

# Composite flavor-singlet scalar in twelve-flavor QCD

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Ref. [arXiv:1305.6006](https://arxiv.org/abs/1305.6006)

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- Calculation of flavor-singlet scalar
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# Introduction

Discovery of “Higgs” particle @ LHC  
 $m_H \sim 126 \text{ GeV}$

Still we have lots of things to understand, such as

- Property of “Higgs” particle  
elementary
- Mechanism of electroweak symmetry breaking  
 $\langle H \rangle \neq 0$
- Gauge hierarchy problem  
fine tuning of  $m_H$

## Standard Model

Beyond Standard Model: SUSY, Little Higgs, Technicolor, ...

# Introduction

Discovery of “Higgs” particle @ LHC  
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- Property of “Higgs” particle

elementary

composite

- Mechanism of electroweak symmetry breaking

$\langle H \rangle \neq 0$

VEV from dynamics

- Gauge hierarchy problem

fine tuning of  $m_H$

no fine tuning

Standard Model

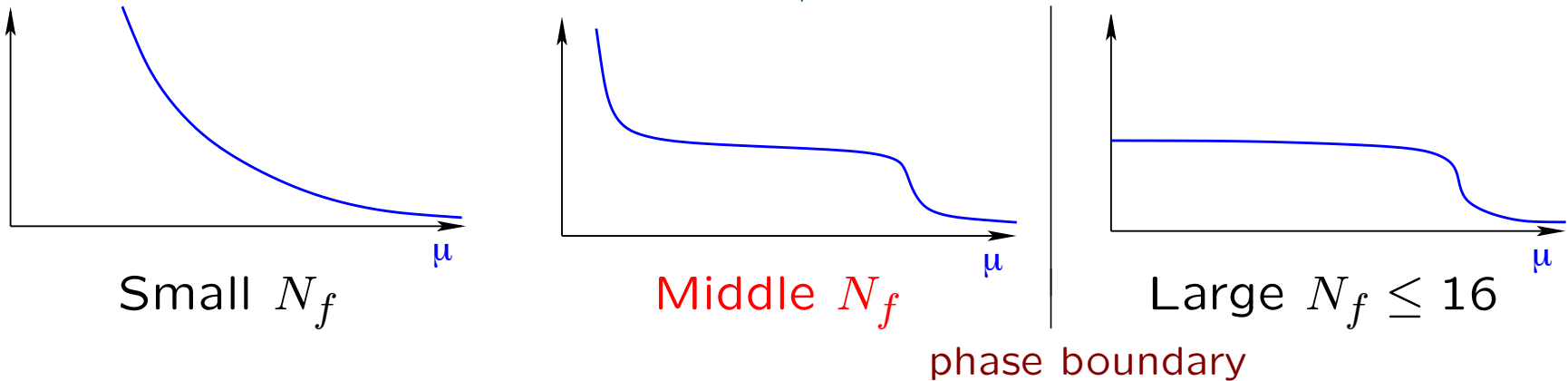
Technicolor: strongly coupled theory

Beyond Standard Model: SUSY, Little Higgs, Technicolor, ...

# Walking technicolor

$N_f$  massless fermions +  $SU(N_{TC})$  gauge at  $\mu_{TC} = O(1)$  TeV

- Spontaneous chiral symmetry breaking
- Slow running (walking) coupling in wide scale range
- Large anomalous mass dimension  $\gamma^* \sim 1$  in walking region



- Composite, light scalar state

$\approx$  Higgs  $\rightarrow$  explain  $M_{\text{Higgs}}/v_{\text{EW}} \sim 0.5$

# Walking technicolor

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Question: Such a theory really exists?

Nonperturbative calculation is important.

→ numerical calculation with lattice gauge theory

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→ numerical calculation with lattice gauge theory

Lattice studies for (approximate) conformal gauge theory:

'92 Iwasaki *et al.*, '92 Brown *et al.*, '97 Damgaard *et al.*,  
'08 Appelquist *et al.*, and various other works



# Purpose of our project

Search for candidate of walking technicolor

Systematic investigation of  $N_f$  dependence

SU(3) gauge theory with  $N_f = 0, 4, 8, 12, 16$  fermions

Common setup for all  $N_f$ : Improved staggered action (HISQ/Tree)

Cheaper calculation cost + small lattice systematic error

'12 Bazakov *et al.*

## Recent works of our group

- Basic physical quantities:  $m_\pi, F_\pi, m_\rho, \langle \bar{\psi}\psi \rangle$   
 $N_f = 12$ : PRD86(2012)054506  
 $N_f = 8$ : PRD87(2013)094511 [Kei-ichi Nagai: 1F]  
 $N_f = 8$  may be candidate of walking theory
- Flavor-singlet scalar in (approximate) conformal theory  
 $N_f = 12$ : arXiv:1305.6006; glueball [Enrico Rinaldi: 1F]  
 $N_f = 8$  [Hiroshi Ohki: 1F]
- $N_f = 8$  S parameter [Yasumichi Aoki: 5F]

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# Purpose of this talk

## Search for candidate of walking technicolor

Why  $N_f = 12$

- Investigated by many groups

'08,'09 Appelquist *et al.*, '10 Deuzeman *et al.*, '10,'12 Hasenfratz,  
'11 Fodor *et al.*, '11 Appelquist *et al.*, '11 DeGrand, '11 Ogawa *et al.*,  
'12 Lin *et al.*, '12 Itou, '12 Jin and Mawhinney, and ...

In our work PRD86(2012)054506

consistent behavior with conformal phase

- Flavor-singlet scalar in conformal theory is not understood well.

