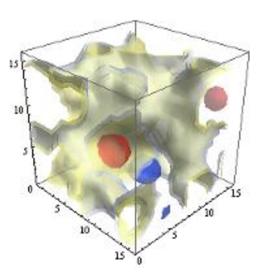
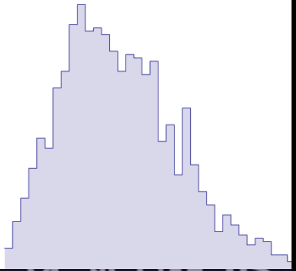
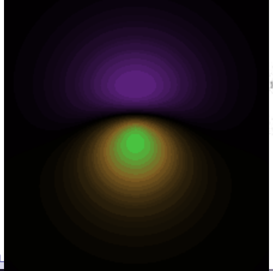
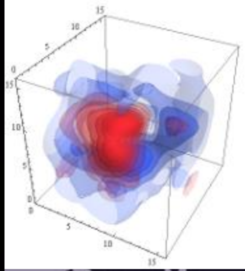
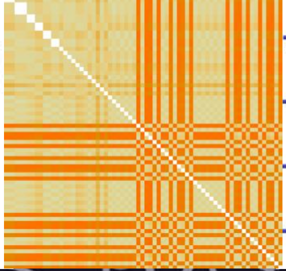
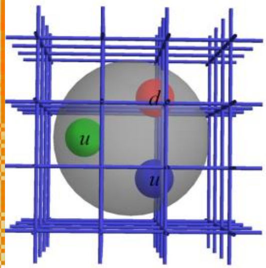


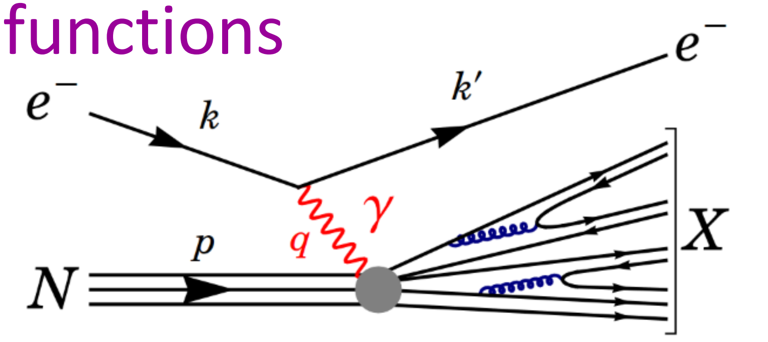
Calculating the *x* Dependence of Hadron PDFs

Huey-Wen Lin
University of Washington

Probing Hadron Structure

§ Structure function/distribution functions

- Deep inelastic scattering (DIS)



$$\sigma \sim L^{\mu\nu} W_{\mu\nu},$$

$$W_{\mu\nu} = i \int d^4 x e^{iqx} \langle N | T\{J^\mu(x), J^\nu(0)\} | N \rangle$$

$$x = Q^2 / 2\, q \cdot P$$

§ Important for QCD and BSM searches

§ Rely on operator product expansion to extract moments

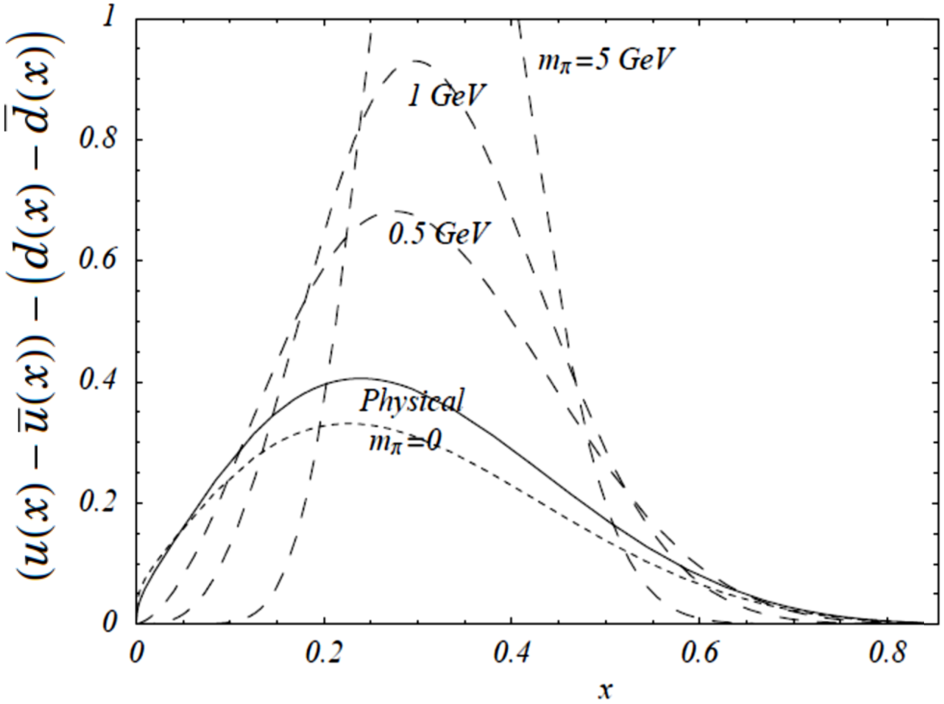
- Moments, e.g. $\langle x^n \rangle_q$, are commonly calculated; must deal with op. mixings, noise increasing with n, etc.
- Hard to get to higher moments ($n > 3$)
- Tricks: subtraction to remove divergent terms, heavy fields, four-point functions… None is practical enough

Limited Access

§ What can we learn about the *x*-distribution?

- Make an ansätz of some smooth form for the distribution and fix the parameters by matching to the lattice moments

$$xq(x) = a\,x^b(1-x)^c\left(1+\epsilon\sqrt{x}+\gamma x\right)$$



Cannot separate valence-quark contribution from sea

New idea needed to access the sea!

W. Detmold et al, Eur.Phys.J.direct C3 (2001) 1-15

The Idea

§ Lightcone quark distribution

Xiangdong Ji, 1305.1539 (PRL soon)

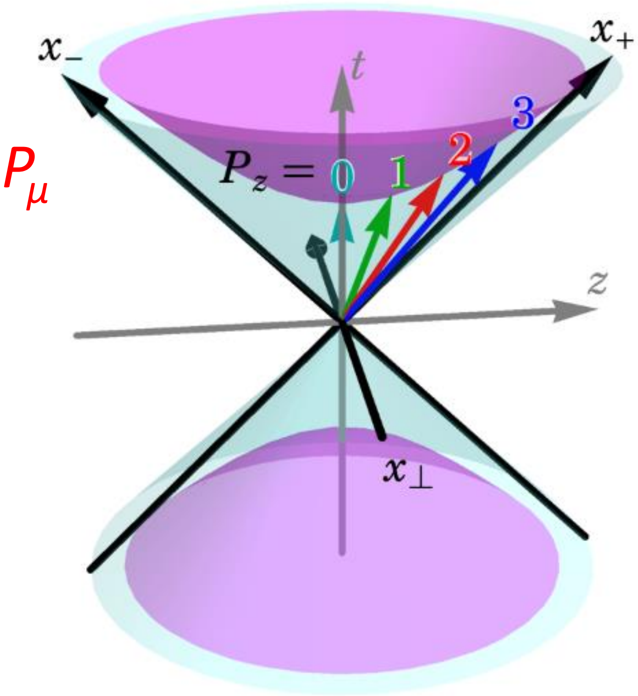
$$q(x,\mu) = \int \frac{d\xi_-}{4\pi} e^{-i\xi_- x P_+} \left\langle P \middle| \overline{\psi}(\xi_-)\gamma_+ \exp\left(-ig\int_0^{\xi_-} d\eta_- A_+(\eta_-)\right)\psi(0) \middle| P \right\rangle$$

