Nucleon generalized form factors from lattice QCD near the physical quark mass

… and an update on our nucleon mass and sigma term data

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Outline

Nucleon generalized form factors

- (t, m_π) dependence: $A_{20}, B_{20}, C_{20},\ \tilde{A}_{20}, \tilde{B}_{20}$
- Comparison of old and new $N_f = 2$ data (impact of smearing)
- Extension of BChPT to full one-loop

Nucleon mass and sigma term

- New data for $m_\pi = 151\ldots, 290, \ldots, 490\,\mathrm{MeV}$
- Pion-Nucleon sigma term
- Nucleon mass fit

Warning: All results are yet preliminary and may change.

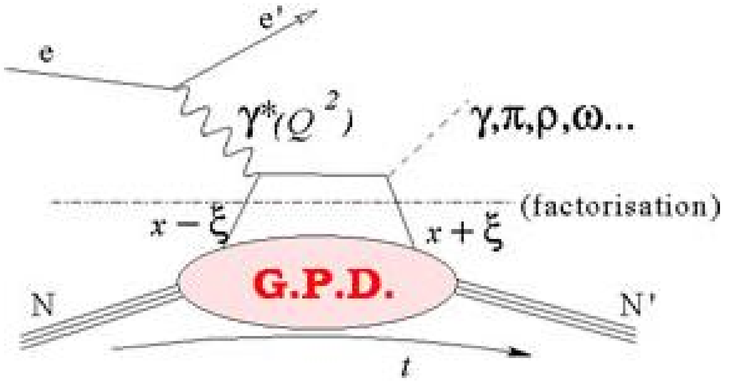
GPDs in a nutshell

Generalized Parton Distributions (GPDs)

- introduced late `90s, for a systematic study of hadron structure
- comprehensive description of the hadron structure **from first principles**

Contain information

- of traditional form factors and parton distributions (limiting cases)
- the quark orbital angular momentum
- various inter- and multi-parton correlations



In particular important for

- spin structure of the proton
- in deeply virtual Compton scattering (DVCS), analysis very demanding, requires **partial modeling** of combined x-, ξ- and t-dependence

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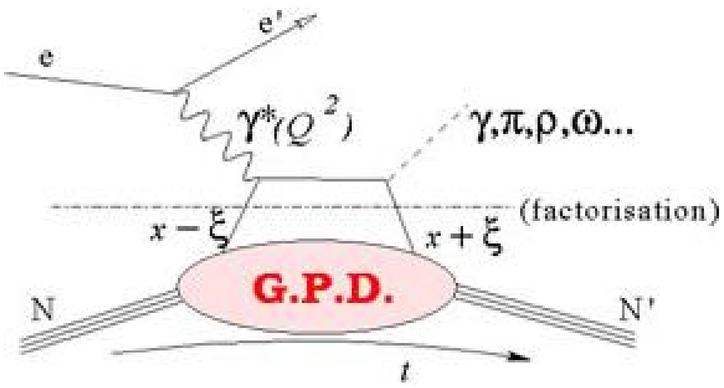


Cross-check with lattice QCD important

GPDs in a nutshell

Nucleon:

- Matrix elements parametrized by 2 vector, 2 axial-vector and 4 tensor GPDs

bilocal operator (see relevant reviews)

$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \mathrm{ht}$$

$$\langle N(P')|O_A^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu\gamma_5 \widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \mathrm{ht}$$

- Kinematic variables: x, ξ, t

momentum transfer

$$-Q^2 = t = -\Delta^2 \quad (\Delta = P' - P)$$

(longitudinal) momentum fraction

longitudinal momentum transfer
(skewness parameter) $\xi = -n \cdot \frac{\Delta}{2}$

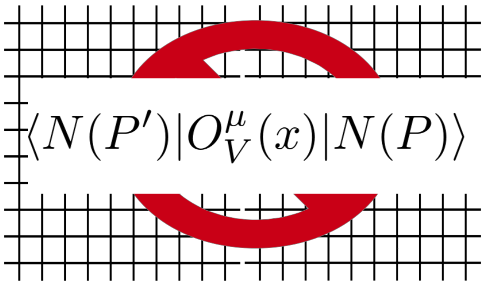
light-cone vector n
such that $n \cdot \overline{P} = 1$

$$\overline{P} = \frac{P' + P}{2}$$

GPDs on the lattice?

Direct determination not possible

- GPDs parametrize matrix elements of **bi-local** operators, separated by a light-like interval



$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \mathrm{ht}$$

- Wick-Rotation makes calculation on Euclidean lattice impossible

Generalized Form Factors

Mellin Moments of GPDs

- can be expressed by polynomials in ξ

$$\int_{-1}^{1} dx x^{n-1} \left[\begin{array}{c} H(x,\xi,t) \\ E(x,\xi,t) \end{array} \right] = \sum_{k=0}^{[(n-1)/2]} (2\xi)^{2k} \left[\begin{array}{c} A_{n,2k}(t) \\ B_{n,2k}(t) \end{array} \right] \pm \delta_{n,\mathrm{even}} (2\xi)^n C_n(t)$$

- expansion coefficients = generalized form factors

Generalized Form Factors

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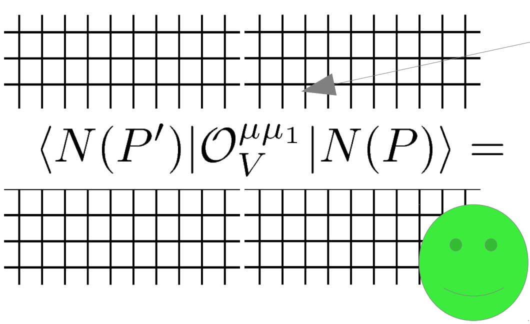
$$\int_{-1}^{1} dx x^{n-1} \begin{bmatrix} H(x,\xi,t) \\ E(x,\xi,t) \end{bmatrix} = \sum_{k=0}^{[(n-1)/2]} (2\xi)^{2k} \begin{bmatrix} A_{n,2k}(t) \\ B_{n,2k}(t) \end{bmatrix} \pm \delta_{n,\mathrm{even}} (2\xi)^n C_n(t)$$

- expansion coefficients = generalized form factors

Lattice QCD

- calculate these form factors via expectation values of **local** operators

$$\mathcal{O}_V^{\mu\mu_1}(z) = \mathcal{S}_{\mu\mu_1}\, \overline{q}(z)\, \gamma^{\mu} i^{\mu_1} q(z) - \mathrm{traces}$$



$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1}|N(P)\rangle = \mathcal{S}_{\mu\mu_1} \overline{U}(P') \left\{ \gamma^{\mu} \overline{P}^{\mu_1} A_{20}(t) + \frac{i\sigma^{\mu\nu}\Delta_{\nu}}{2m_N} \overline{P}^{\mu_1} B_{20}(t) + \frac{\Delta^{\mu}\Delta^{\mu_1}}{m_N} C_{20}(t) \right\} U(P)$$

We have

Data

- Nucleon (generalized) form factors for $n = 1{,}2{,}3$

$$\int_{-1}^{1} dx x^{n-1} \left[\begin{array}{c} H(x,\xi,t) \\ E(x,\xi,t) \end{array} \right] = \sum_{k=0}^{[(n-1)/2]} (2\xi)^{2k} \left[\begin{array}{c} A_{n,2k}(t) \\ B_{n,2k}(t) \end{array} \right] \pm \delta_{n,\mathrm{even}} (2\xi)^n C_n(t)$$

- Also axial and tensor GFFs and pion GFFs
- 3pt-functions with no, one and two derivatives → ratios of 3-point and 2-point functions
- Two lattice spacings, pion masses: 150...490 MeV