1. SU(2) Adjoint  $N_f = 2$  glueball: '09 Del Debbio *et al.*
2. SU(3)  $N_f = 12$  meson: '12 Jin and Mawhinney

## Purpose of this work

Understand properties of flavor-singlet scalar in  $N_f = 12$

regarded as pilot study of more interesting  $N_f = 8$  theory

# Difficulty of flavor-singlet scalar meson

- Flavor non-singlet scalar meson  $S_{NS}(t) = \sum_{\vec{x}} \bar{\psi}_a(\vec{x}, t) \psi_b(\vec{x}, t)$  ( $a \neq b$ )

$$\langle 0 | S_{NS}(t) S_{NS}^\dagger(0) | 0 \rangle = \left\langle \text{Diagram} \right\rangle = -C(t)$$

c.f.  $m_\pi, F_\pi$  from non-singlet pseudoscalar

$$O(10) \text{ configurations} \times O(1) D^{-1}[U](x, y)$$

- Flavor-singlet scalar meson  $S(t) = \sum_{\vec{x}} \bar{\psi}_a(\vec{x}, t) \psi_a(\vec{x}, t)$

$$\langle 0 | S(t) S^\dagger(0) | 0 \rangle = -C(t) + 3D(t) \text{ (disconnected)}$$

$$D(t) = \left\langle \text{Diagram 1} \quad \text{Diagram 2} \right\rangle - \left\langle \text{Diagram 3} \right\rangle^2$$

Much harder but essential for flavor-singlet

$$O(10000) \text{ configurations} \times O(100) D^{-1}[U](x, x)$$

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Much harder but essential for flavor-singlet

$$O(10000) \text{ configurations} \times O(10) D^{-1}[U](x, x)$$

using noise reduction method

'97 Venkataraman and Kilcup

# Flavor-singlet scalar in $N_f = 12$ QCD

arXiv:1305.6006

## Simulation parameters

- $\beta = 4$  HISQ/Tree action

Consistent with conformal phase

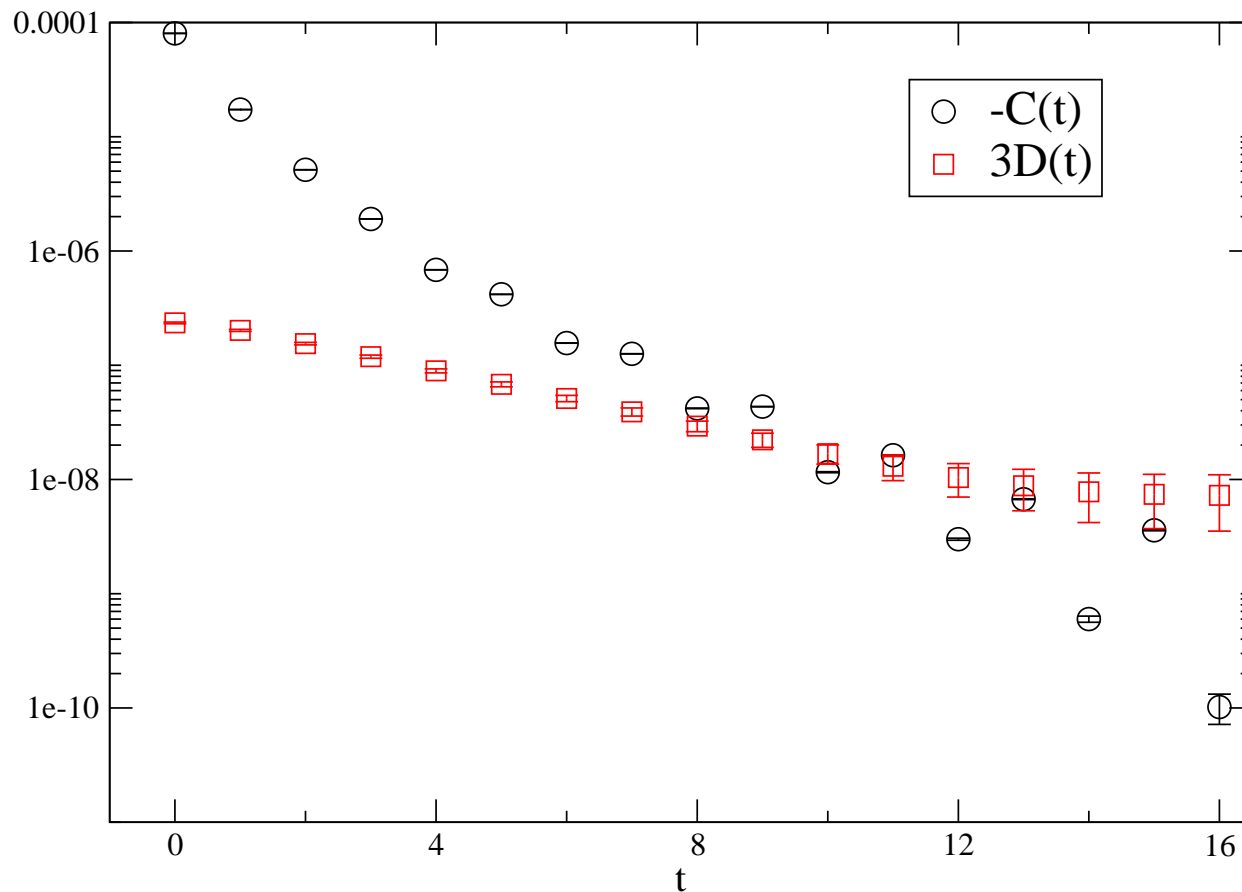
LatKMI; PRD86(2012)054506

- Huge number of configurations  
measuring every 2 tarj.
- Four  $m_f$ s on more than two volumes
- Noise reduction method with  $N_r = 64$
- Local meson operator of  $(1 \otimes 1)$

$L, T$	$m_f$	confs
24,32	0.05	11000
	0.06	14000
	0.08	15000
	0.10	9000
30,40	0.05	10000
	0.06	15000
	0.08	15000
	0.10	4000
36,48	0.05	5000
	0.06	6000

Machines:  $\varphi$  at KMI, CX400 at Kyushu Univ.