Renormalization scale μ

Lightcone coordinate $\xi_\pm = (t \pm z)/\sqrt{2}$

Nucleon momentum P_μ

Gluon potential A_+



§ Approaching lightcone with large P

- Just another limit to take, like taking $a \to 0$

The Idea

§ Finite-momentum quark distribution

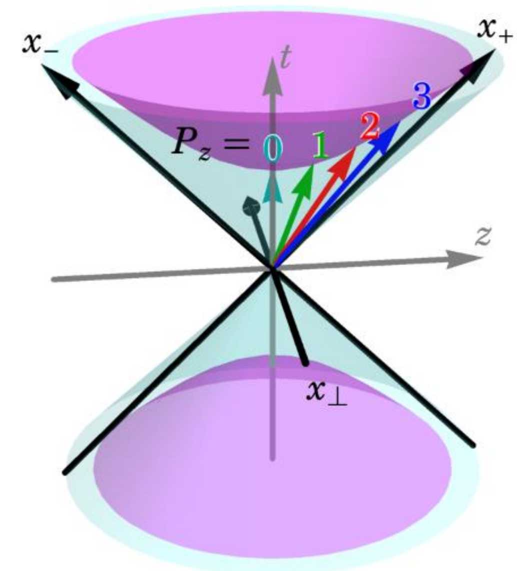
$$q(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \overline{\psi}(z)\,\gamma_z \exp\!\left(-ig\int_0^z dz'\,A_z(z')\right)\psi(0) \middle| P \right\rangle + \mathcal{O}\left(\Lambda_{\rm QCD}^2/P_z^2,\, M_N^2/P_z^2\right)$$

$x = k_z/P_z$

Lattice z coordinate

Nucleon momentum $P_\mu = \{P_0, 0, 0, P_z\}$

Product of lattice gauge links



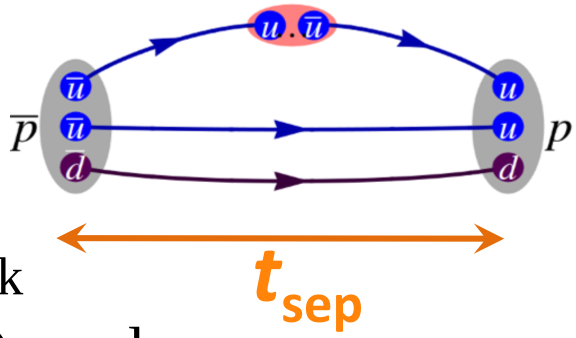
- In $P_z \longrightarrow \infty$ limit, parton distribution is recovered
- For finite P_z, corrections are needed

Xiangdong Ji, 1305.1539 (PRL soon)

Some Lattice Details

§ Exploratory study

- $N_f = 2+1+1$ clover/HISQ lattices (MILC)
  $M_\pi \approx$ **310** MeV, $a \approx$ **0.12** fm ($L \approx 2.88$ fm)
- Isovector only ("disconnected" suppressed)
  gives us flavor asymmetry between up and down quark
- 2 source-sink separation ($t_{sep} \approx 0.96$ and 1.2 fm) used



§ Properties known on these lattices

- Lattice Z_Γ for bilinear operator ~ 1
  (with HYP-smearing)
- $M_\pi L \approx 4.6$ large enough to avoid finite-volume effects

Hyak @ UW

§ Feasible with today's computational resources!

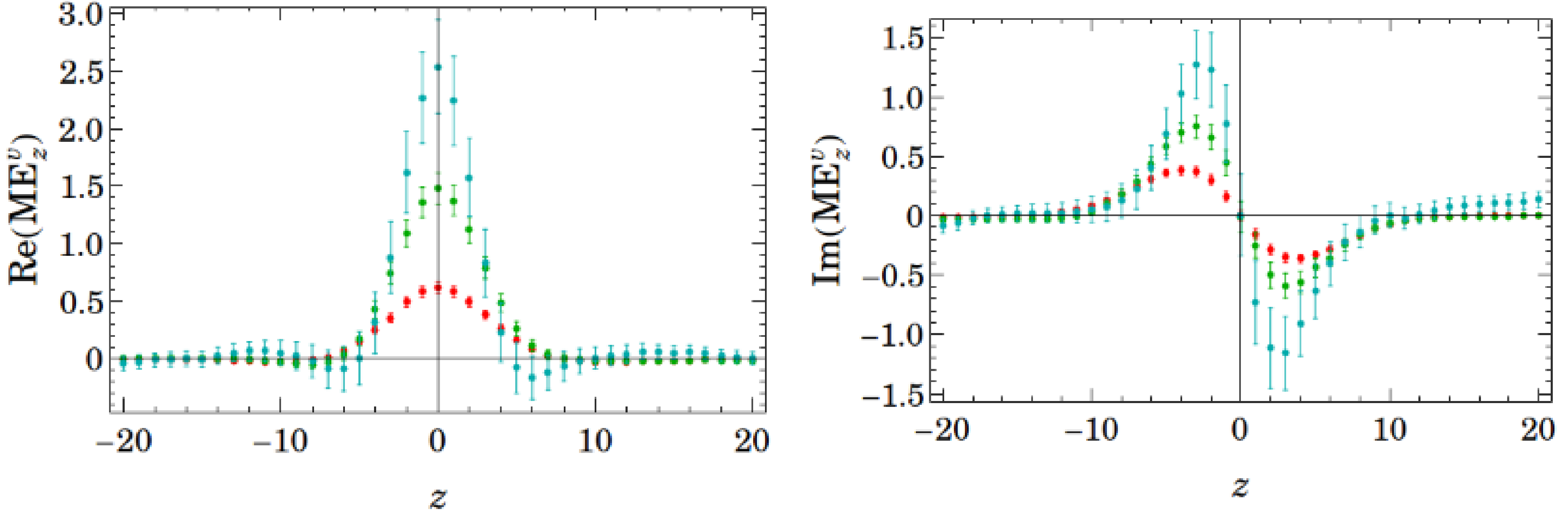
- O(hour) rewriting three-point insertion code (Chroma)
- 8/16 nodes on UW Hyak cluster

Quark Distribution

§ Exploratory study

$$\left\langle P \,\middle|\, \overline{\psi}(z)\,\gamma_z \exp\!\left(-i g \int_0^z dz'\, A_z(z')\right) \psi(0) \,\middle|\, P \right\rangle$$

- How many links are needed?