This talk

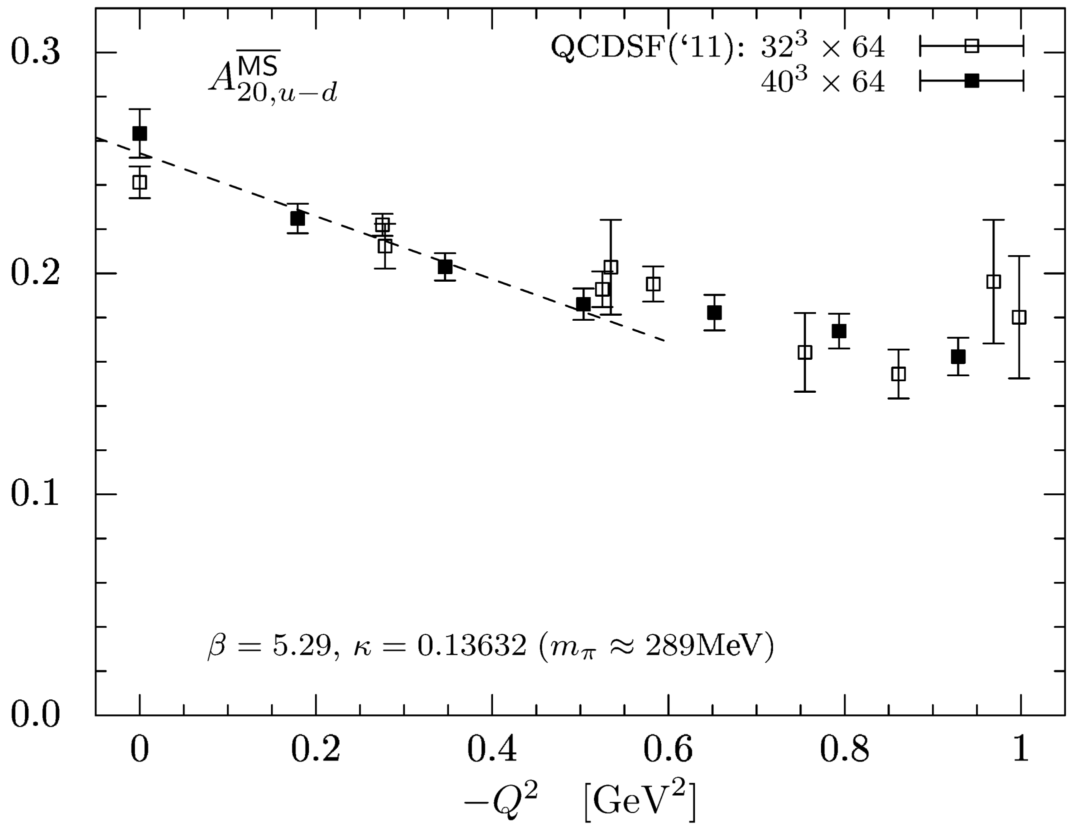
- Flash some of our $n = 2$ GFFs results (one derivative)

Isovector GFFs

t-dependence of $A_{20}(t)$

- Lattice2011 data
  (Jacobi-Smearing)

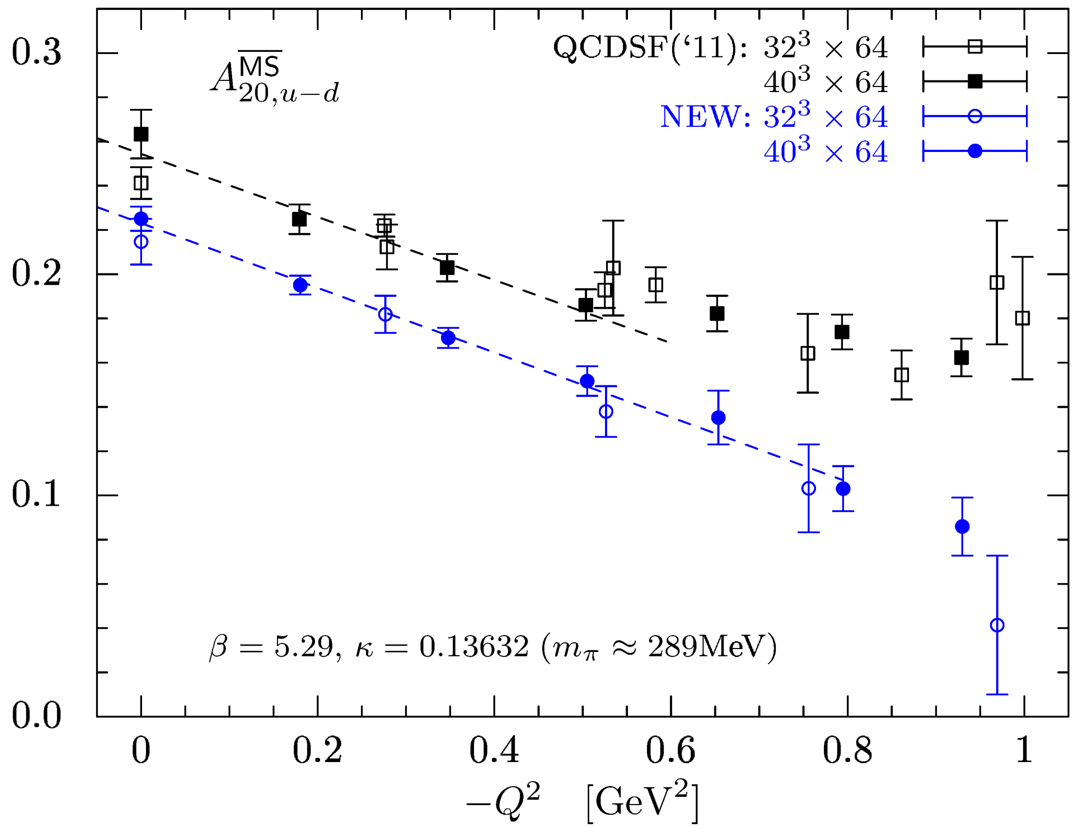
arXiv:1203.6579



Isovector GFFs

t-dependence of $A_{20}(t)$

- Lattice2011 data
  (Jacobi-Smearing)
- New data
  (Wuppertal smearing)
- Constant offset at small t
- Consistent with findings for $\langle x \rangle$
- Slopes are similar
- Deviations (and error) increase with t



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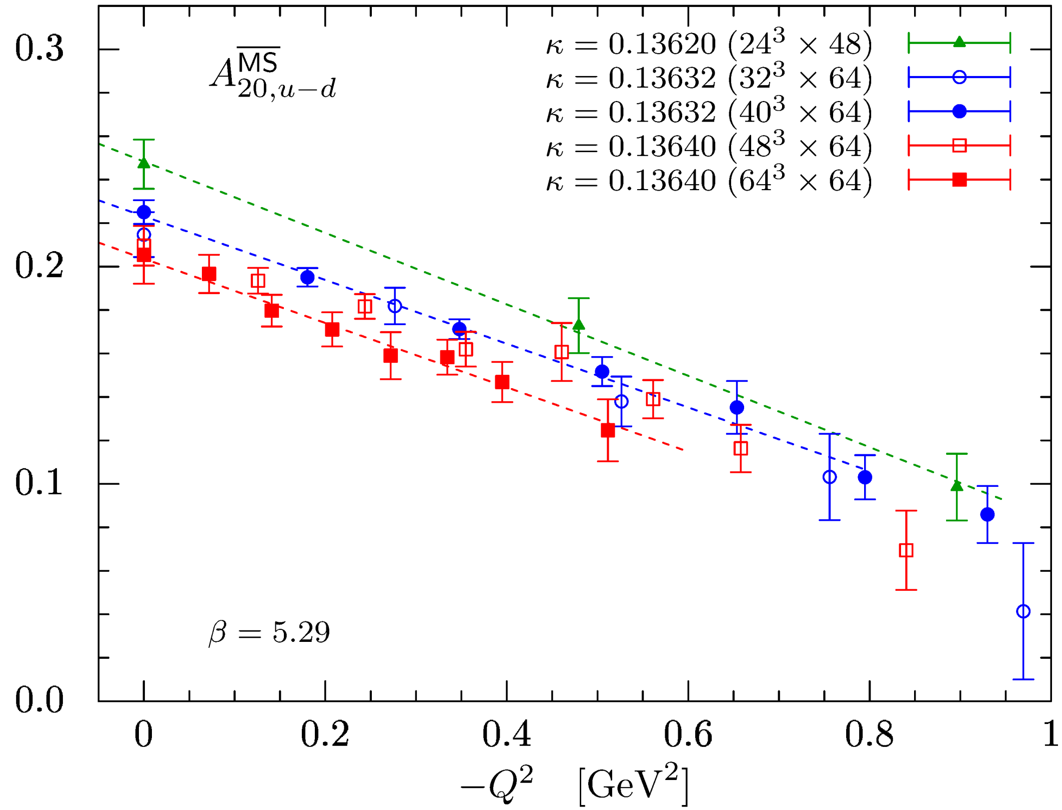


