_T^{u-d}

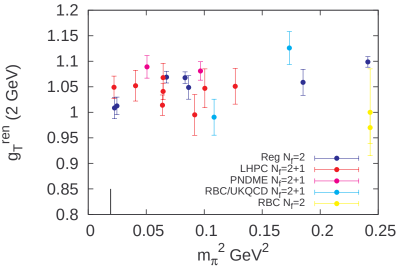




- No significant dependence on β, V or m_q.

g_S^{u-d} and g_T^{u-d}: comparison with other groups





Outlook

Work in progress to investigate the systematic uncertainties in calculating $\langle x \rangle_{u-d}$ and g_A.

- Analysis needs to be finalised - increased statistics at $m_\pi = 150$ MeV, $Lm_\pi = 3.5$.
- Excited states contamination can be brought under control.
- Volume effects for g_A - generating $Lm_\pi = 6.7$ at $m_\pi = 290$ MeV, finite V extrapolation.
- g_A and $\langle x \rangle_{u-d}$, possible finite a effects - generating $\beta = 5.20$ ensemble.

Results for g_S and g_T consistent with other groups.

Calculation of disconnected contributions in progress.