- Lattice momenta discretized by finite size of volume

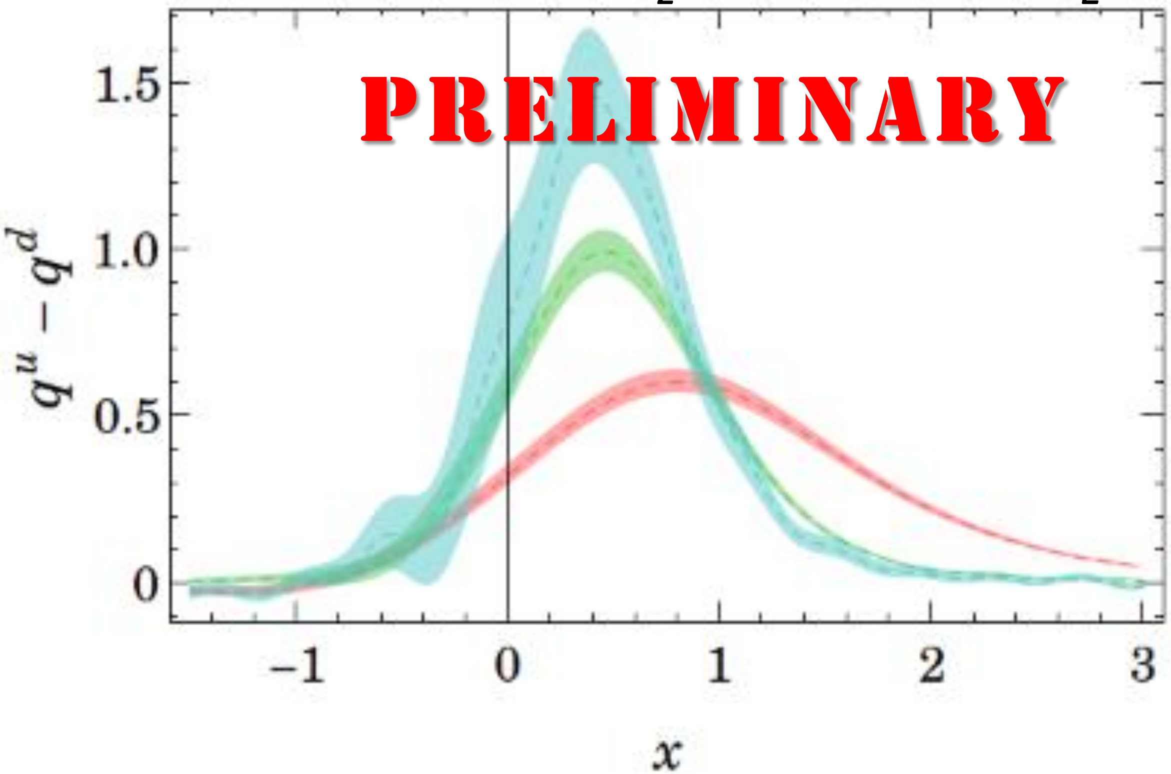
$P_z \in \{1, 2, 3\}\, {}^{2\pi}/_{L}$

Quark Distribution

§ Exploratory study

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \overline{\psi}(z)\, \gamma_z \exp\!\left(-ig\int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$

$P_z \in \{1, 2, 3\}\, ^{2\pi}/_L$



Uncorrected bare lattice results

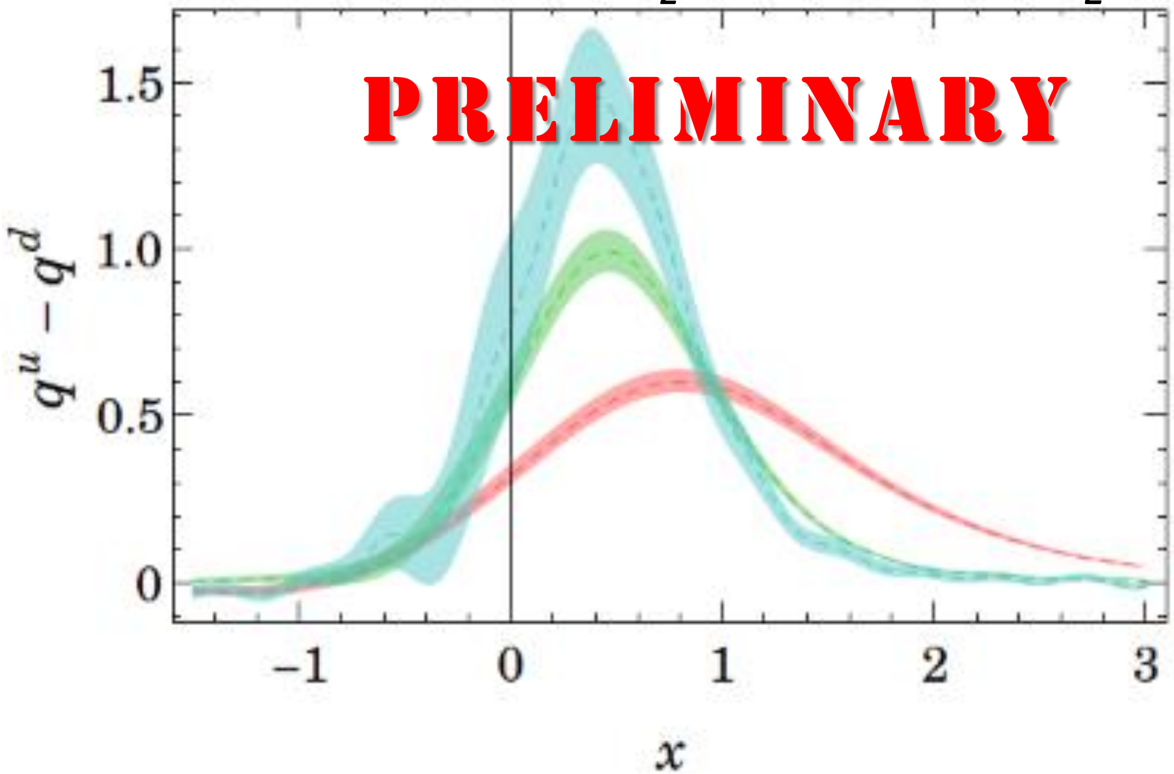
$x = k_z/P_z$

Quark Distribution

§ Exploratory study

$$\int \frac{dz}{4\pi}\, e^{-izk_z} \left\langle P \left| \overline{\psi}(z)\, \gamma_z \exp\!\left(-ig\int_0^z dz'\, A_z(z')\right) \psi(0) \right| P \right\rangle$$

$P_z \in \{1, 2, 3\}\, {}^{2\pi}/_{L}$



Distribution gets sharper as P_z increases

Artifacts due to finite P_z on the lattice

Improvement?

Work out leading-P_z corrections

Quark Distribution

§ Back to the continuum

Xiangdong Ji, 1305.1539 (PRL soon)

$$q(x,\mu) = q_{\text{FP}}(x,\mu,P_z) + O\left(\Lambda_{\text{QCD}}^2 / P_z^2\right) + O\left(M_N^2 / P_z^2\right) + O(\alpha_s)$$

What we want

What we calculate on the lattice

$P_z \in \{1, 2, 3\}\, {}^{2\pi}/_{L}$

Smaller P_z correction but complicated twist-4 operator (extrapolate it away)

J.-H. Zhang, Y. Zhao, J.-W. Chen et al. (in preparation)

Dominant correction (for nucleon) ; known scaling form

J.-W. Chen

Not included yet; O(20%) systematics

Quark Distribution

§ Back to the continuum

Xiangdong Ji, 1305.1539 (PRL soon)

$$q(x,\mu) = q_{\mathrm{FP}}(x,\mu,P_z) + O\left(\Lambda_{\mathrm{QCD}}^2 / P_z^2\right) + O\left(M_N^2 / P_z^2\right) + O(\alpha_s)$$

What we want

What we calculate on the lattice
$P_z \in \{1, 2, 3\}\, 2\pi/L$

Dominant correction (for nucleon) ; known scaling form
J.-W. Chen

Not included yet; O(20%) systematics

Smaller P_z correction but complicated twist-4 operator (extrapolate it away)

J.-H. Zhang, Y. Zhao, J.-W. Chen et al. (in preparation)