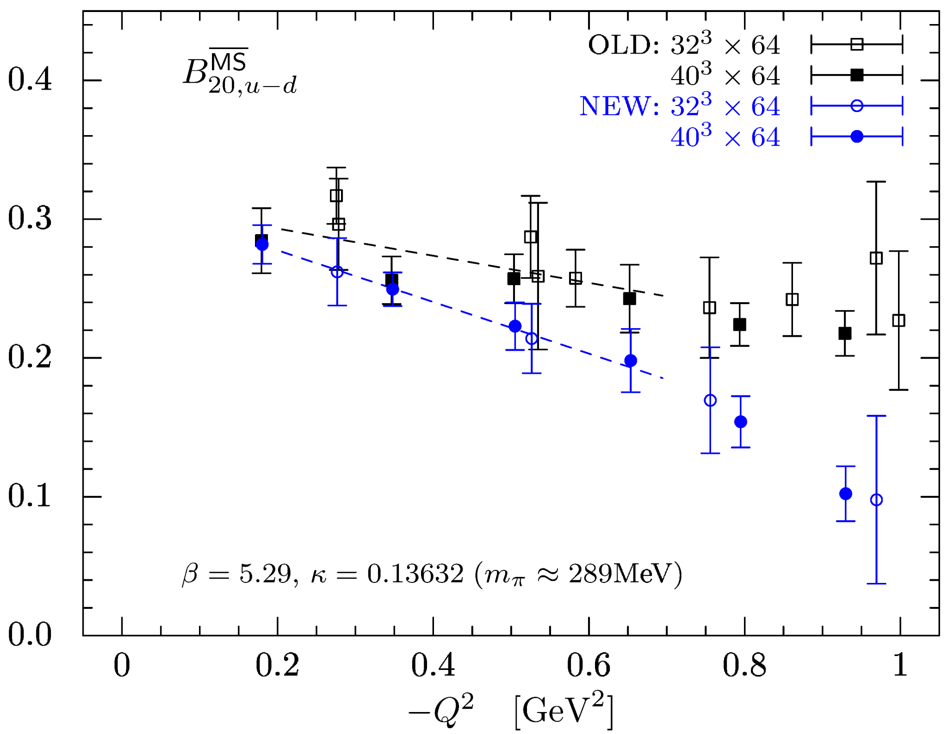
Varying m_π

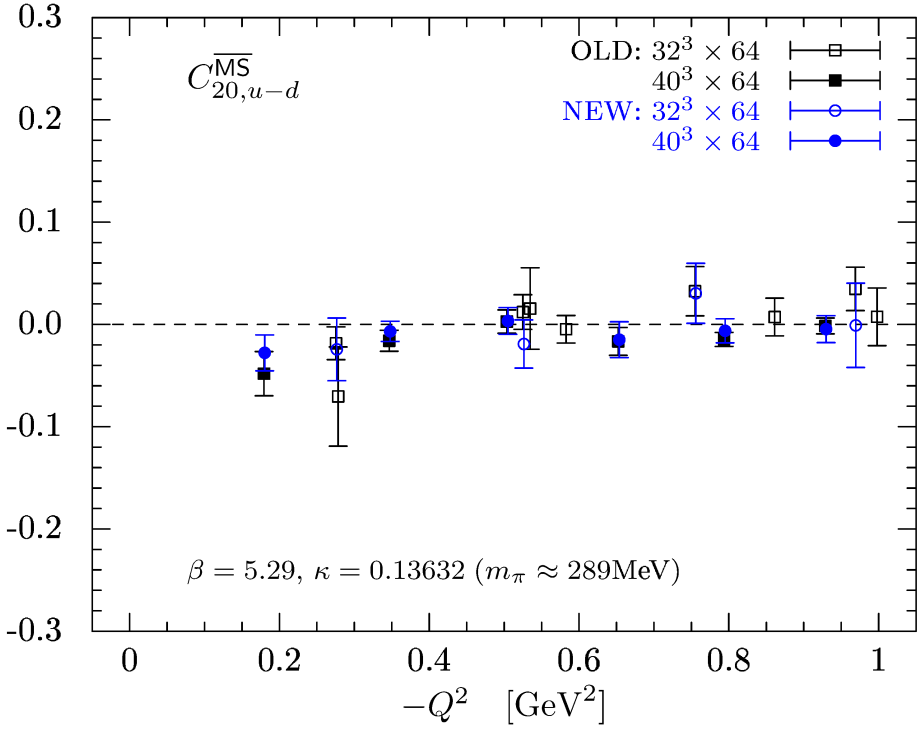
- $151\,\mathrm{MeV}$, $285\,\mathrm{MeV}$, $429\,\mathrm{MeV}$
- data shifted downwards with m_π

Isovector GFFs

t-dependence of Vector GFFs: B_{20} and C_{20}

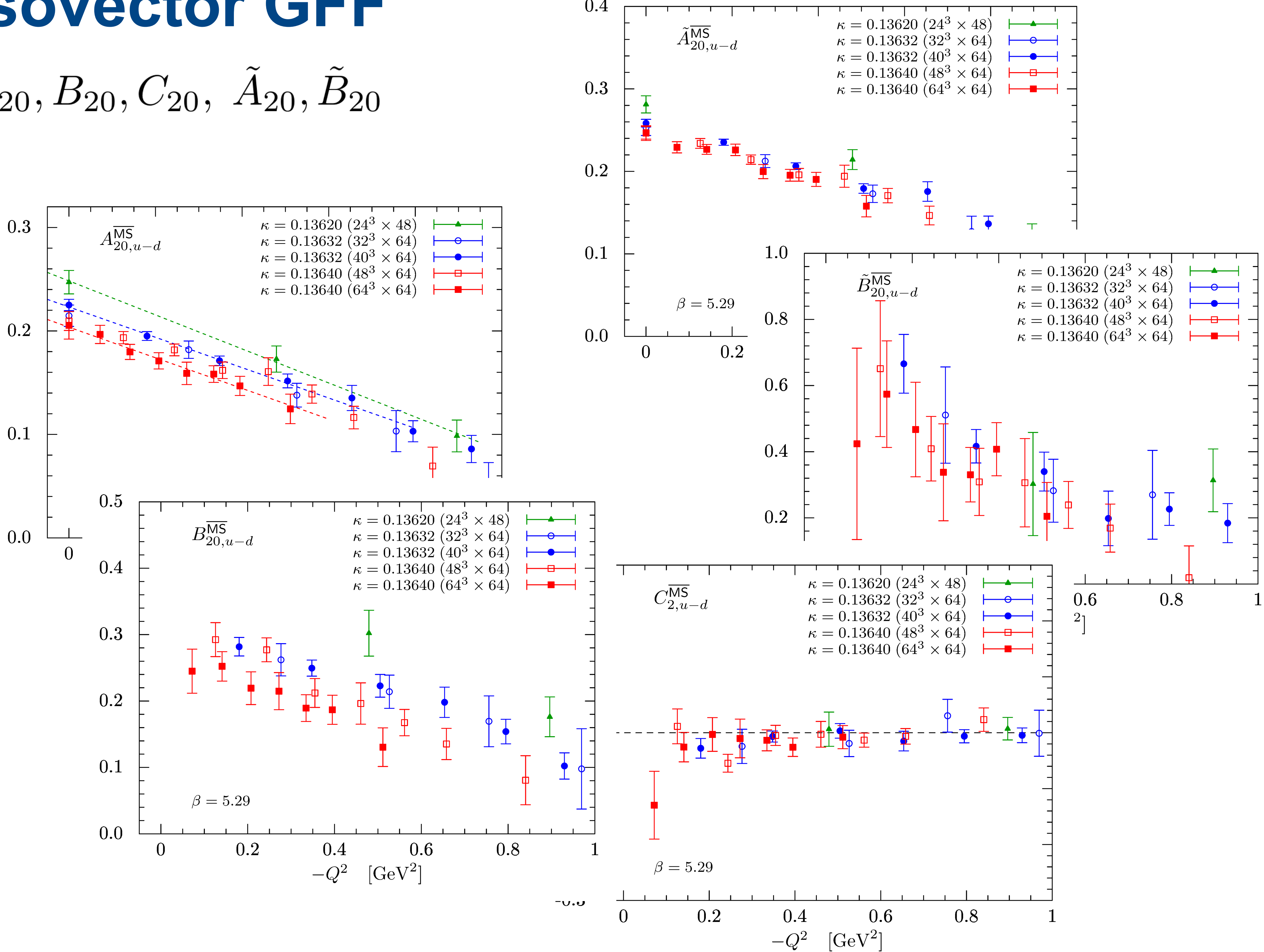
- Lattice2011 and new data: for B_{20} deviations increase with t