# Correlators in $N_f = 12$ ( $m_f = 0.06, L = 24$ with $N_{\text{conf}} = 14000$ )

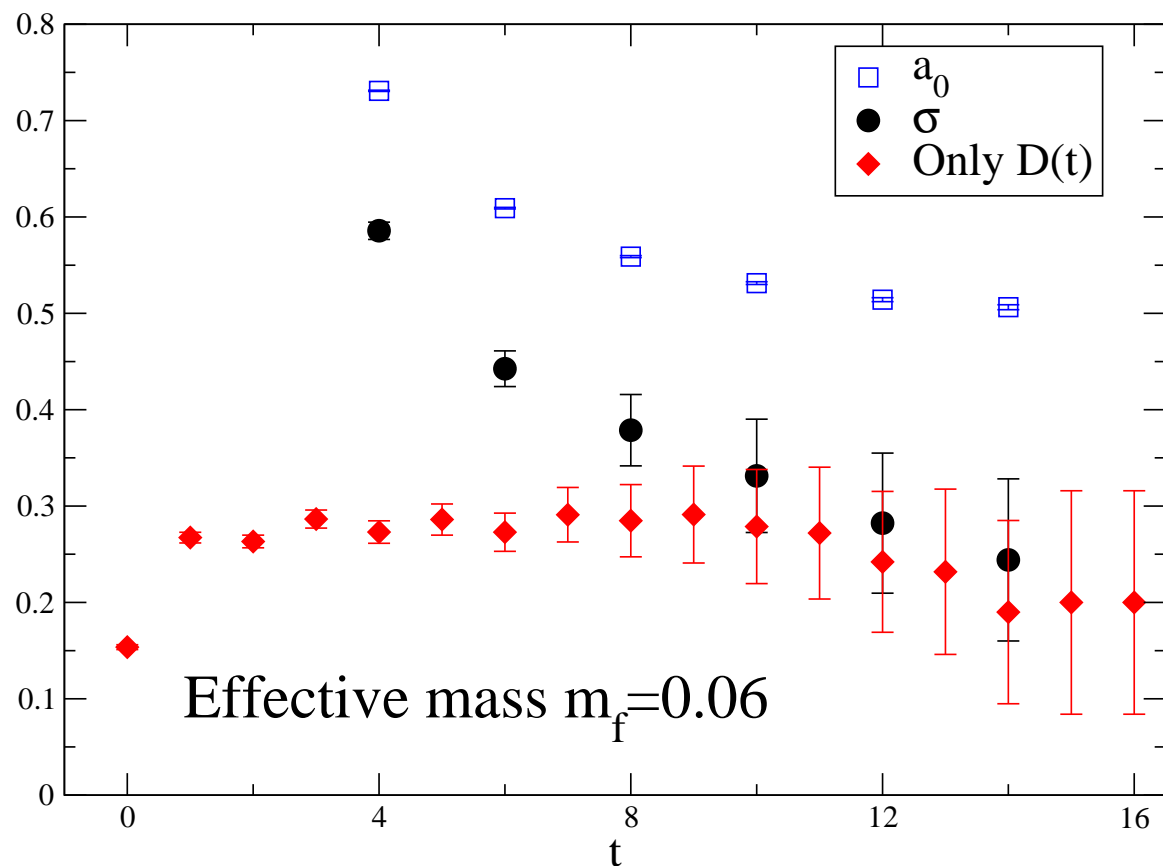


$-C(t)$  oscillates, but  $D(t)$  does not

cancellation: species-singlet and non-singlet  $\pi_{5C}$  in  $D(t)$

thanks to small taste symmetry breaking; PRD86(2012)054506

# Effective mass in $N_f = 12$ ( $m_f = 0.06, L = 24$ with $N_{\text{conf}} = 14000$ )



Non-singlet scalar

$a_0: -C_+(t)$

Singlet scalar

$\sigma: 3D_+(t) - C_+(t)$

$\sigma: D(t)$  i.e.  $m_\sigma < m_{a_0}$

Consistent  $m_\sigma$