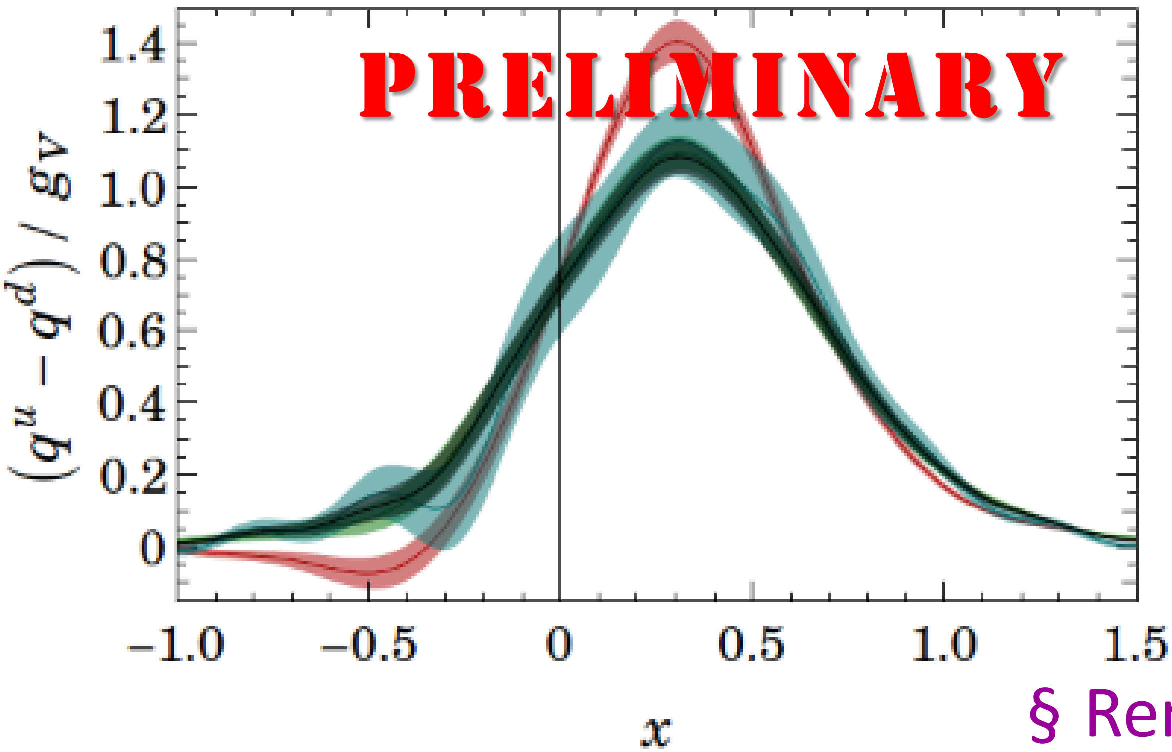
§ Changes in x, and q(x)

Quark Distribution

§ Exploratory study

- Take ratios (partially cancel statistical and systematic uncertainty)