Isovector GFF

$$A_{20}, B_{20}, C_{20},\ \tilde{A}_{20}, \tilde{B}_{20}$$



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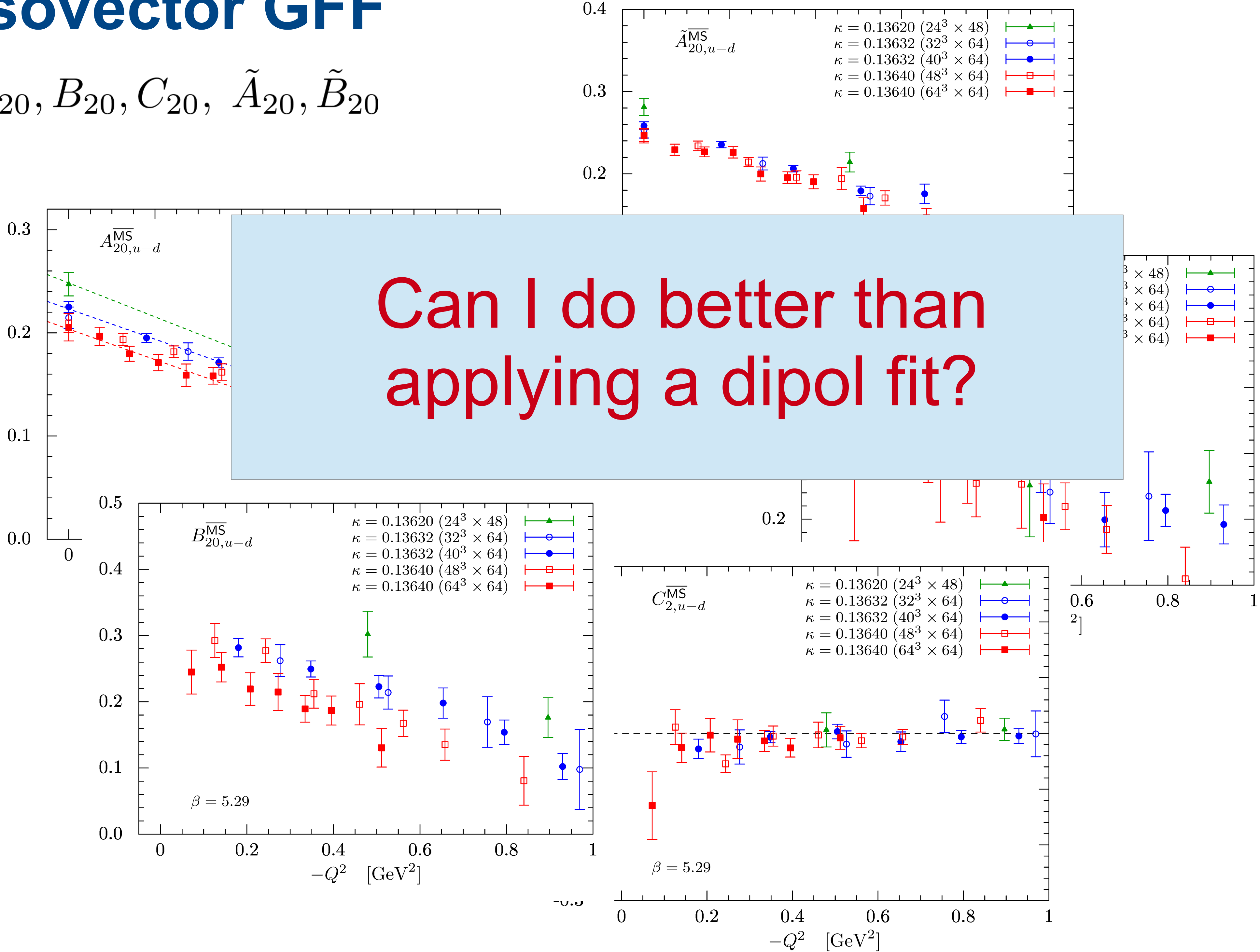

Baryon Chiral Perturbation theory

Up to now

[M. Dorati, T. A. Gail, T. R. Hemmert (2008)]

- **leading** 1-loop (2nd order) calculation of the nucleon-to-nucleon matrix element (twist-2 tensor operator) was available
- some 3rd order contributions were included

New: full 1-loop expressions (BChPT)

[P. Wein, P. Bruns, A. Schäfer (2013)]

- nucleon-to-nucleon matrix elements (tensor) and $\rightarrow A_{20}, B_{20}, C_{20}$
- for the pseudo tensor operators $\rightarrow \tilde{A}_{20}, \tilde{B}_{20}$

Advantage

- allows fit of the combined (Q^2, m_π) -dependence
  simultaneous fit of all 5 GFFs
- parameters enters different expression for form factors
- Disadvantage: 16 unknown parameters, valid only for small $m_\pi, -Q^2$

First attempt of a simultaneous fit

Observations

- it works (from a practical point of view)
- small Q^2-dependence almost linear
- validity range of expressions unclear
- current fit parameters certainly beyond their physical values (unknown)

Next

- systematic study $\longrightarrow$ stay tuned!

Isoscalar GFFs ....soon

Data available

- for connect part so far (see backup slide)
- disconnected is being produced (next slides: first data for scalar case)

Why?

- In the forward limit related to total angular momentum of quark in the nucleon (Ji's sum rule)

$$J^q = \frac{1}{2}\int_{-1}^{1} dx x(H(x,\xi,0)+E(x,\xi,0)) \equiv \frac{1}{2}\left(A_{20}(t=0)+B_{20}(t=0)\right).$$

- Can also compute orbital angular momentum $L^q = J^q - s^q$

  when combined with the quark spin contributions to the nucleon
  (= forward value of the axial form factor)

$$s^q = \frac{1}{2}\int_{-1}^{1} dx \tilde{H}(x,\xi,0) = \frac{1}{2}\tilde{A}_{10}(t=0)$$

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- Comparison of old and new $N_f = 2$ data (impact of smearing)
- Progress on BChPT

Nucleon mass and sigma term

- New data for $m_\pi = 151\ldots, 290, \ldots, 490\,\mathrm{MeV}$
- Pion-Nucleon sigma term
- Nucleon mass fit

Warning: All results are yet preliminary and may change.

Nucleon mass and sigma terms

Nucleon

- spontaneous χSB generates most of nucleon's mass
- small fraction from sea + valence quarks