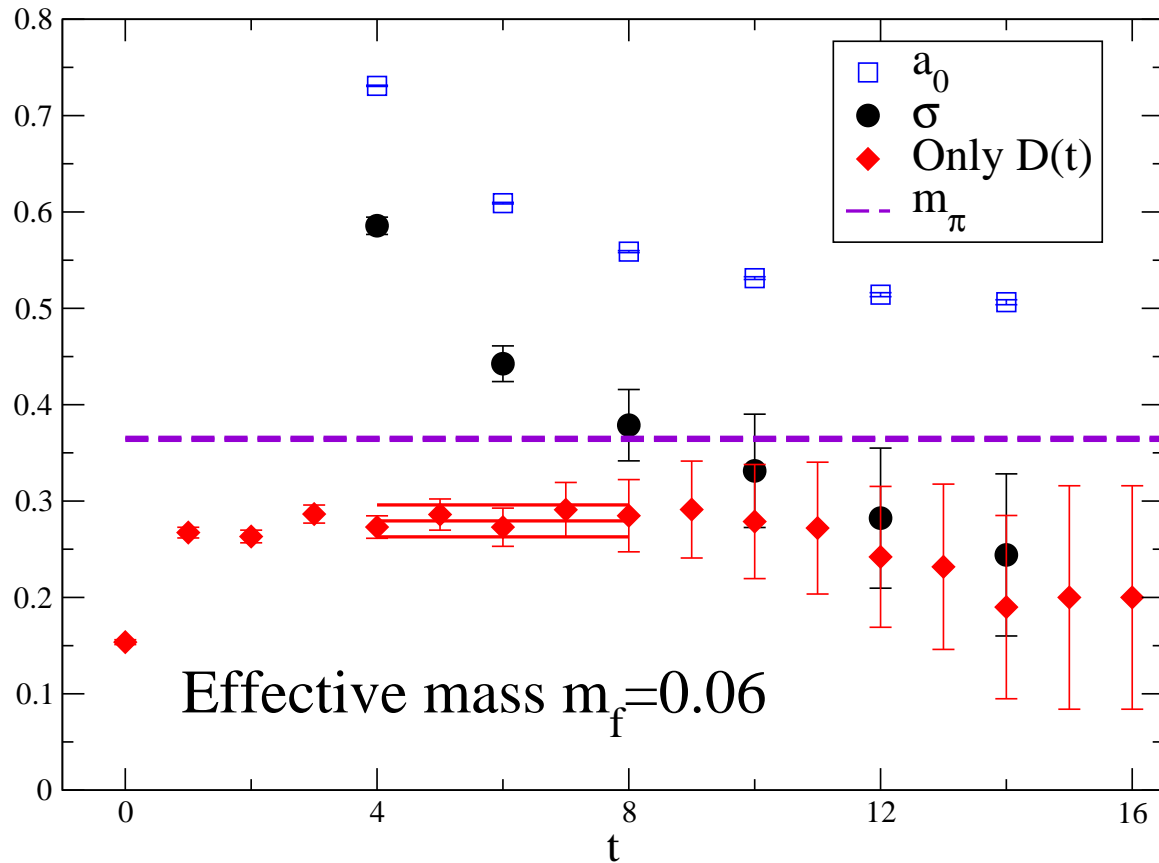
with smaller error

$$X_+(t) = 2X(t) + X(t+1) + X(t-1)$$

Good signal of  $m_\sigma$  from  $D(t)$



# Effective mass in $N_f = 12$ ( $m_f = 0.06, L = 24$ with $N_{\text{conf}} = 14000$ )



Non-singlet scalar

$a_0: -C_+(t)$

Singlet scalar

$\sigma: 3D_+(t) - C_+(t)$

$\sigma: D(t)$  i.e.  $m_\sigma < m_{a_0}$

Consistent  $m_\sigma$

with smaller error

$m_\sigma < m_\pi$  at  $m_f = 0.06$

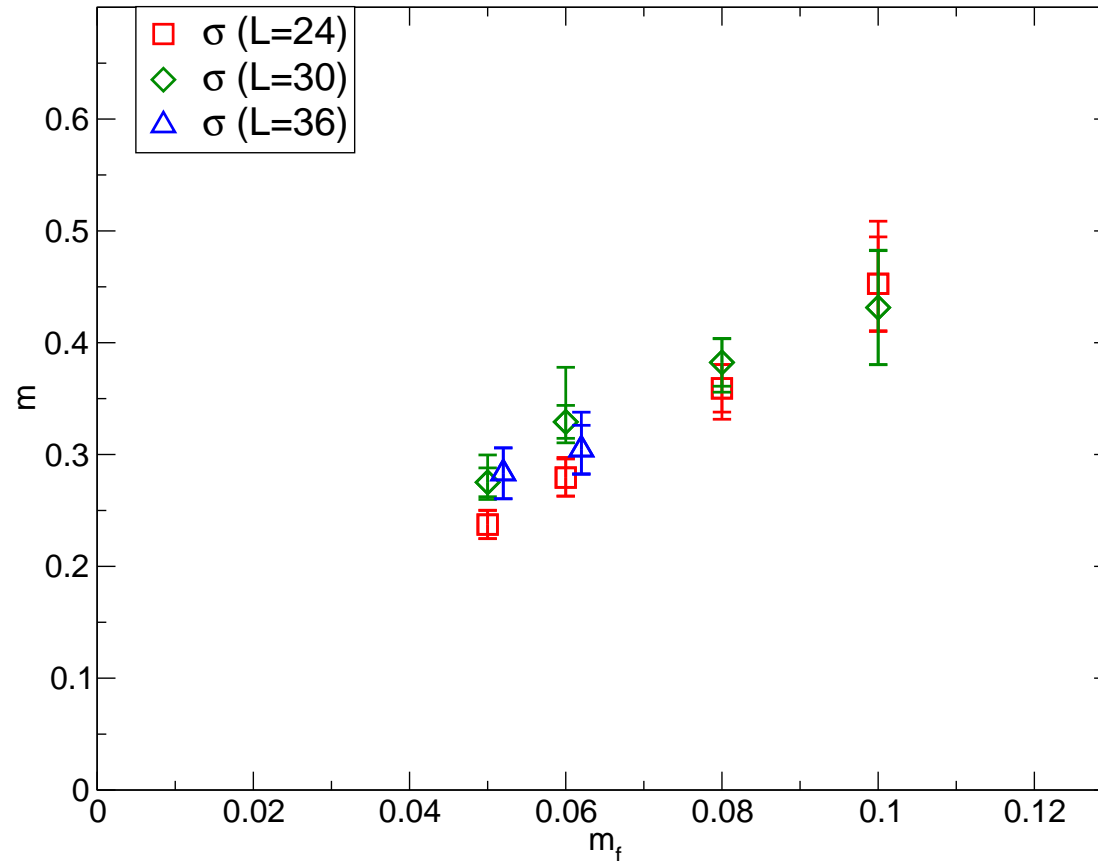
$$X_+(t) = 2X(t) + X(t+1) + X(t-1)$$

Good signal of  $m_\sigma$  from  $D(t)$