$$q_{\mathrm{norm}}(x,\mu,P_z)=\frac{q(x,\mu,P_z)}{\int dx\,q(x,\mu,P_z)}$$



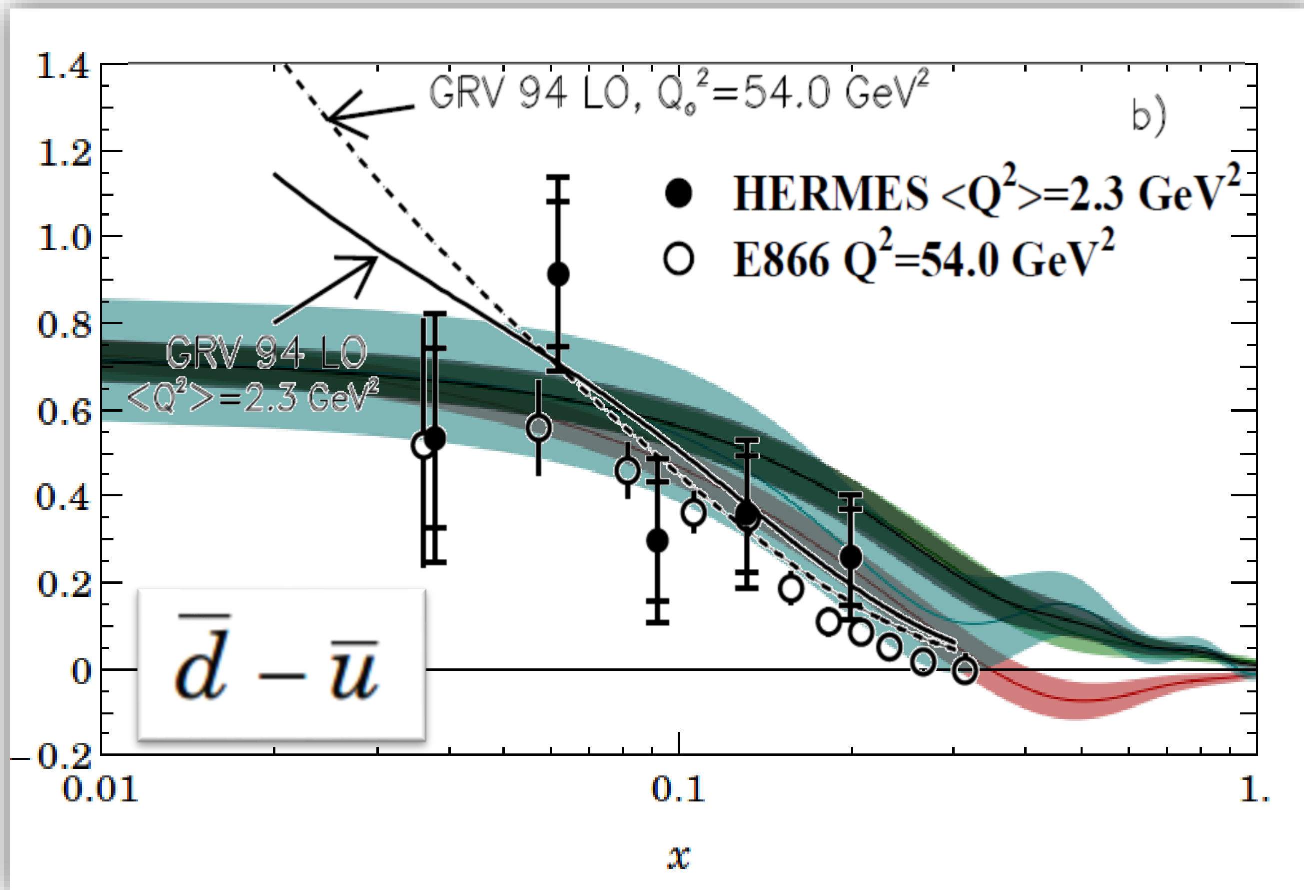
Removing $O(M_N^2/4P_z^2)$ errors

No significant finite-momentum effect seen for P_z>1

§ Renormalization needed

Quark Distribution

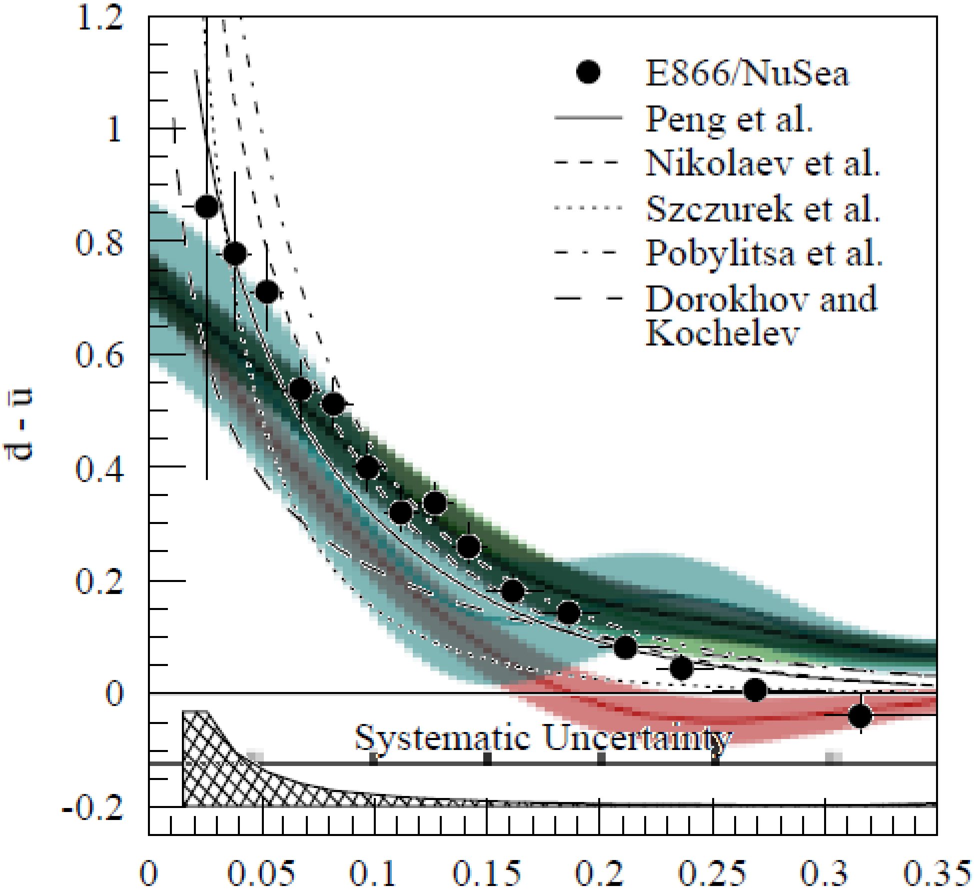
§ Compare with experiments



K. Ackersta et al. (HERMES Collaboration), Phys.Rev.Lett. 81, 5519 (1998)

Quark Distribution

§ Compare with experiments



Compared with E866

Too good to be true?

Lost resolution in small-*x* region

Future improvement to have larger lattice volume

$$\int dx\, \frac{\overline{u}(x) - \overline{d}(x)}{g_V} \approx 0.196\,(28)$$

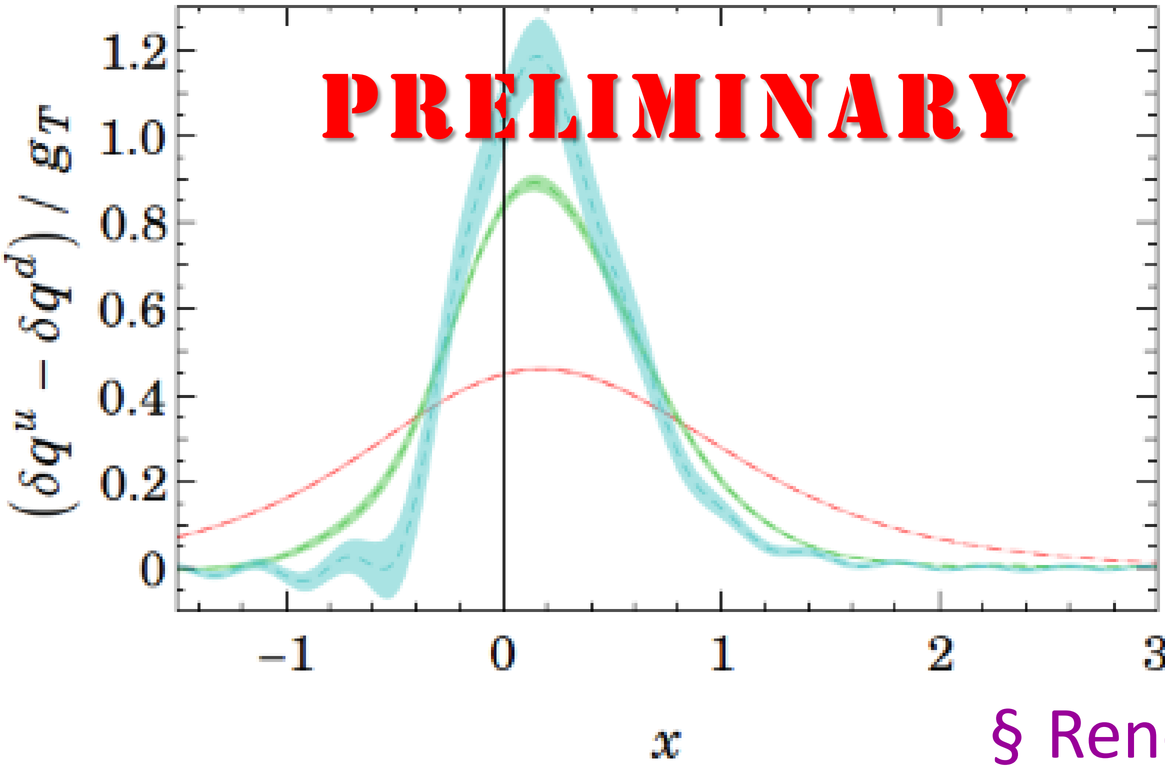
Experiment	x range	$\int_0^1[\overline{d}(x)-\overline{u}(x)]dx$
E866	$0.015<x<0.35$	0.118 ± 0.012
NMC	$0.004<x<0.80$	0.148 ± 0.039
HERMES	$0.020<x<0.30$	0.16 ± 0.03

R: Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

Transversity Distribution

§ Exploratory study

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \overline{\psi}(z)\, \sigma_{xy} \exp\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$



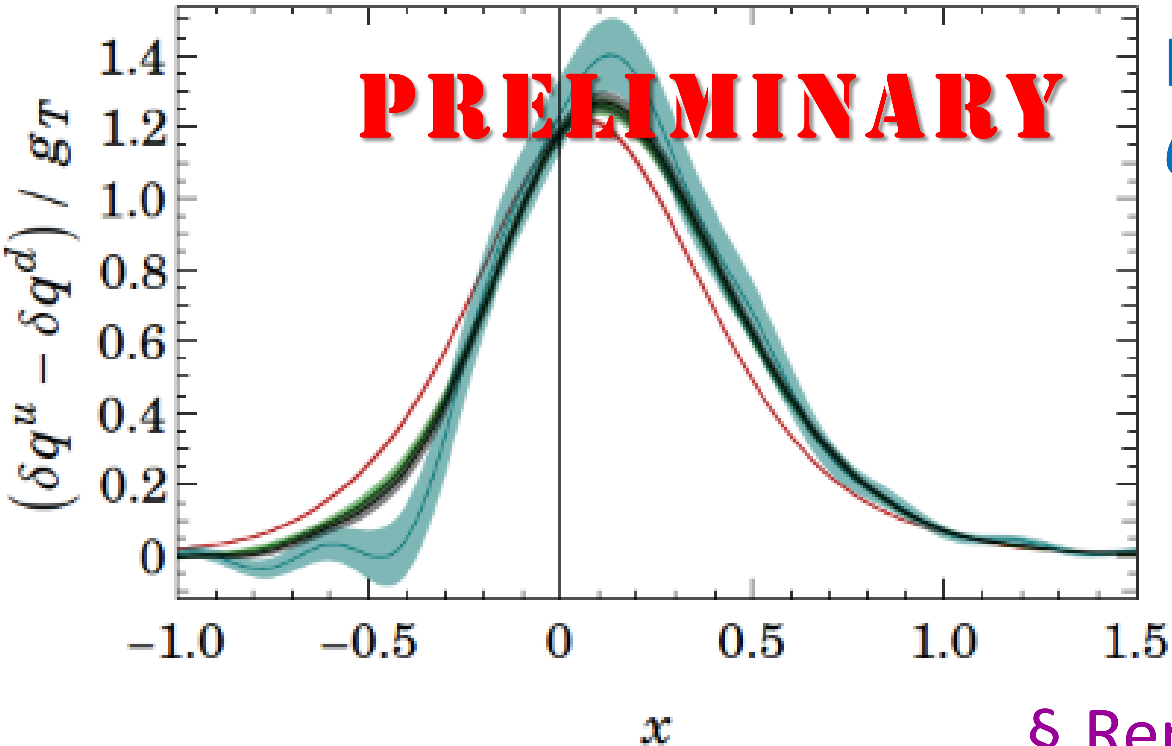
Uncorrected bare lattice results

§ Renormalization needed

Transversity Distribution

§ Exploratory study

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \overline{\psi}(z)\, \sigma_{xy} \exp\left(-ig\int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$



