Effective θ term in QCD induced by CP-odd electromagnetic background fields.

Marco Mariti

University of Pisa, Italy

Lattice 2013
31^{st} International Symposium on Lattice Field Theory
Mainz, Germany

based on M. D'Elia, M. Mariti and F. Negro, Phys. Rev. Lett. **110**, 082002 (2013)
and on a work in progress with C.Bonati and G.Cossu

In QCD it is possible to introduce the so called θ-term:

$$\mathcal{L} = \mathcal{L}_{QCD} - i\theta \frac{g^2}{64\pi^2} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} \quad \text{where} \quad \tilde{G}^a_{\mu\nu}(x) = \epsilon_{\mu\nu\rho\sigma} G^{a\rho\sigma}(x)$$

$\rightarrow P$ and CP symmetry violation. However, no experimental evidence of such violations in strong interactions!

From the experimental upper limit on neutron electric dipole moment $|d_n| < 2.9 \times 10^{-26} e$ cm [1], θ value has an upper bound : $|\theta| \lesssim 10^{-9}$.
Nevertheless, with the introduction of θ dependence in QCD one can address the $U(1)_A$ problem, explaining the mass of the η' meson.

In QED the CP-violating analog term is: $F_{\mu\nu}\tilde{F}_{\mu\nu} \propto \mathbf{E} \cdot \mathbf{B}$.
However in QED it gives no physical effects, while in QCD this term brings contributions due to non-perturbative gauge fields configurations
$\rightarrow$ **Instantons**.

[1] C.A. Baker *et al.*, Phys. Rev. Lett. **97**, 131801 (2006)

Introduction

Quarks carry simultaneously electric and color charge. It is natural to ask if external electromagnetic fields can influence, via quark loop effects, the gluonic configurations.

Question:

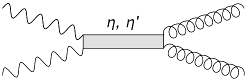
- Is it possible to induce a CP-breaking term in the strong sector, i.e. a θ-*like* term, starting from a CP-violating term in the e.m. sector?

Why we ask it:

- From this effect we can obtain information on the effective **QED-QCD interactions**, in particular on the pseudoscalar channel.
- Maybe relevant for the **Chiral Magnetic Effect** which is basically the inverse (local) CP-breaking induction, i.e. strong $\rightarrow$ e.m.

Gluons can interact with external e.m. fields via effective interactions. One can write the pseudoscalar QED-QCD interaction[1]:

$$\mathcal{L}_{eff} = \kappa_0 \alpha \alpha_S (\mathbf{E} \cdot \mathbf{B})(\mathbf{E}^a \cdot \mathbf{B}^a)$$



where the interaction can be seen as mediated by virtual mesons η and η'. One can write the following effective Lagrangean term:

$$\mathcal{L}_{eff} = \theta_{eff} \frac{g^2}{64\pi^2} G^a_{\mu\nu}(x) \tilde{G}^a_{\mu\nu}(x)$$

where θ_{eff} is an odd function of $\mathbf{E} \cdot \mathbf{B}$ given by:

$$\theta_{eff} \simeq \chi_{CP} e^2 \mathbf{E} \cdot \mathbf{B} + O((\mathbf{E} \cdot \mathbf{B})^3)$$

$\rightarrow \chi_{CP}$ can be interpreted as the *susceptibility of QCD vacuum to* CP*-breaking electromagnetic fields.*

The purpose of our work is to furnish a first determination of χ_{CP} based on lattice QCD techniques, performing Monte Carlo simulations in presence of uniform e.m. background fields, such that $\mathbf{E} \cdot \mathbf{B} \neq 0$.