# $m_f$ dependence of $m_\sigma$ in $N_f = 12$

arXiv:1305.6006

$m_\sigma$  from fit of  $D(t)$  with  $t = 4-8$



Reasonable signals with almost 10% statistical error

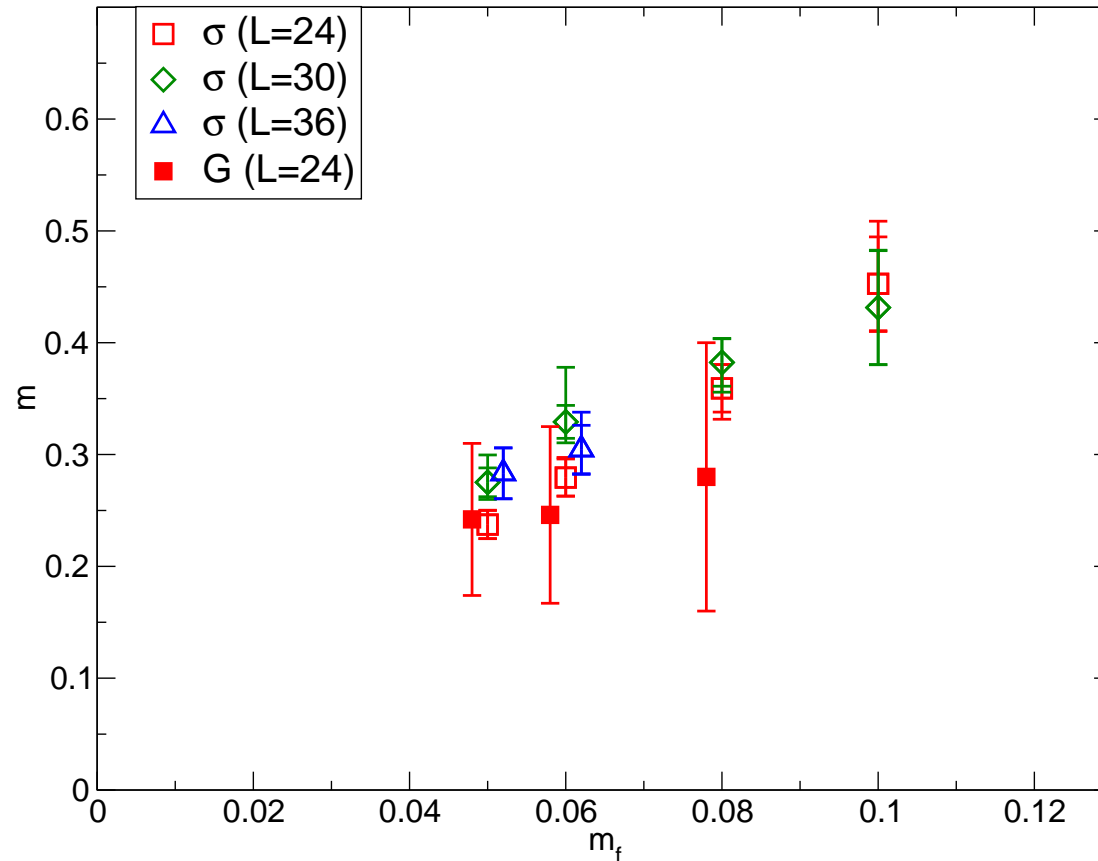
Systematic error from fit range dependence of  $D(t)$

Finite volume effect under control  $\leftarrow$  2 larger volumes agree

# $m_f$ dependence of $m_\sigma$ in $N_f = 12$

arXiv:1305.6006

$m_\sigma$  from fit of  $D(t)$  with  $t = 4-8$



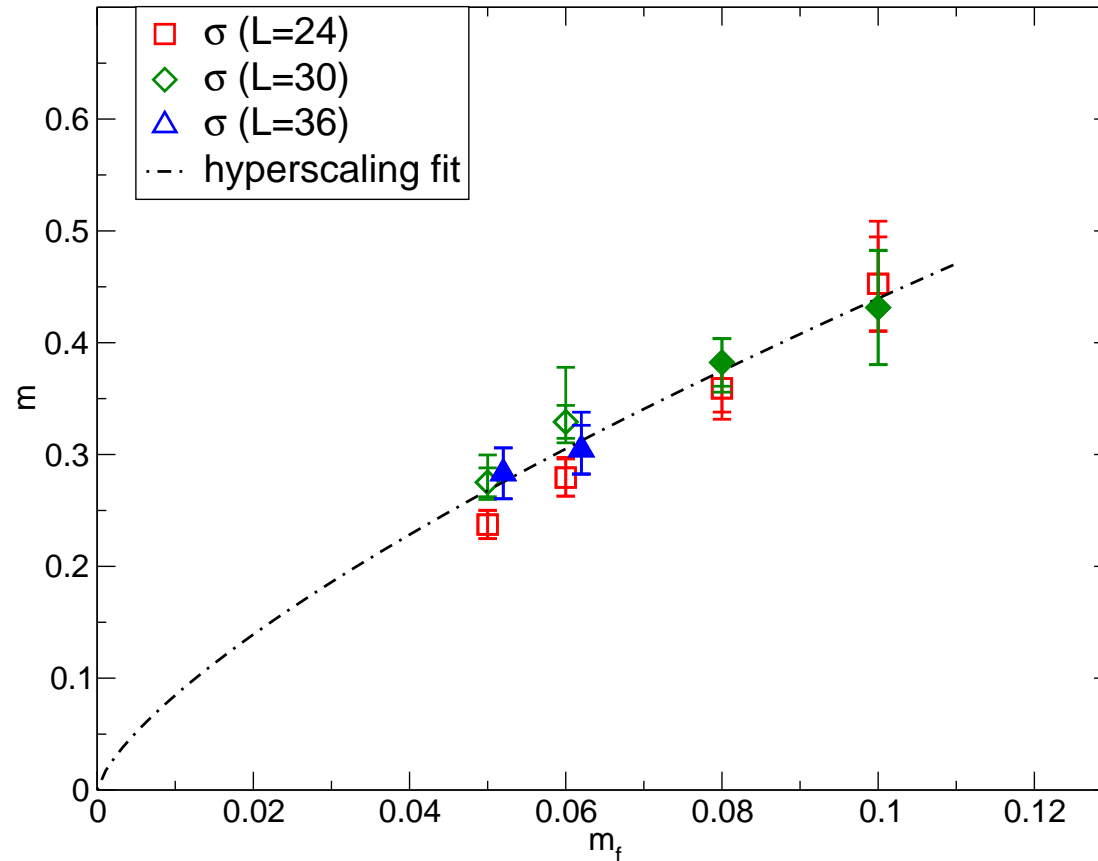
Consistent mass from glueball operator calculation