Removing $O(M_N^2/4P_z^2)$ errors

§ Renormalization needed

Transversity Distribution

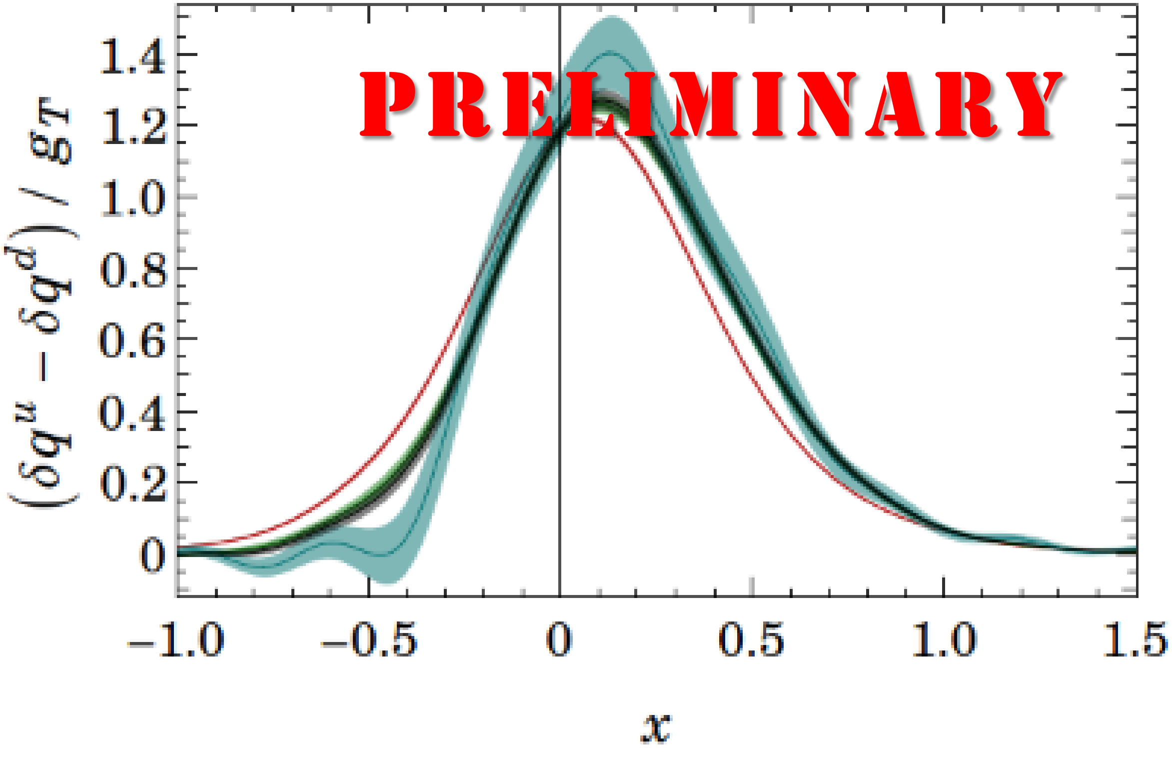
§ Exploratory study

- We found $\delta\overline{u} < \delta\overline{d}$ with large sea asymmetry
- Chiral quark-soliton model

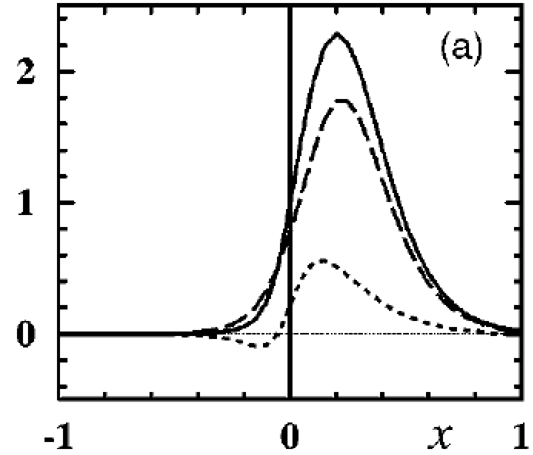
$$\int dx\, \frac{\delta\overline{u}(x) - \delta\overline{d}(x)}{g_T} \approx -0.320\,(18)$$

$$\int dx\left(\delta\overline{u}(x) - \delta\overline{d}(x)\right) \approx -0.082$$

B. Dressler et al., hep-ph/9809487



CQS model

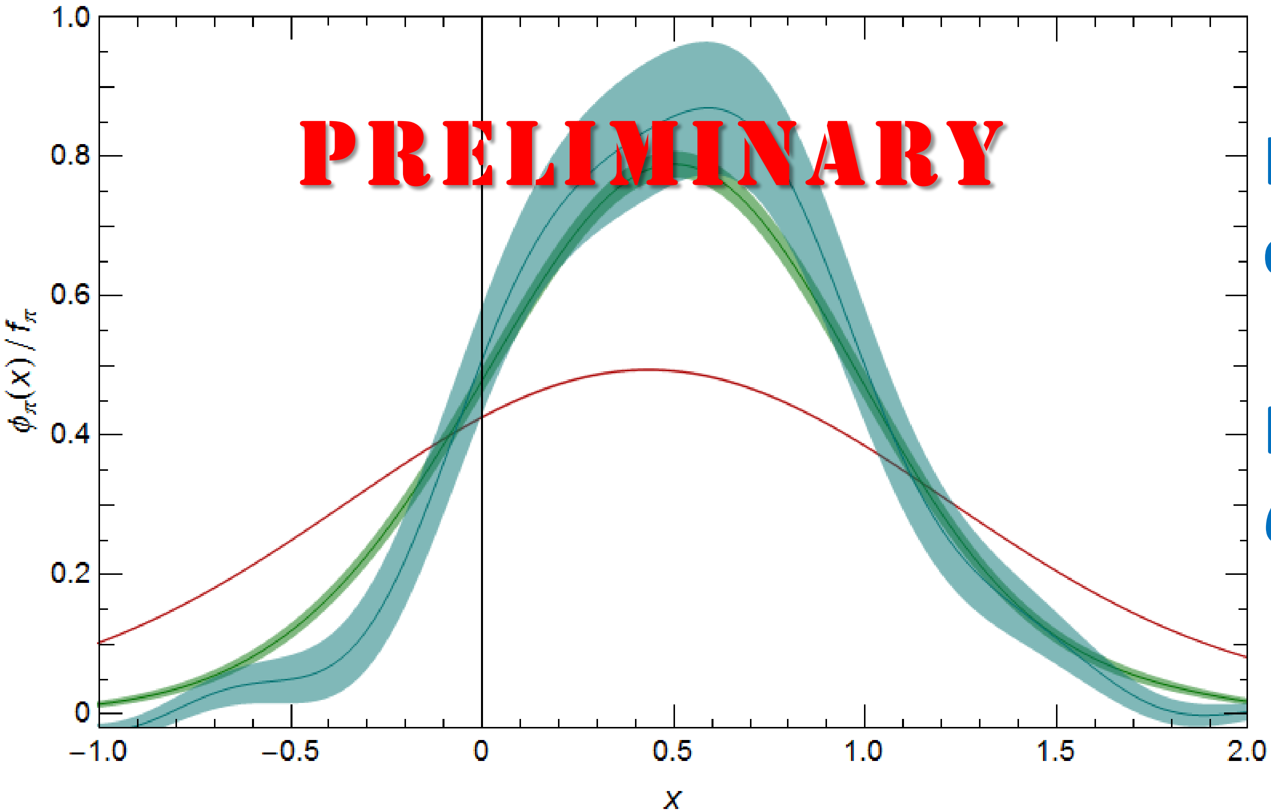


P. Schweitzer et al. PRD 64, 034013 (2001)

Pion Distribution Amplitude

§ Exploratory study

$$\int \frac{dz}{2\pi} e^{-izk_z} \left\langle 0 \left| \overline{d}(z)\, \gamma_z\, \gamma_5 \exp\!\left(-ig\int_0^z dz'\, A_z(z')\right) u(0) \right| \pi^+(P) \right\rangle$$