[1] M. Asakawa, A. Majumder, B. Muller, Phys. Rev. **C** 81, (2010) 064912

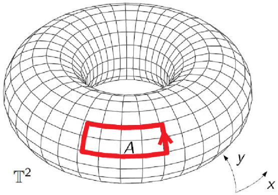
External e.m. fields on the Lattice

We can introduce abelian gauge fields modifying the QCD covariant derivative, adding proper $U(1)$ phases:

$$D_\mu \psi \to \frac{1}{2a}\Big(U_\mu(i) u_\mu(i) \psi(i+\hat{\mu}) - U_\mu^\dagger(i-\hat{\mu}) u_\mu^*(i-\hat{\mu}) \psi(i-\hat{\mu})\Big)$$

Because we use boundary conditions on the lattice, we deal with a compact manifold $\to$ the phase acquired along a closed path with an orthogonal homogeneous e.m. field is (e.g. $\mathbf{B} = B\hat{z}$):

$$\oint a_\mu dx_\mu = AB \quad \text{or} \quad \oint a_\mu dx_\mu = (A - l_x l_y) B$$



from which we get the quantization condition:

$$e^{iqBA} = e^{iqB(A - l_x l_y)} \to qB = \frac{2\pi b}{l_x l_y} = \frac{2\pi b}{a^2 L_x L_y} \;, \quad b \in \mathbb{Z}$$

For an e.m. field with a single non-vanishing component, $F_{\mu\nu} = F$, we used:

$$u_\nu(n) = e^{i a^2 q F \, n_\mu} \quad u_\mu(n)|_{n_\mu = L_\mu} = e^{-i \, a^2 q L_\mu F \, n_\nu} \quad \text{otherwise} \quad u_\rho(n) = 1$$

Simulation setup

We consider Lattice QCD with $N_f = 2$ using rooted staggered fermions:

$$Z(T,B) \equiv \int D[U] e^{-S_g} \det M^{\frac{1}{4}}[B,q_u] \det M^{\frac{1}{4}}[B,q_d]$$

$$M_{i,j} = am\delta_{i,j} + \frac{1}{2}\sum_{\nu=1}^{4} \eta_\nu(i) \left(u_\nu(i)\ U_\nu(i)\delta_{i,j-\hat{\nu}} - u^*_\nu(i-\hat{\nu})\ U^\dagger_\nu(i-\hat{\nu})\delta_{i,j+\hat{\nu}} \right)$$

and S_g is the standard Wilson action.
In Monte Carlo simulations one computes the mean value of an observable $\mathcal{O}$ using:

$$\langle \hat{\mathcal{O}} \rangle = \int D[U] P[U] \hat{\mathcal{O}}[U] \quad \text{where} \quad P[U] = \frac{e^{-S_g[U]} \text{det} M^{\frac{1}{4}}(q_u) \det M^{\frac{1}{4}}(q_d)}{Z}$$

This require $\mathbf{P}[\mathbf{U}]$ **real**! But if we take a real $\mathbf{E} = -\nabla A_t$ in Minkowski space, after performing the rotation to Euclidean space, A_μ acquires an imaginary component and $\text{det} M$ assumes complex value$\rightarrow$ sign problem in P[U]!
We use imaginary electric fields $\mathbf{E} = i\mathbf{E}_I$ in Minkowski space, then exploiting analytic continuation.

Measure of θ_{eff}

Since $\mathbf{E} = i\mathbf{E}_I$ we expect to induce an imaginary effective parameter: $\theta_{eff} = i\theta_{Ieff}$. This should introduce a factor $e^{\theta_{Ieff}Q}$ in the probability distribution of gluon fields $\rightarrow$ We expect a **shift of the topological charge distribution**.

Using $\quad Z(\theta_I) = e^{-VF(\theta_I)} \quad$ where $\quad F(\theta_I) = F(0) + \frac{\chi}{2}\theta_I^2 + O(\theta_I^4)$

the expectation value of the topological charge in presence of θ_{Ieff} will be:

$$\langle Q\rangle_{\theta_{Ieff}} = V\frac{d}{d\theta_{Ieff}}F(\theta_{Ieff}) \simeq V\chi\theta_{Ieff} + \mathcal{O}(\theta_{Ieff}^3)$$

where χ is the topological susceptibility:

$$\chi = \int d^4x \langle q(x)q(0)\rangle_{\theta_{Ieff}=0} = \frac{\langle Q^2\rangle_{\theta_{Ieff}=0}}{V}$$

So we can determine θ_{Ieff} produced by a given e.m. background using:

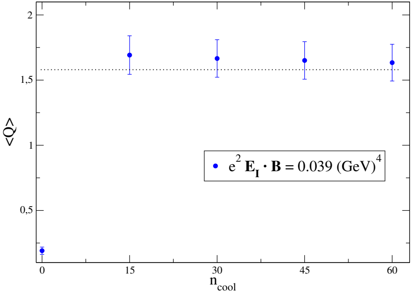
$$\theta_{Ieff} \simeq \frac{\langle Q\rangle(\mathbf{E}_I, \mathbf{B})}{\langle Q^2\rangle_{\theta_{Ieff}=0}} + O((\mathbf{E}_I \cdot \mathbf{B})^3)$$

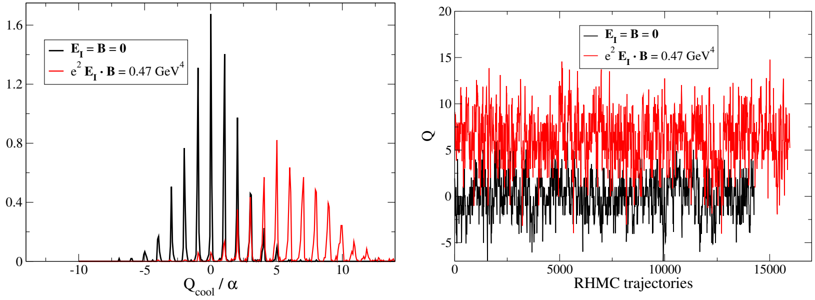
Measure of θ_{eff}

We determined Q using the **field theoretical method**, defining:

$$q_L(x) = \frac{-1}{2^9 \pi^2} \sum_{\mu\nu\rho\sigma=\pm 1}^{\pm 4} \epsilon_{\mu\nu\rho\sigma} \mathrm{Tr}\left(\Pi_{\mu\nu}(x)\Pi_{\rho\sigma}(x)\right)$$

- **Cooling** procedure and approximation to the closer integer.
- Verified the stability of the procedure over n_{cool}.
- $n_{cool} = 30, 60$ used.
- Check the results for a subset of configurations with **overlap** fermions.



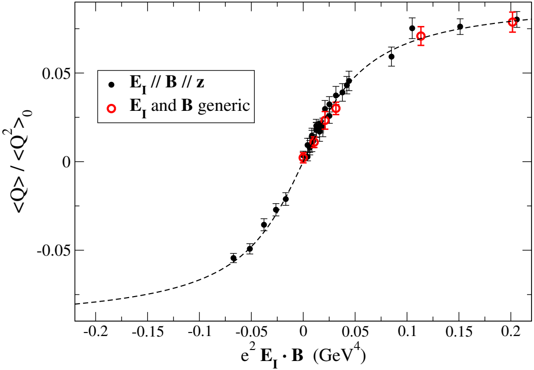


We observed, as expected, a shift in the topological charge distribution due to the e.m. term $\mathbf{E}_I \cdot \mathbf{B}$.

In figures we show the shift for $e^2\mathbf{E}_I \cdot \mathbf{B} = 0.47$ GeV^4, both from the topological charge distribution and from the Monte Carlo histories.

To investigate the dependence of $\langle Q\rangle(\mathbf{E}_I, \mathbf{B})$, we plot θ_{Ieff} for non-parallel $\mathbf{E}_I$, $\mathbf{B}$. For example we plotted:

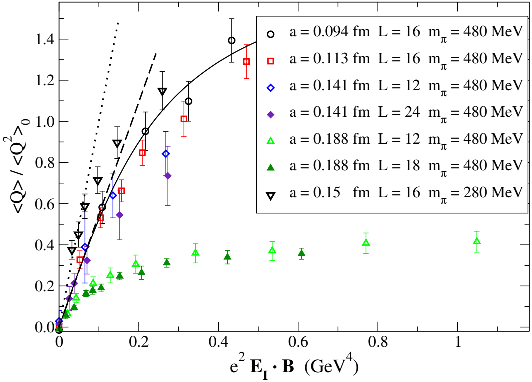
$\mathbf{n_E}$	$\mathbf{n_B}$
2,-8,-2	8,1,4
1,2,3	3,2,1
1,3,4	2,2,3
5,5,5	1,2,3
6,6,6	6,6,6
8,8,8	8,8,8



Within errors, θ_{Ieff} is a function of $\mathbf{E}_I \cdot \mathbf{B}$ alone: odd, linear for small e.m. fields, $atan$-like for larger fields.