Sigma terms

$$\sigma_q = m_q \langle N | \bar{q} q | N \rangle$$

- parametrizes individual quark contribution to nucleon mass

$$\Longrightarrow f = \frac{\sigma_q}{M_N}$$

Pion-Nucleon sigma term

$$\sigma_{\pi N} = m_l \langle N | \bar{u} u + \bar{d} d | N \rangle$$

$$m_l = \frac{m_u + m_d}{2}$$

- contribution of light quarks

$$M_N = M_0 + \sigma_{\pi N} + \ldots$$

- phenomenology does not give a clear answer on $\sigma_{\pi N}$

$64(7)\mathrm{MeV}$

$59(7)\mathrm{MeV}$

$45(8)\mathrm{MeV}$

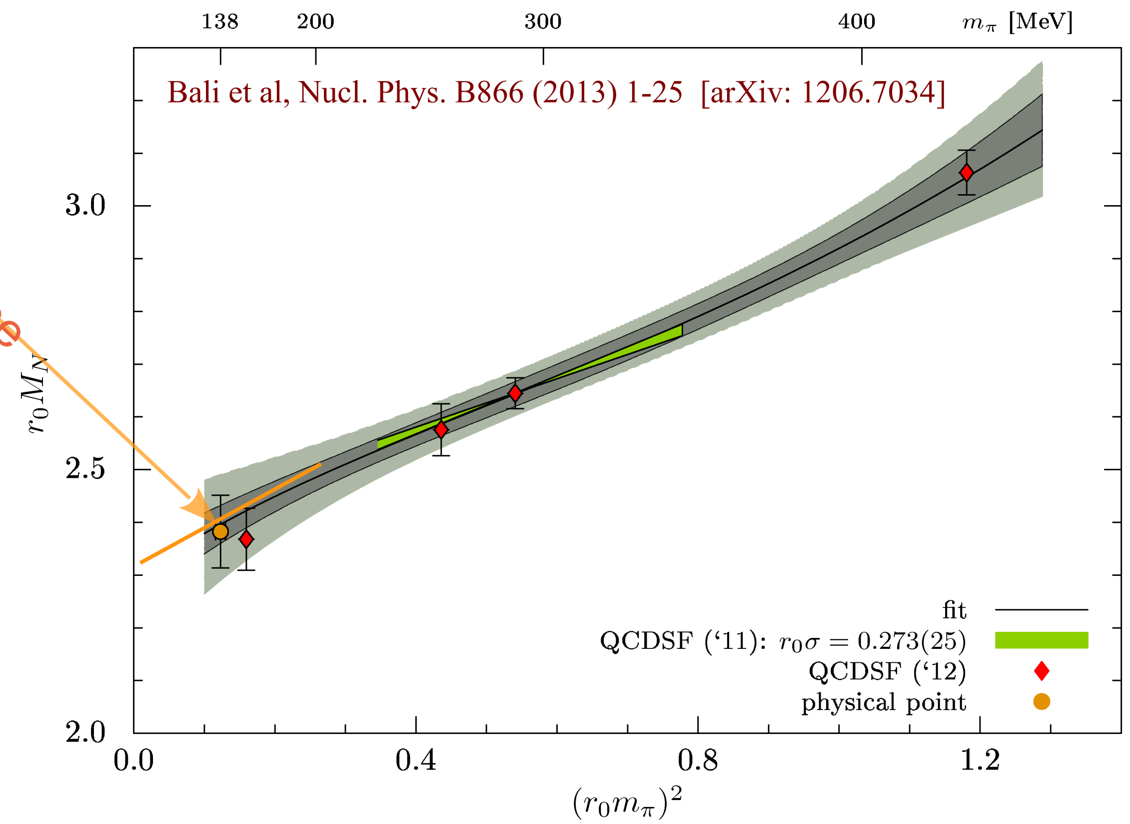
Pion-Nucleon sigma term on the lattice

(A) indirectly via Feynman-Hellmann theorem

$$\sigma_{\pi N} = m_\pi^2 \frac{\partial M_N}{\partial m_\pi^2}$$

- LQCD allows to study $M_N(m_\pi)$
- expensive close to physical point

slope



Pion-Nucleon sigma term on the lattice

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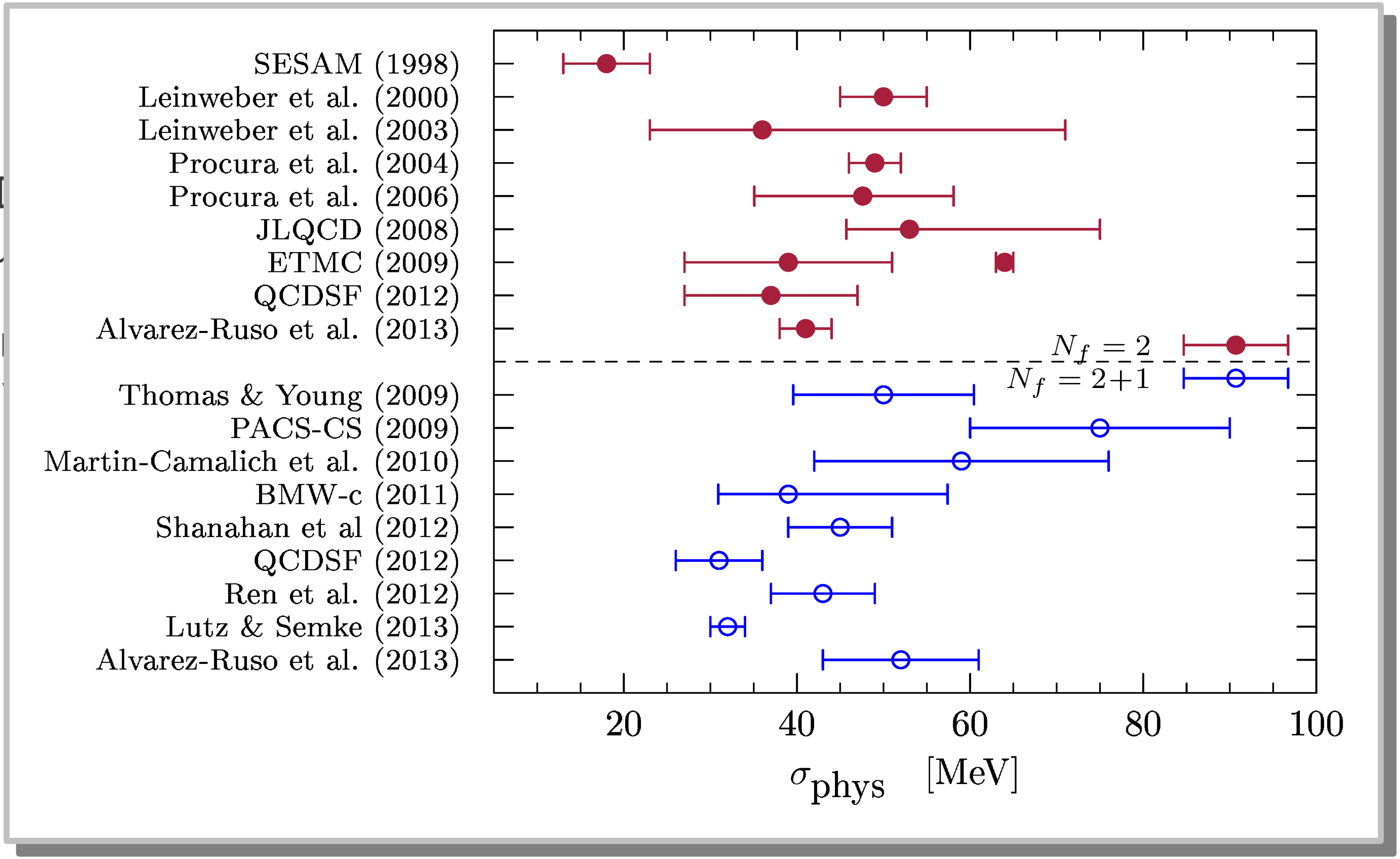
$\sigma_{\pi N}$

– LQCD
to stu

– expe
to ph



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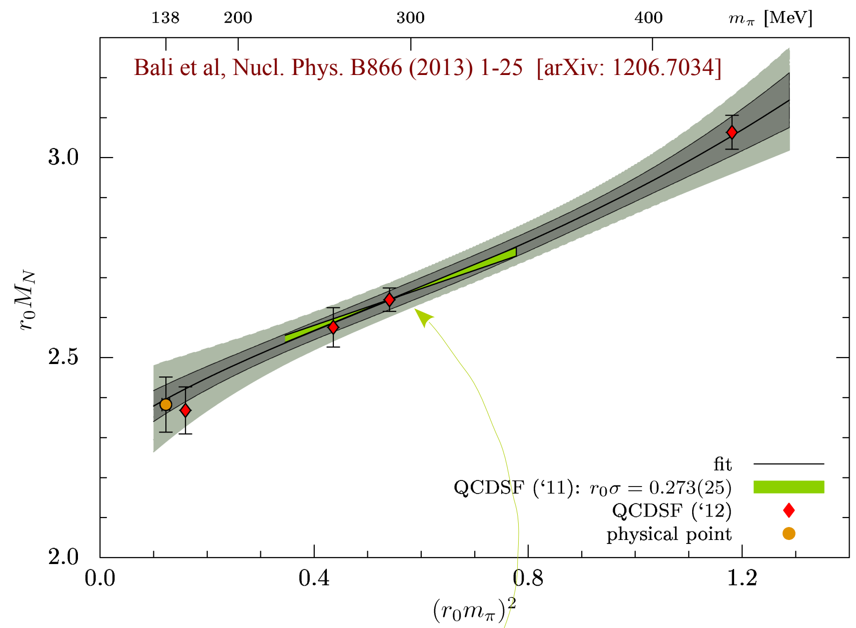


(B) directly

- Computational intensive (**disconnected diagrams**)
- became feasible in recent years (single quark mass / lattice spacing)

[Bali, Collins et al. (2011)]

$$\sigma_{\pi N}(290\text{MeV}) = 108(10)\text{MeV} \qquad N_f = 2$$

[Drach, Dinter et al. (2012)]

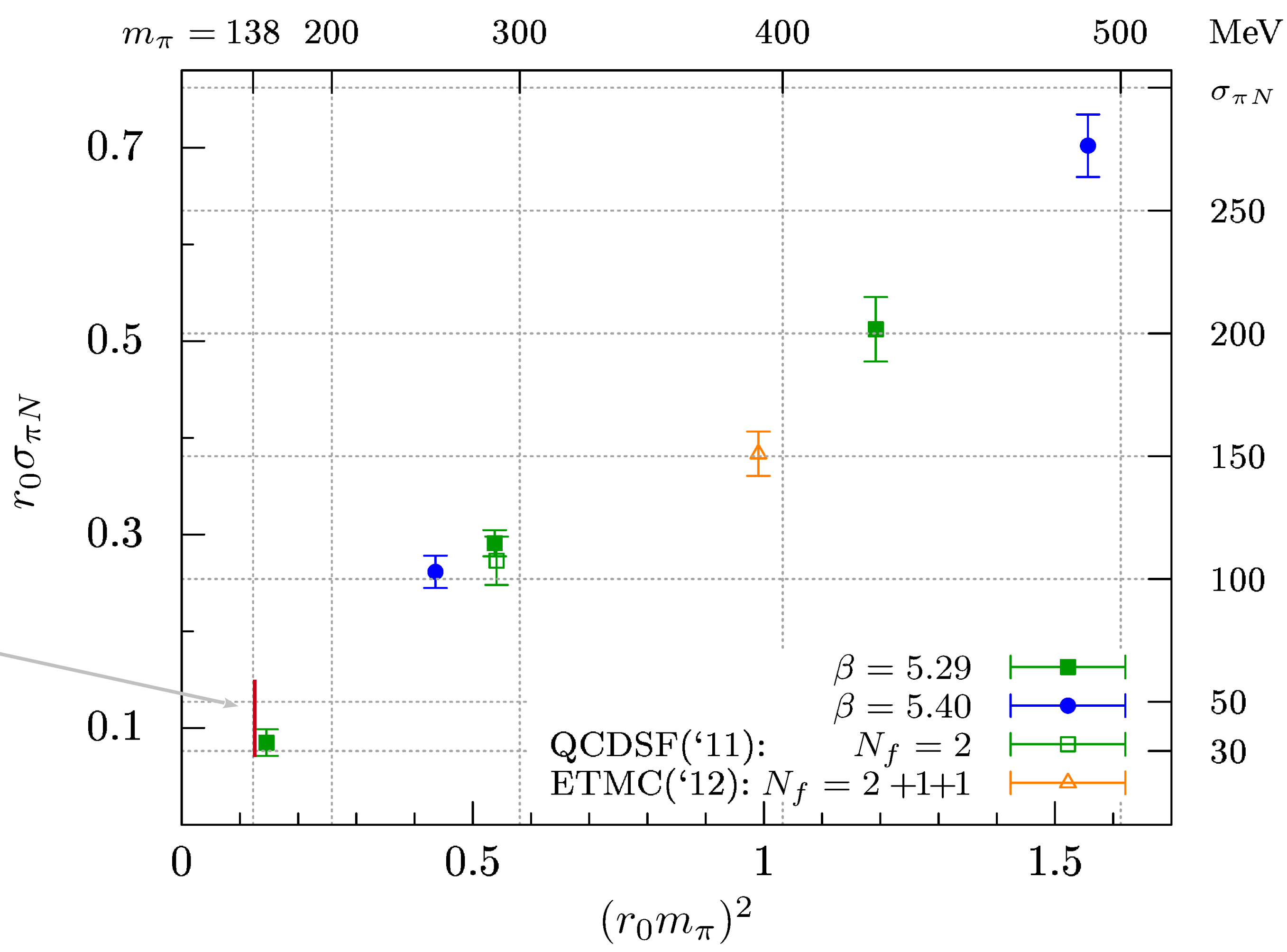
$$\sigma_{\pi N}(390\text{MeV}) = 151(8)(4)\text{MeV} \qquad N_f = 2{+}1{+}1$$