Leading mass correction applied

Dominated by $O(\Lambda_{QCD}^2/P_z^2)$ errors

$P_z \in \{1, 2, 3\}\, ^{2\pi}/_L$

A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful

Summary and Outlook

Exciting time for hadron structure on the lattice

§ Overcoming longstanding obstacle to *x*-distribution

- New idea by Ji for studying full x dependence of PDFs
- Promising results on unpolarized and polarized sea asymmetry compared with experiments, even at non-physical pion mass

§ Caveats

- Not a precision calculation *yet*
- Need to complete the other p_z corrections
  (on-going; possibly done in a couple weeks)
- Systematics due to large momenta (some ideas to improve it)

§ Need improvement for large-momentum sources

- Better overlapping boosted hadron smearing (asymmetric source)
- Applications: large-q form factors, hadronic and flavor physics, …

Summary and Outlook

Exciting time for hadron structure on the lattice

§ We hope this exploratory study motivates others to give Ji's method a try

- More details in the upcoming paper(s), such as P_z corrections from various sources

§ Hope to see many calculations and more ideas

- Like $g_{S,T}$ calculations with PNDME
- Many more quantities to study: strange/charm/beauty sea distributions, gluons, TMD…

§ "Working" workshop in Shanghai this fall

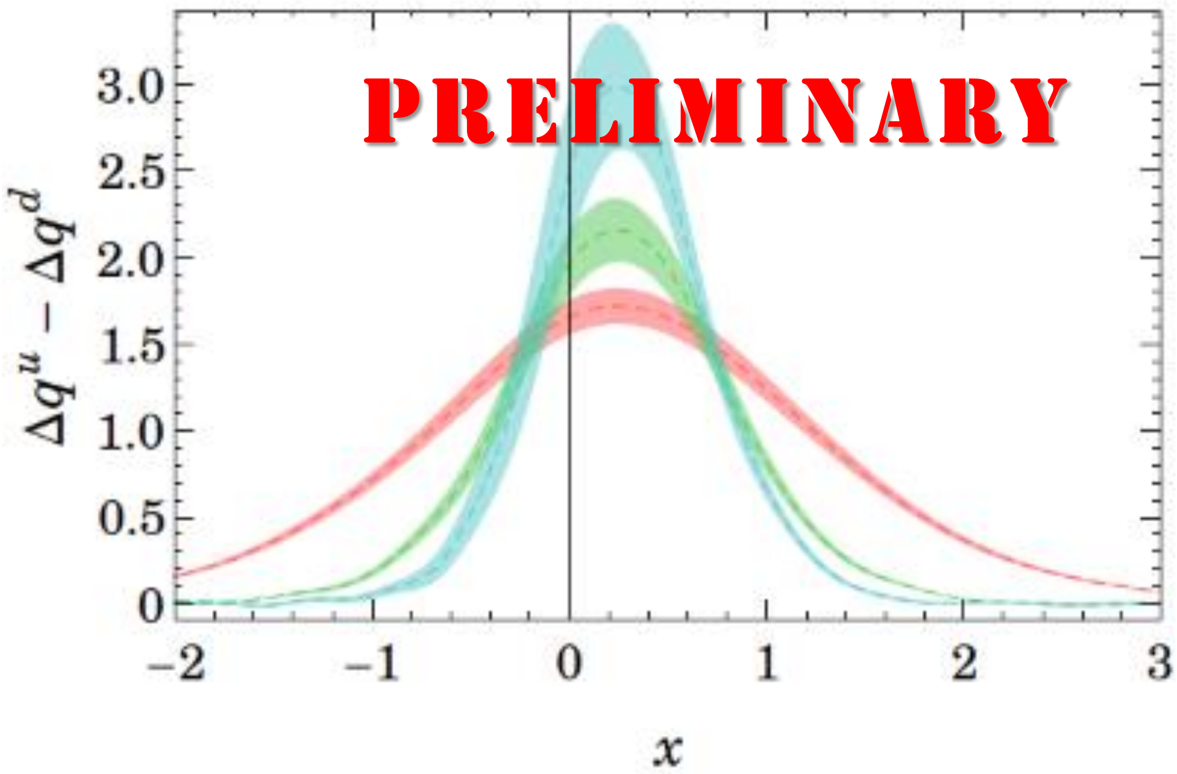
- More details will be announced via latticenews

Backup Slides

Helicity Distribution

§ Exploratory study

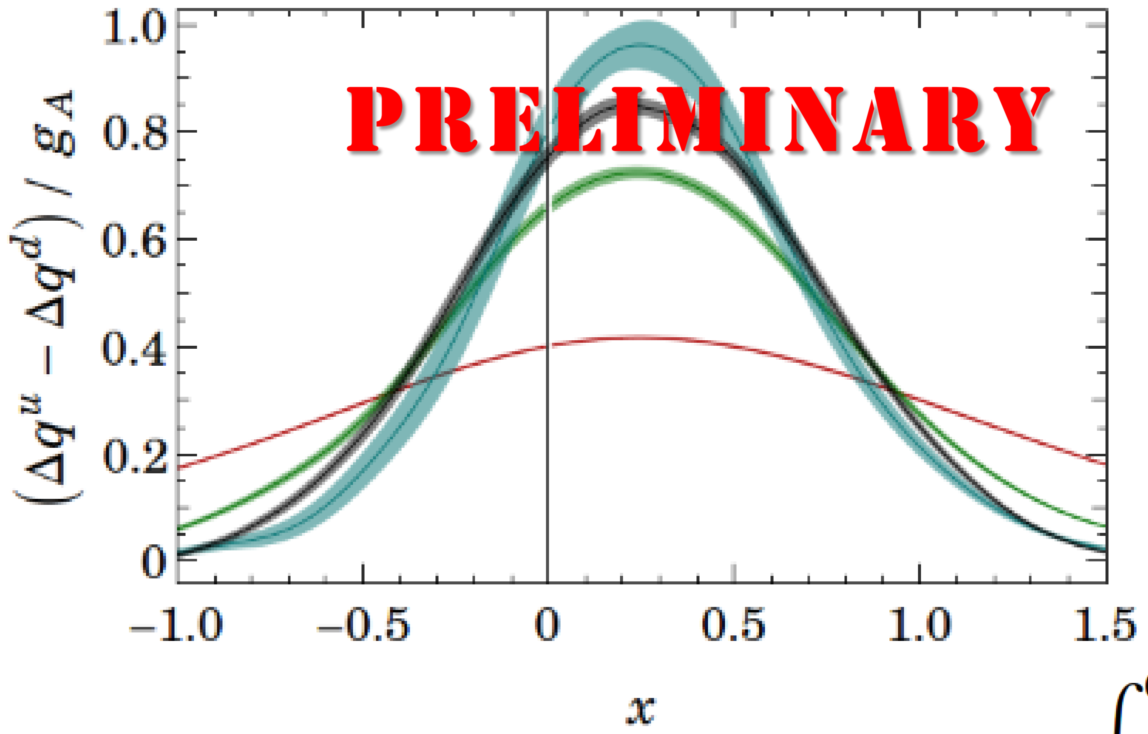
$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \overline{\psi}(z)\, \gamma_z\, \gamma_5 \exp\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$



Helicity Distribution

§ Exploratory study

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \overline{\psi}(z)\, \gamma_z\, \gamma_5 \exp\!\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \right| P \right\rangle$$