[Enrico Rinaldi: 1F]

# $m_f$ dependence of $m_\sigma$ in $N_f = 12$

arXiv:1305.6006

$m_\sigma$  from fit of  $D(t)$  with  $t = 4-8$



Hyperscaling test with fixed  $\gamma$  using target volume at each  $m_f$

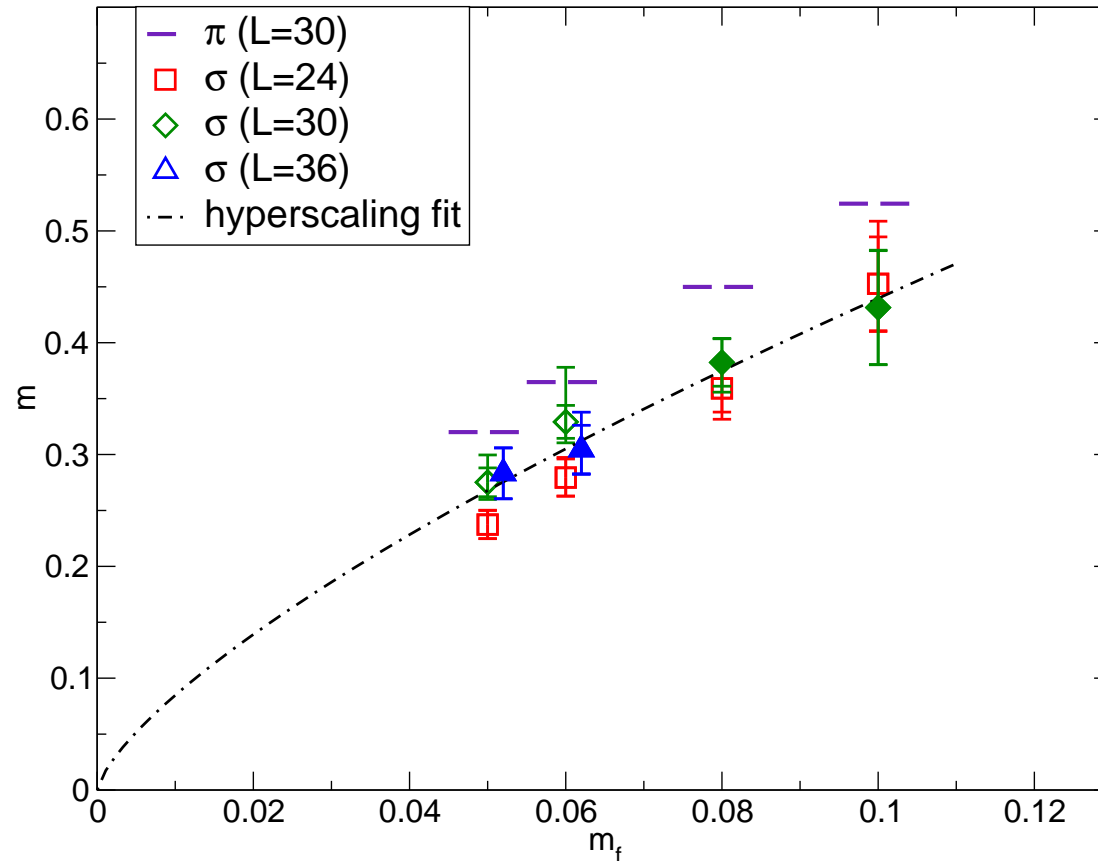
$$m_\sigma = C m_f^{1/(1+\gamma)} \text{ with } \gamma = 0.414 \text{ from hyperscaling of } m_\pi$$

Consistent hyperscaling as  $m_\pi$

# $m_f$ dependence of $m_\sigma$ in $N_f = 12$

arXiv:1305.6006

$m_\sigma$  from fit of  $D(t)$  with  $t = 4-8$



Lighter than  $\pi$  in all  $m_f$

Much different from usual QCD

Conformal symmetry may make  $\sigma$  light

# Summary

Flavor-singlet scalar is important in walking technicolor theory.

However, difficult due to huge noise in lattice simulation

⇒ Noise reduction method and Huge  $N_{\text{conf}} \mathcal{O}(10000)$

Results of  $N_f = 12$  QCD (consistent with conformal phase)

- Consistent behavior with hyperscaling
- $m_\sigma < m_\pi$ ; much different from small  $N_f$  QCD
- Conformal symmetry may make  $\sigma$  light

Encouraging results for light  $\sigma$  in walking theory

## Future perspectives

Candidate of walking theory:  $N_f = 8$  QCD [Kei-ichi Nagai: 1F]