Pion-Nucleon sigma term on the lattice

New direct calculations of $\sigma_{\pi N}$

- pion masses: $151\ldots490\text{MeV}$
- **preliminary results**

  agree with findings from indirect calculation

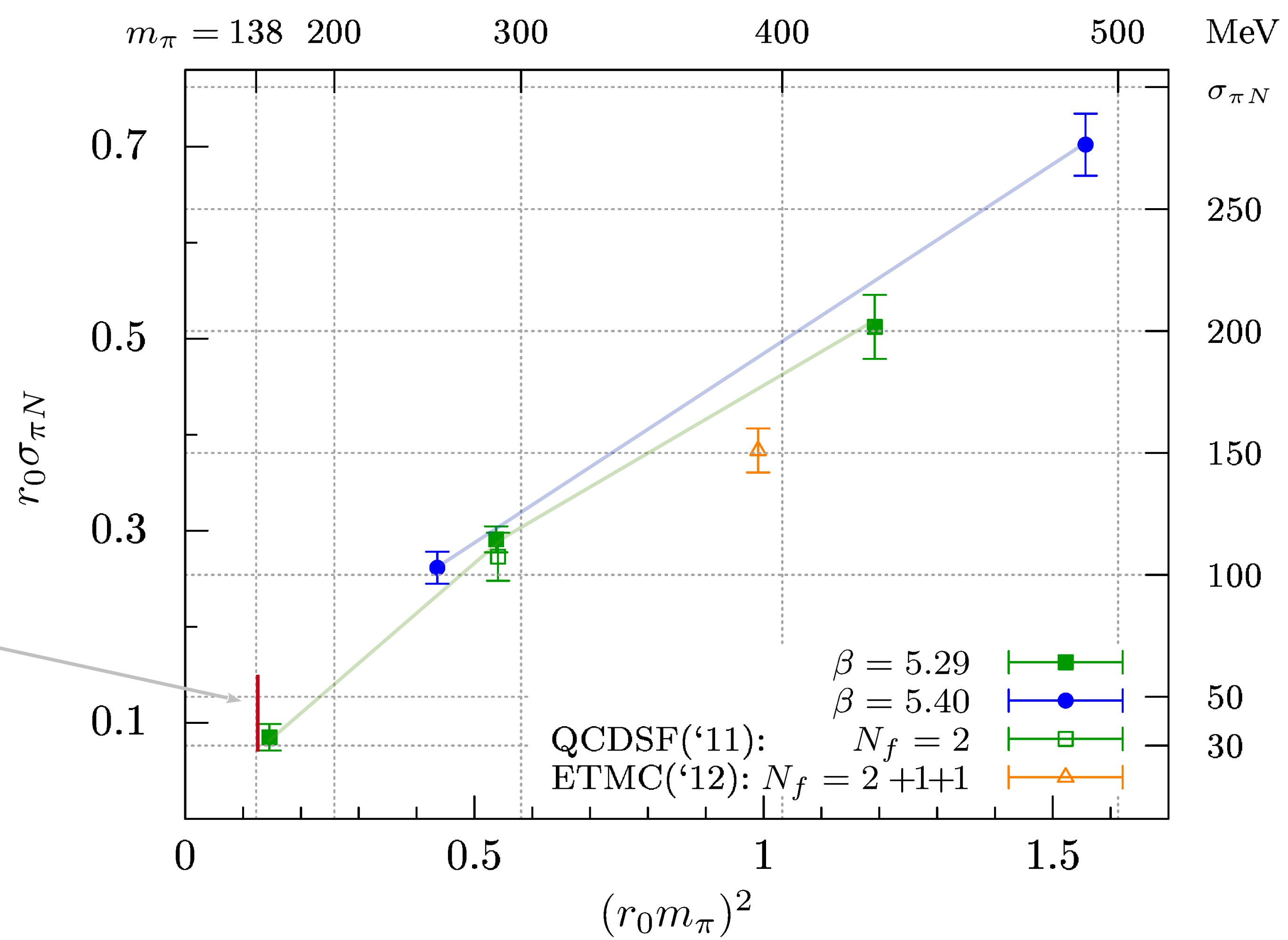


Pion-Nucleon sigma term on the lattice

New direct calculations of $\sigma_{\pi N}$

- pion masses: $151 \ldots 490\text{MeV}$
- **preliminary results**

  agree with findings from indirect calculation
- Lattice spacing and flavor dependence might be an issue



Physical scale and low energy constants

Baryon Chiral Perturbation theory (BChPT)

Nucleon mass

$$M_N = M_0 - 4c_1 m_\pi^2 - \frac{3g_A^2 m_\pi^3}{32\pi F_\pi^2} + 4e_1^r m_\pi^4 \\ + \frac{m_\pi^4}{8\pi^2 F_\pi^2}\left[\frac{3c_2}{16} - \frac{3g_A^2}{8M_0} + \log\frac{m_\pi}{\lambda}\left(8c_1 - \frac{3c_2}{4} - 3c_3 - \frac{3g_A^2}{4M_0}\right)\right]$$

Sigma term

$$\sigma = -4c_1 m_\pi^2 - \frac{9g_A^2 m_\pi^3}{64\pi F_\pi^2} + m_\pi^4\left[8e_1^r - \frac{8c_1 l_3^r}{F_\pi^2} + \frac{3c_1}{8\pi^2 F_\pi^2} - \frac{3c_3}{16\pi^2 F_\pi^2} \\ - \frac{9g_A^2}{64\pi^2 M_0 F_\pi^2} + \frac{1}{4\pi^2 F_\pi^2}\log\frac{m_\pi}{\lambda}\left(7c_1 - \frac{3c_2}{4} - 3c_3 - \frac{3g_A^2}{4M_0}\right)\right] .$$

Physical scale and low energy constants

Baryon Chiral Perturbation theory (BchPT)

Desired: a_{M_N} [fm] $(r_0$ [fm]$)$

$M_0, c_1, c_2, c_3, e_1^r, \bar{l}_3$

Nucleon mass

$$M_N = M_0 - 4c_1 m_\pi^2 - \frac{3g_A^2 m_\pi^3}{32\pi F_\pi^2} + 4e_1^r m_\pi^4 + \frac{m_\pi^4}{8\pi^2 F_\pi^2}\left[\frac{3c_2}{16} - \frac{3g_A^2}{8M_0} + \log\frac{m_\pi}{\lambda}\left(8c_1 - \frac{3c_2}{4} - 3c_3 - \frac{3g_A^2}{4M_0}\right)\right]$$

Sigma term

$$\sigma = -4c_1 m_\pi^2 - \frac{9g_A^2 m_\pi^3}{64\pi F_\pi^2} + m_\pi^4\left[8e_1^r - \frac{8c_1 l_3^r}{F_\pi^2} + \frac{3c_1}{8\pi^2 F_\pi^2} - \frac{3c_3}{16\pi^2 F_\pi^2} - \frac{9g_A^2}{64\pi^2 M_0 F_\pi^2} + \frac{1}{4\pi^2 F_\pi^2}\log\frac{m_\pi}{\lambda}\left(7c_1 - \frac{3c_2}{4} - 3c_3 - \frac{3g_A^2}{4M_0}\right)\right].$$

$$l_3^r \equiv -\frac{1}{64\pi^2}\left(\bar{l}_3 + 2\log\frac{m_\pi^{\text{phys}}}{\lambda}\right)$$