Corrected to $O(P_z^{-2})$

Gray band shows extrapolation of P_z^{-4} terms

Large $O(P_z^{-4})$ seen but well fit by extrapolation

$$\int_{-\infty}^{0} dx\, \frac{\Delta u(x) - \Delta d(x)}{g_A} \approx 0.315\,(11)$$

Helicity Distribution

§ Model: e.g. chiral quark-soliton model

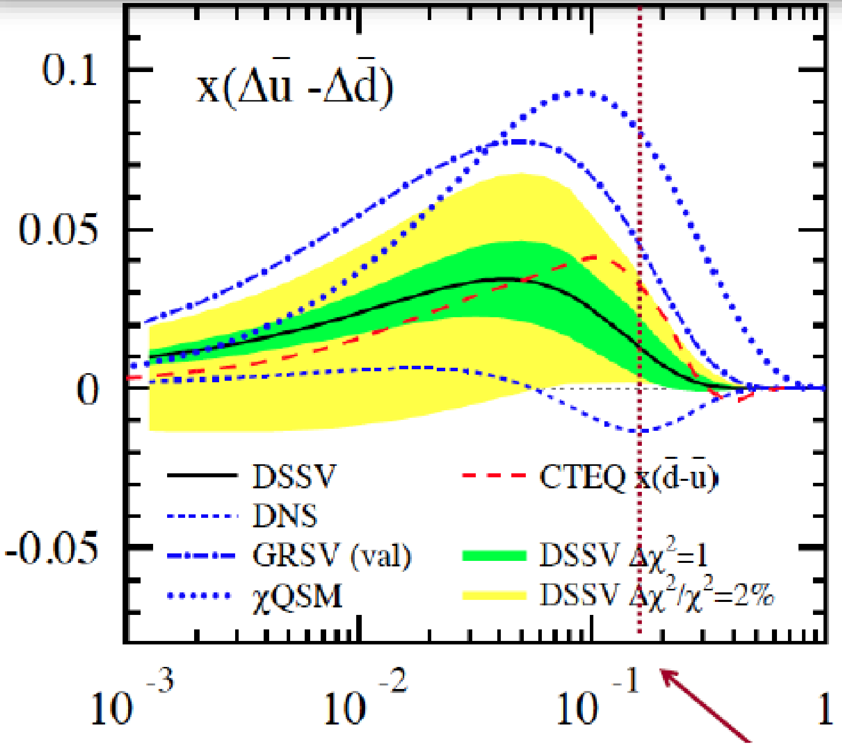
$$\int dx\,\left(\Delta\bar{u}(x)-\Delta\bar{d}(x)\right)\approx 0.31$$

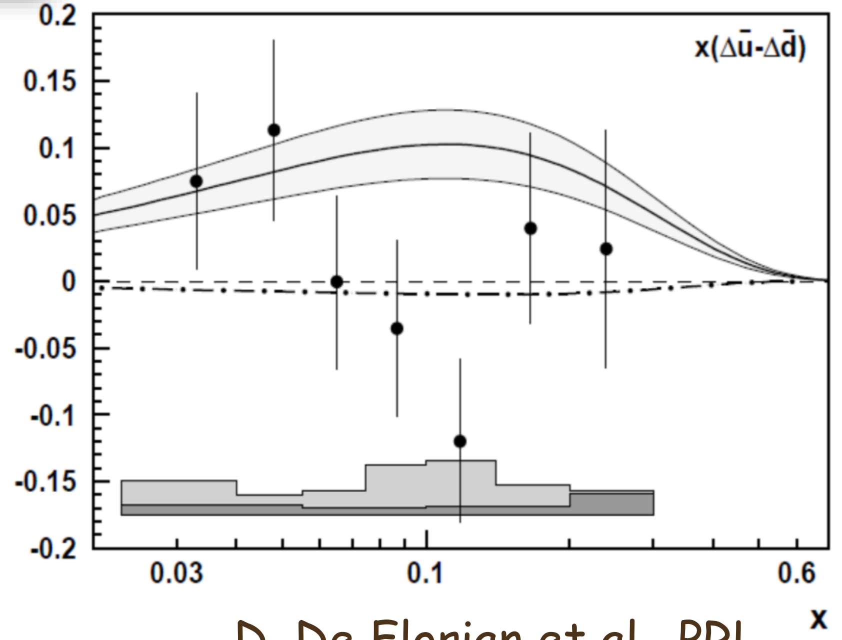
B. Dressler et al. hep-ph/9809487

§ Experimental comparison

$$\int dx\,\left(\Delta\bar{u}(x)-\Delta\bar{d}(x)\right)\approx 0.090\,(91)$$

A. Airapetian et al. (HERMES), Phys.Rev. D71, 012003 (2005)





D. De Florian et al., PRL 101 (2008) 072001

Helicity Distribution

§ **Model:** e.g. chiral quark-soliton model

$$\int dx\,\left(\Delta\bar{u}(x) - \Delta\bar{d}(x)\right) \approx 0.31$$

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§ **Experimental comparison**

$$\int dx\,\left(\Delta\bar{u}(x) - \Delta\bar{d}(x)\right) \approx 0.090\,(91)$$

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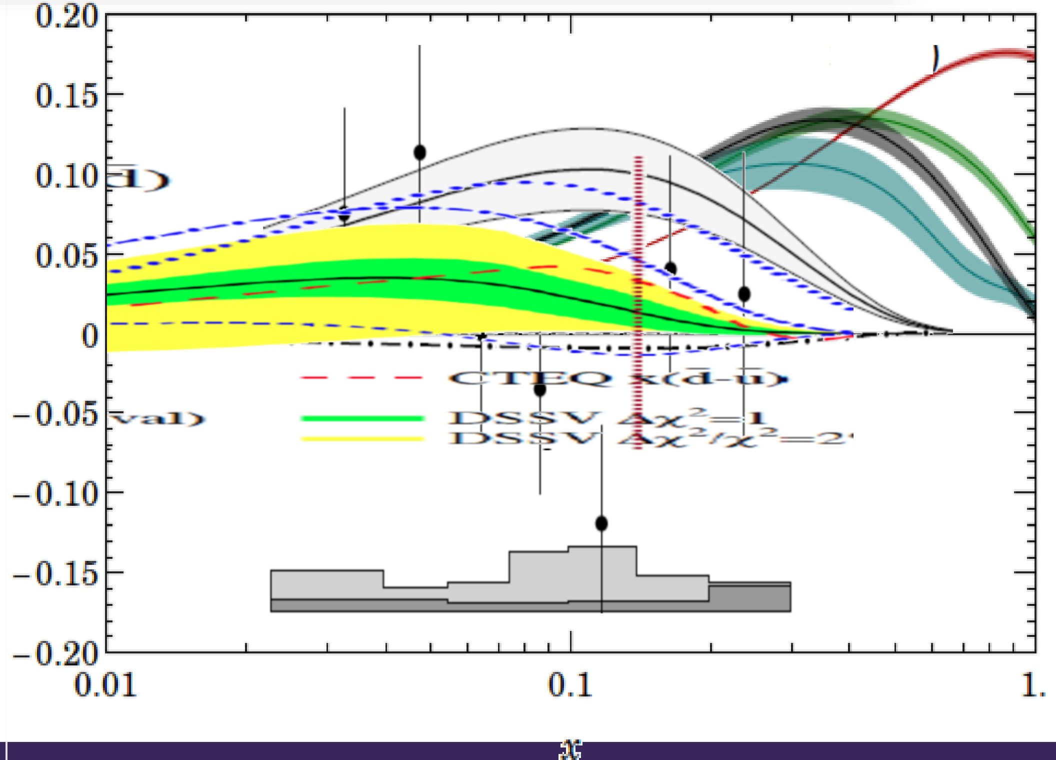

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D. De Florian et al., PRL 101 (2008) 072001