Important to study flavor-singlet scalar, if  $m_\sigma \sim F_\pi$

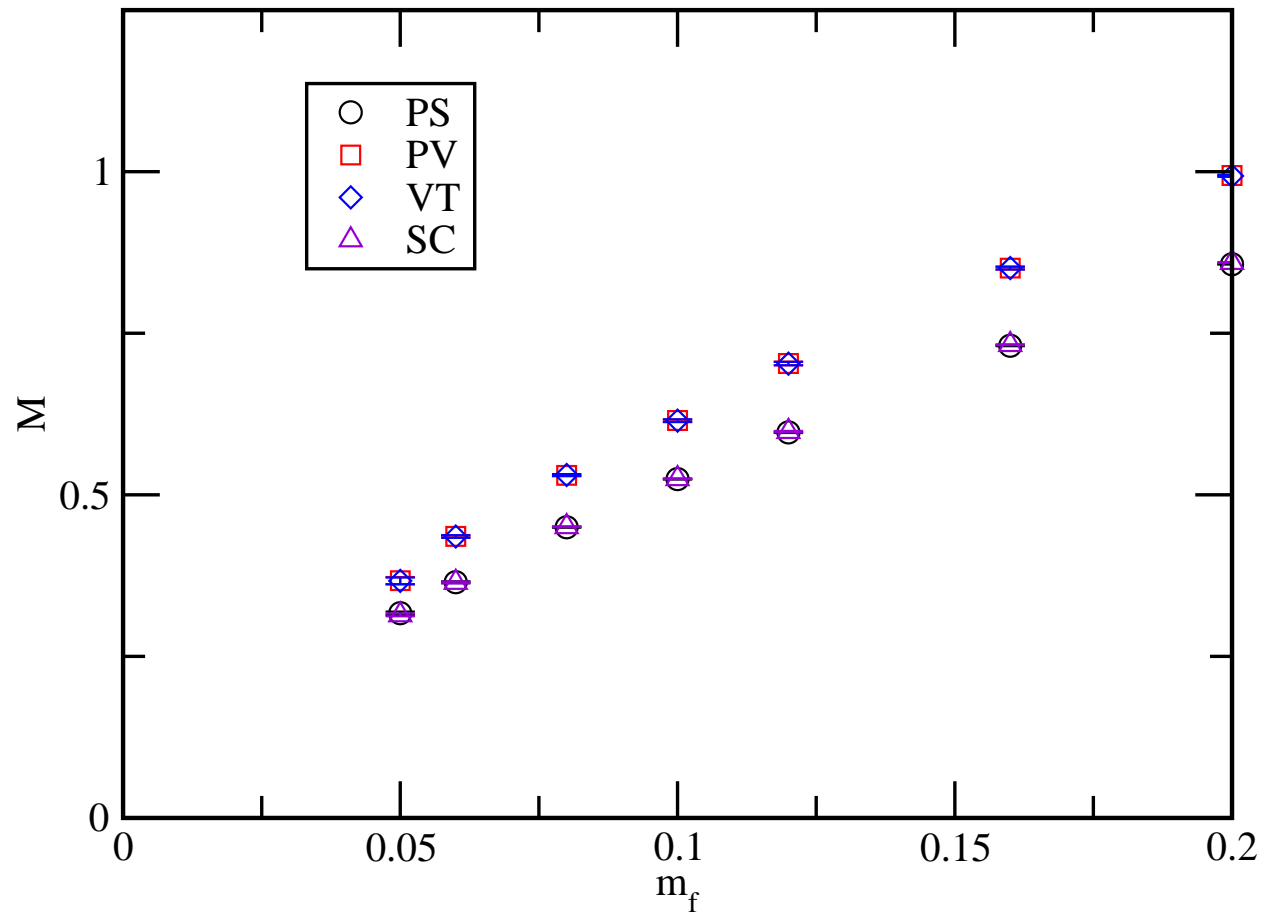
Preliminary result of  $N_f = 8$  QCD [Hiroshi Ohki: 1F]

Back up

# $N_f = 12$ taste symmetry breaking effect

LatKMI; PRD86(2012)054506

$0^-$ : PS, SC;  $1^-$ : PV, VT



Small taste symmetry breaking in meson masses



## States in $D(t)$

$$A_H(t) = A_H \exp(-M_H t)$$

Connected part

$$-C(t) = A_{a_0}(t) + (-1)^t A_{\pi_{SC}}(t)$$

Connected + disconnected

$$N_f D(t) - C(t) = A_\sigma(t) + (-1)^t A_{\pi_{\overline{SC}}}(t)$$

$$\xrightarrow{\text{taste symmetric limit}} \pi_{\overline{SC}} = \pi_{SC} = \pi_{PS}$$

$\pi_{\overline{SC}}$ : Species-singlet but taste-non-singlet  $0^-$

$\eta$  in PRD76:094504(2007)

disconnected part

$$N_f D(t) = A_\sigma(t) - A_{a_0}(t) + (-1)^t \left( A_{\pi_{\overline{SC}}}(t) - A_{\pi_{SC}}(t) \right)$$