Our approach: Simultaneous fit

as in Bali et al, Nucl. Phys. B866 (2013) 1-25 [1206.7034]

$$r_0 = 0.501(10)(11)\mathrm{fm}$$

with 2012 data

Fit

- nucleon mass and sigma term data plus their volume dependence
- all data in units of r_0 : $\hat{M}_N \equiv r_0 M_N,\ \hat{L} \equiv L/r_0, \ldots$

χ^2 function

$$\chi^2 = \chi^2_N + \chi^2_\sigma$$

$$\chi^2_\sigma = \sum_i^{ndata} \frac{\left(\widehat{\sigma}_{\pi N}(\hat{m}_\pi) + \Delta\widehat{\sigma}_{\pi N}(\hat{m}_\pi, \hat{L}) - \hat{\sigma}_i\right)^2}{e_i^2}$$

$$\chi^2_N = \sum_i^{ndata} \frac{\left(\widehat{M}_N(\hat{m}_\pi) + \Delta\widehat{M}_N(\hat{m}_\pi, \hat{L}) - \hat{M}_i\right)^2}{e_i^2}$$

BChPT parameters

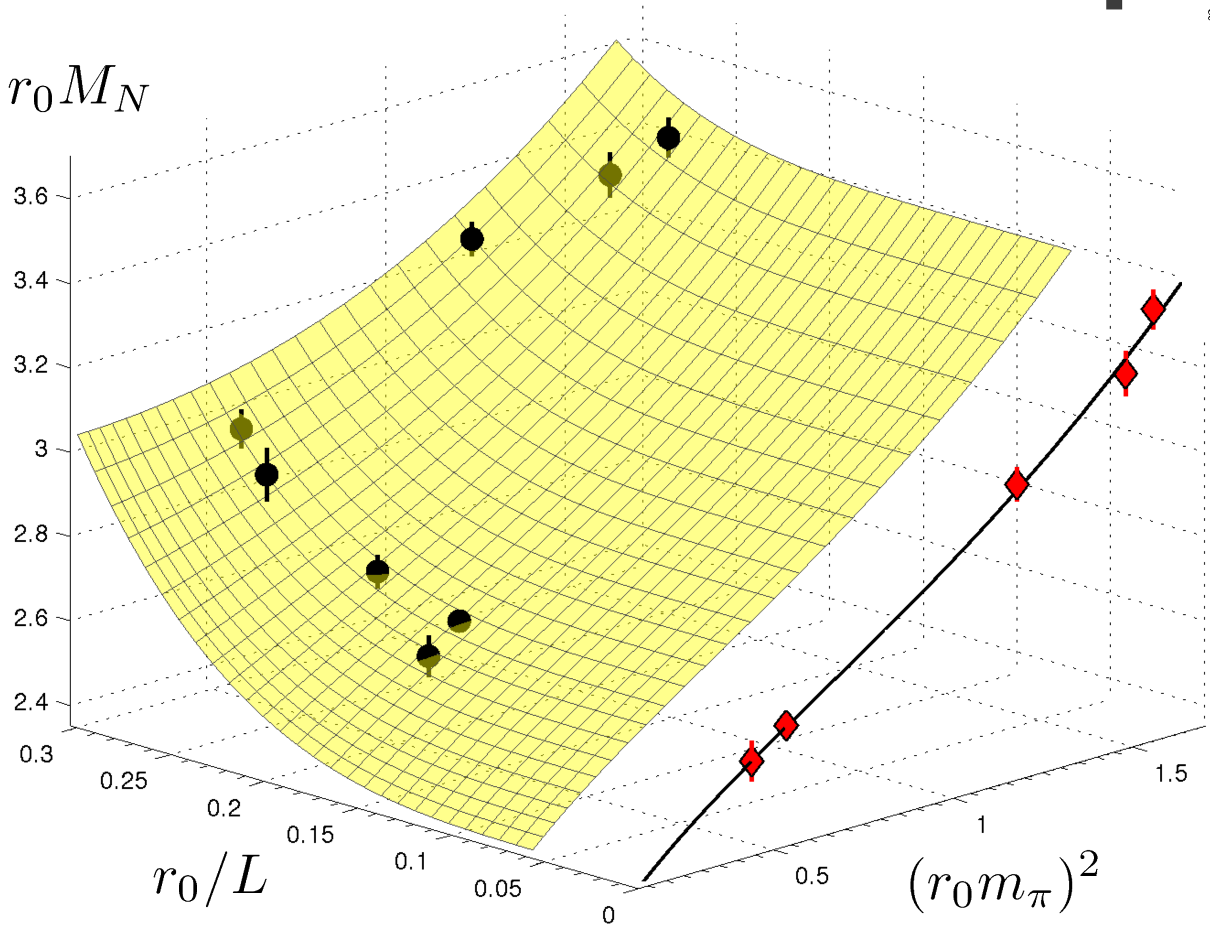
M_0, c_1, c_2, c_3

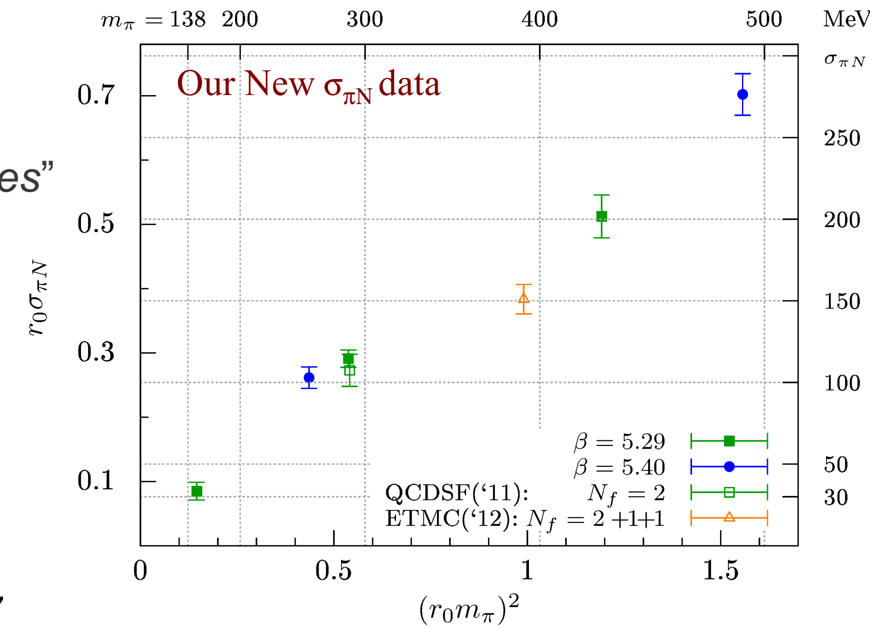
also enter volume correction

Illustration of fit

“derivatives”