Results

We show θ_{Ieff} for various volumes and lattice spacings, with $m_\pi = 480$ MeV and $m_\pi = 280$ MeV.



- θ_{Ieff} does not depend on the volume V;
- χ_{CP} is independent of the lattice spacing, for $a \lesssim 0.141$ fm;
- χ_{CP} appears to increase for decreasing value of m_π.

$Q_{SU(3)} + Q_{U(1)}$

The induced θ_{Ieff} violate explicitly CP, leading to gluonic configurations with $\langle Q \rangle \neq 0$. But how that happens?

Naively, these configurations should have an higher value of S_g $\rightarrow$ Suppressed in the path integral! Moreover, in the chiral limit the generated zero modes should kill these configurations in the path integral.

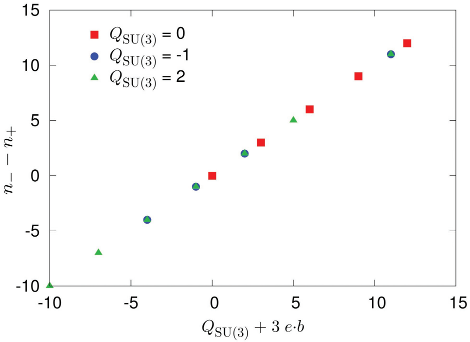
The reason for this behavior reside in the modification of the axial anomaly equation with the inclusion of the $U(1)$ term:

$$n_- - n_+ = Q_{tot} \equiv Q_{SU(3)} + N_C \; e \cdot b$$

So the number of zero modes depends both on the presence of $Q_{SU(3)}$ and on the abelian part.
We verified explicitly this relation using overlap fermions to count the zero modes in controlled configurations.

$Q_{SU(3)} + Q_{U(1)}$

- We used configurations with known topological content and given external fields.
- We measured zero modes with overlap fermions



We explicitly verified the relation

$$n_- - n_+ = Q_{tot} \equiv Q_{SU(3)} + N_c \; e \cdot b$$

with $N_C = 3$, for various external e.m. fields and three different values of the topological charge $Q_{SU(3)}$.
So, at least in the chiral limit, relevant gluonic configurations must have nontrivial $Q_{SU(3)}$ in reaction to the external e.m. fields.

Conclusions

We quote $\chi_{CP} = (5.47 \pm 0.78)$ GeV^{-4} for $m_\pi = 480$ MeV. Preliminary results suggest an increase of χ_{CP} for smaller mass, with $\chi_{CP} \simeq 10$ GeV^{-4} for $m_\pi = 280$ MeV.

This value can be compared with the estimates for the effective pseudoscalar QED-QCD interaction[1], from which $\chi_{CP} \simeq 3$ GeV^{-4}, which is smaller, but of the same order.

- We simulate $N_f = 2$.
- No dynamical QED.

Future developments:

- Going to physical masses, with $N_f = 2 + 1$.
- Extend the analysis to finite T, in particular around T_c, where a smaller χ_{CP} is expected[1].

[1] M. Asakawa, A. Majumder and B. Müller, Phys. Rev. C **81**, 064912 (2010).

Tank you for the attention.

Overlap check

We checked our topological charge measurement with staggered fermions for a subset of configurations:

- $a = 0.094$ fm, $E_I \cdot B = 0.22$ GeV^4
- Set of 30 decorrelated configurations

We obtained $\frac{\langle Q_{ov} \rangle}{\langle Q_{15} \rangle} = 0.81(8)$

- Q_{ov}: $n_- - n+$
- Q_{15}: topological charge evaluated with $q_L(x)$ after 15 cooling sweeps.

External fields effects on a

We verified that the lattice spacing is not modified by the introduction of external fields:

We measured the Sommer parameter for two set of configurations :

- 24^4 lattice, $a = 0.094$ fm, with and without e.m. fields.
- $E_I \cdot B = 0 \rightarrow \frac{r_0}{a} = 5.36(10)$
- $E_I \cdot B = 0.12$ GeV^4 $\rightarrow \frac{r_0}{a} = 5.07(12)$