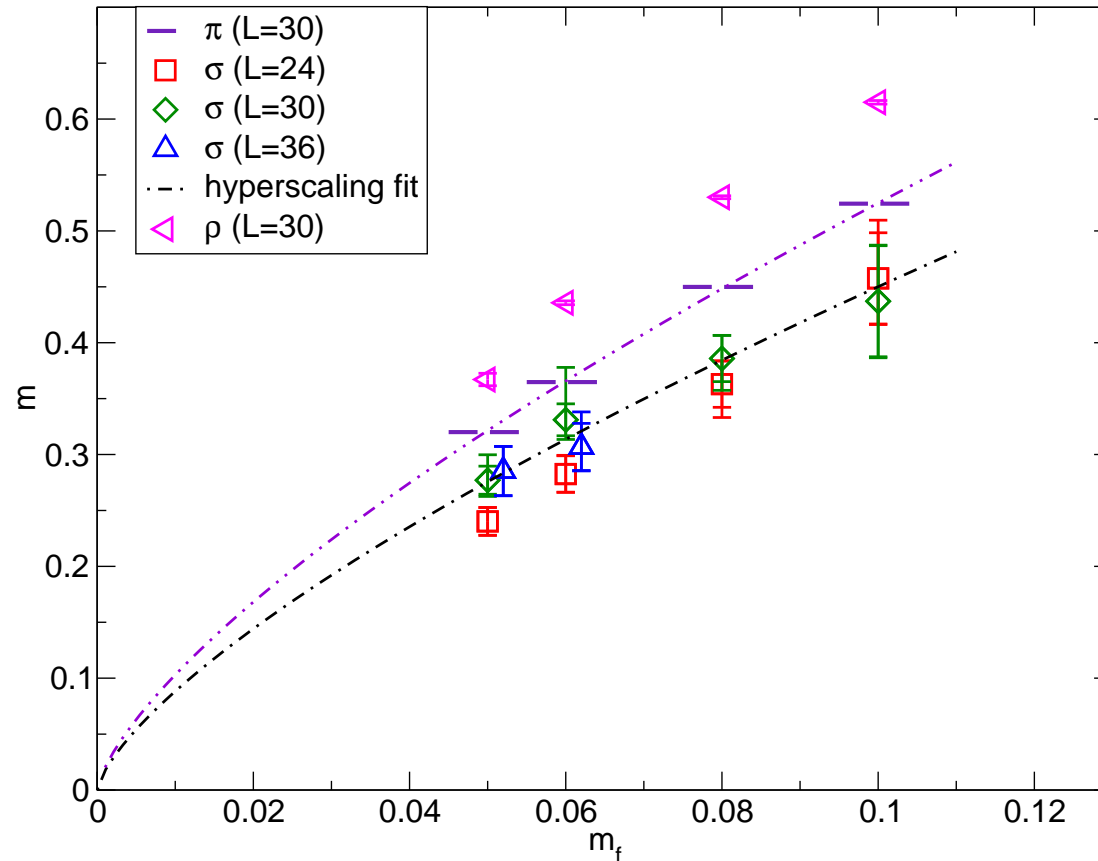
$$\xrightarrow{\text{taste symmetric limit}} A_\sigma(t) - A_{a_0}(t)$$

$\Rightarrow$  small oscillation in  $D(t)$  if good taste symmetry

# $m_f$ dependence of $m_\sigma$ in $N_f = 12$

arXiv:1305.6006

$m_\sigma$  from fit of  $D(t)$  with  $t = 4-8$



Lighter than  $\pi$  in all  $m_f$

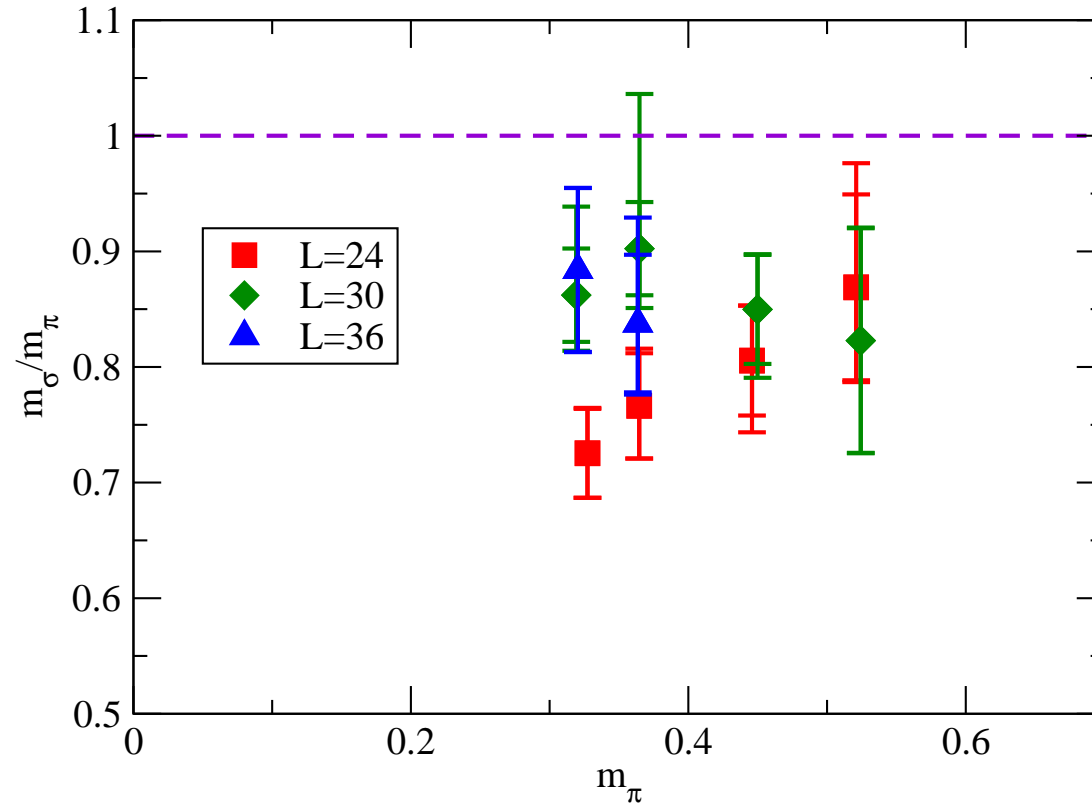
Much different from usual QCD

Conformal symmetry may make  $\sigma$  light

# $m_f$ dependence of $m_\sigma/m_\pi$

arXiv:1305.6006

$m_\sigma$  from fit of  $D(t)$  with  $t = 4-8$



# Discussion

## Why flavor-singlet scalar calculation is possible?

- Nice noise reduction method
- Huge  $N_{\text{conf}}$
- Small  $m_\sigma \rightarrow$  slow exponential damp of correlator
- Small taste symmetry breaking  $\leftarrow$  improved action, large  $N_f$ , etc.