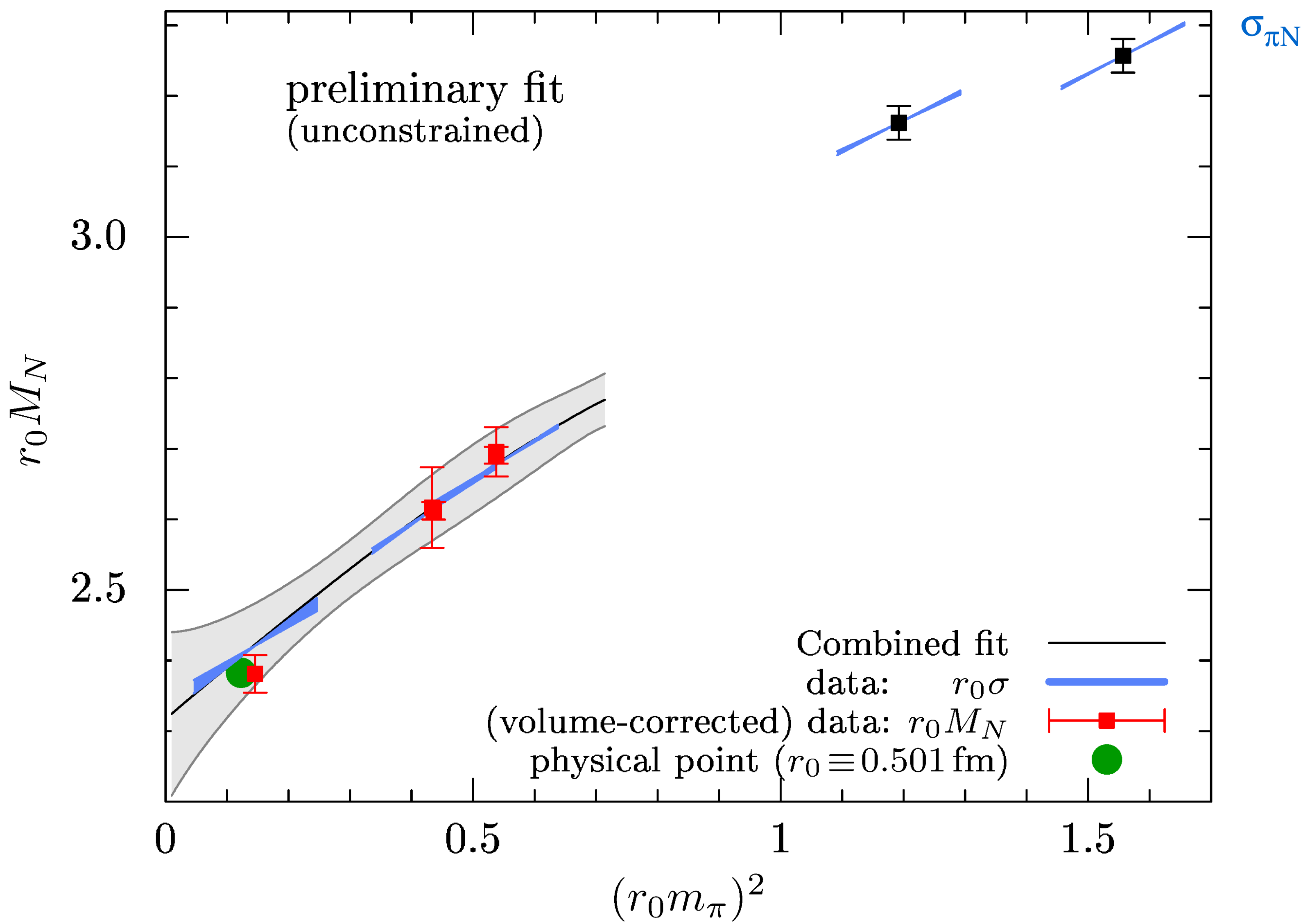
“values” +





Picture from arXiv: 1206.7034

First look at (new) combined fit



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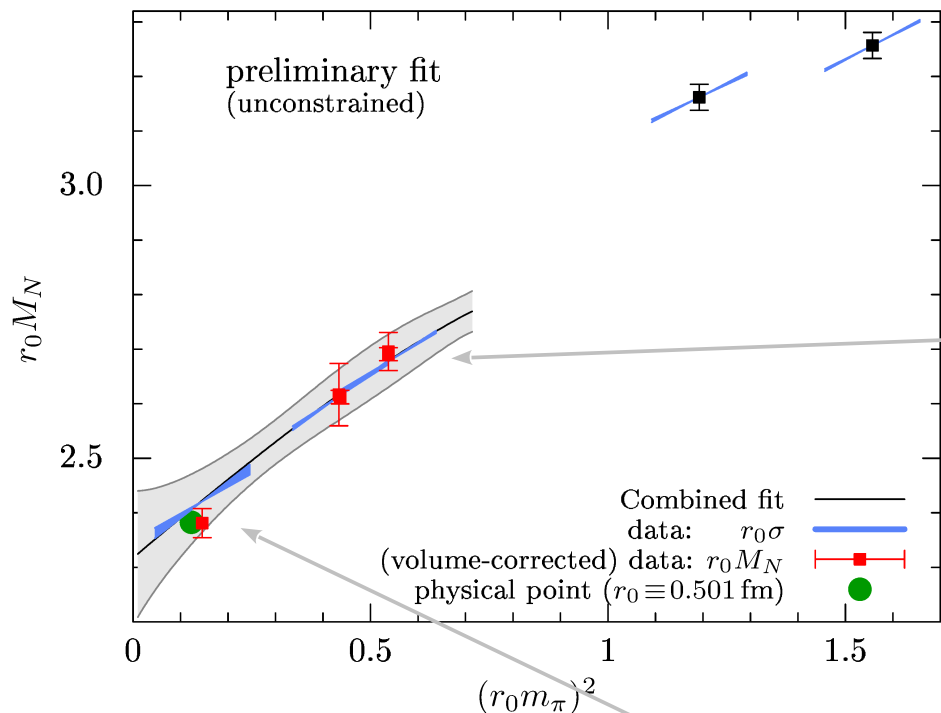

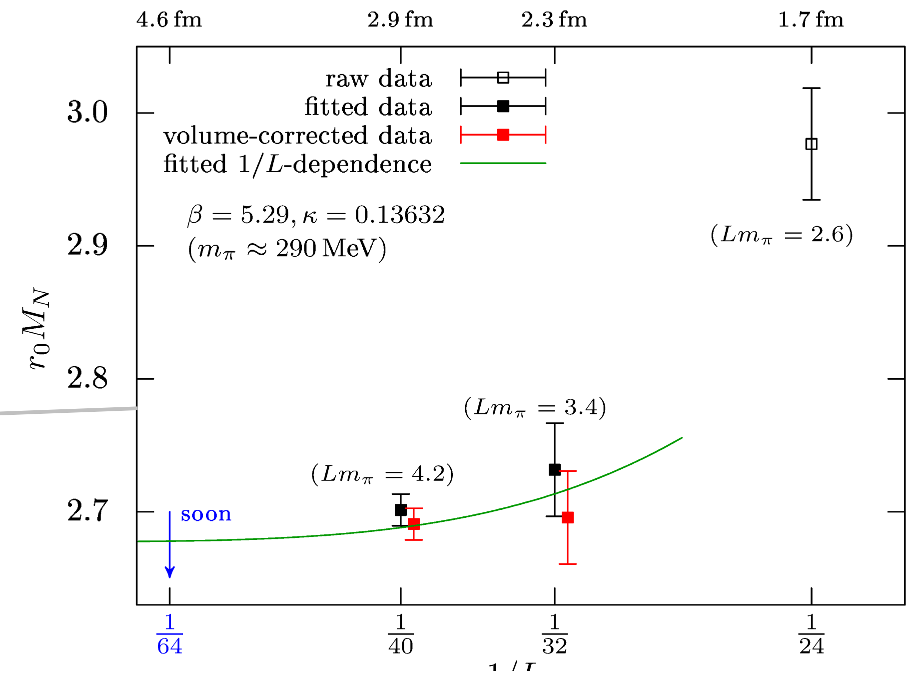

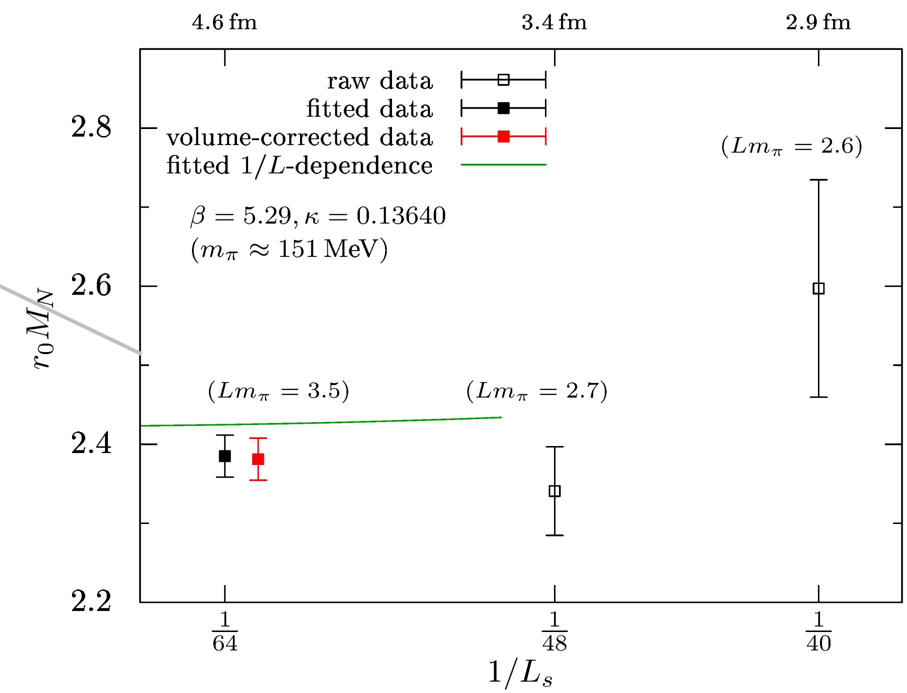

At each point

- mass value and slope

Some points

- data for different volumes
- fitted volume dependence agrees with that of data

Summary

Nucleon form factors

- performed reanalysis of the QCDSF + Regensburg $N_f = 2$ ensembles
- added a new ensemble at m_π =150 MeV (64^4)
- used improved smearing to reduce excited-state contaminations
- Q^2-dependence changes (form factor dependent)
- have: 3-point functions with no, one and two derivatives
- full analysis will follow

Pion-Nucleon sigma term

- extended our calculation of $\sigma_{\pi N}$ to m_π =150...490 MeV
- will allow for improved estimate of physical $\sigma_{\pi N}$
- almost no extrapolation needed

Also

- helps much also for our nucleon mass fits ($\rightarrow$ estimating r_0...)

Warning: All results are yet preliminary and may change.

Thank you for your attention

Isoscalar GFFs

Some example plots for connected part