Surface worm algorithm for Abelian gauge-Higgs systems at finite density

Y. Delgado, C. Gattringer, A. Schmidt

Karl-Franzens-Universität Graz

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Der Wissenschaftsfonds.

Study of Strongly Interacting Matter

HadronPhysics

ITN STRONGnet

UNI GRAZ

Gauge-Higgs systems at finite density

- At finite density: $e^{-S(\mu)}$ is complex for $\mu > 0$.

- The complex phase problem is **solved** using: Dual representation.

- We generalized the worm algorithm to update the new type of dual variables:

  - Z_3 and U(1) gauge-Higgs models coupled to one scalar field:
    C. Gattringer and A. Schmidt (PRD 2012).
    YD, C. Gattringer, A. Schmidt, Comput. Phys. Commun. (2013).

  - U(1) gauge-Higgs model with two flavors of opposite charge:
    YD, C. Gattringer, A. Schmidt, 1307.6120 [hep-lat].

The Z_3 gauge-Higgs model

- Conventional representation

$$S_G = -\frac{\beta}{2}\sum_x \sum_{\nu<\rho} \left[U_{x,\nu\rho} + U^*_{x,\nu\rho}\right] ,$$

$$S_H = -\kappa \sum_{x,\nu} \left[e^{\mu\delta_{\nu,4}} \phi_x^* \, U_{x,\nu} \, \phi_{x+\hat{\nu}} + e^{-\mu\delta_{\nu,4}} \phi_x^* \, U^*_{x-\hat{\nu},\nu} \, \phi_{x-\hat{\nu}}\right] ,$$

$$U_{x,\nu}, \ \phi_x \in \ Z_3 = \{1, e^{i2\pi/3}, e^{-i2\pi/3}\} .$$

- Dual representation

$$Z \ \propto \sum_{\{p,k\}} \mathcal{W}[p,k] \, \mathcal{C}_S[k] \, \mathcal{C}_L[p,k] ,$$

$$p_{x,\nu\rho} \in \{-1, 0, +1\} , \ k_{x,\nu} \in \{-1, 0, +1\}$$

C. Gattringer and A. Schmidt (PRD 2012).

Constraints

- Triality function:

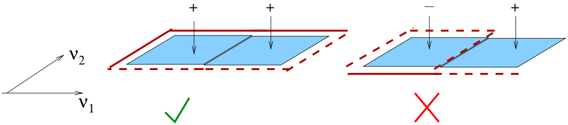
$$T(n) = \begin{cases} 1 & n \text{ mod } 3 = 0 \\ 0 & \text{else} \end{cases}$$

- Site constraint ⇒ matter loops

$$\mathcal{C}_S[k] = \prod_x T\left(\sum_{\nu=1}^{4} [k_{x,\nu} - k_{x-\hat{\nu},\nu}]\right)$$

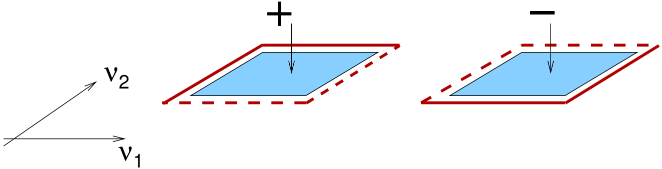
- Link constraint ⇒ gauge surfaces

$$\mathcal{C}_L[p,k] = \prod_x \prod_{\nu=1}^{4} T\left(\sum_{\rho:\nu<\rho} [p_{x,\nu\rho} - p_{x-\hat{\rho},\nu\rho}] - \sum_{\rho:\nu>\rho} [p_{x,\rho\nu} - p_{x-\hat{\rho},\rho\nu}] + k_{x,\nu}\right)$$

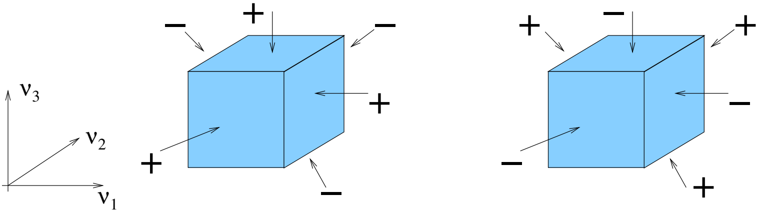


Local Metropolis Update

- Plaquette update:

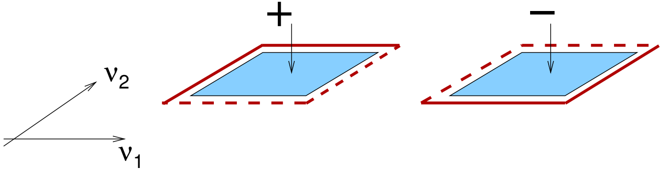


- Cube update:

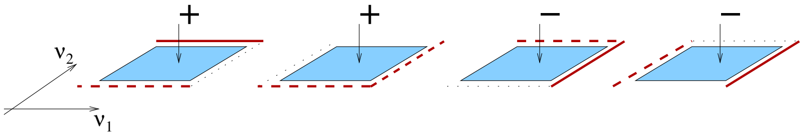


Elements of the SWA

- Take smallest unit of the local update:

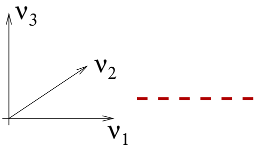


- Relax the constraints in 2 elements → segments



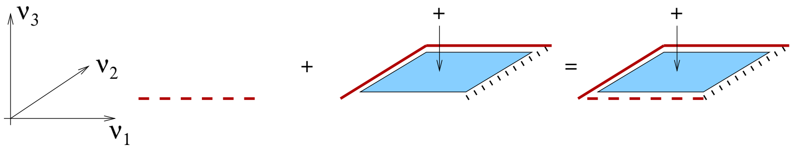
Updating scheme

1. One link is inserted at a random position of the lattice L_0.



Updating scheme

1. One link is inserted at a random position of the lattice L_0.
2. The worm may insert a new segment at L_v, healing the constraints at this position and then move to one of other three links of the segment.



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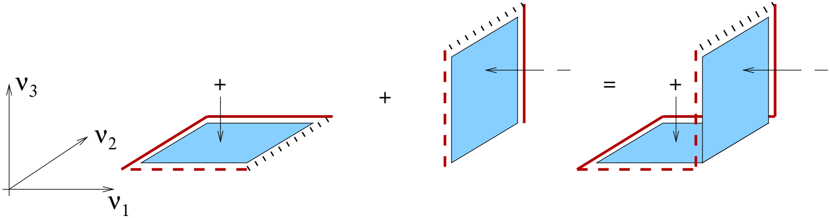

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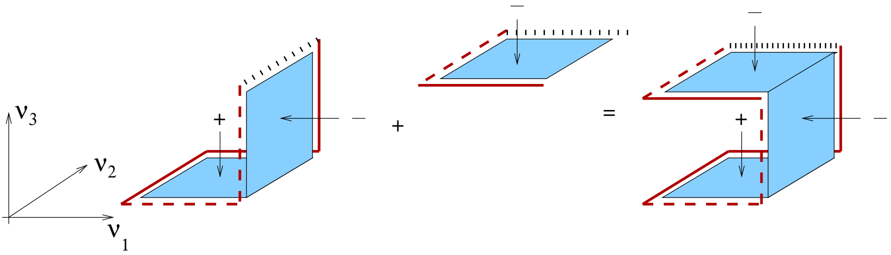

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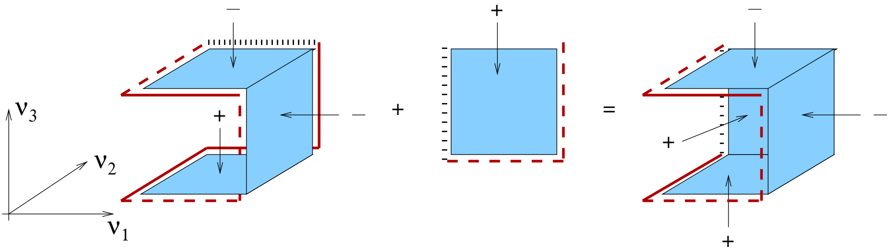
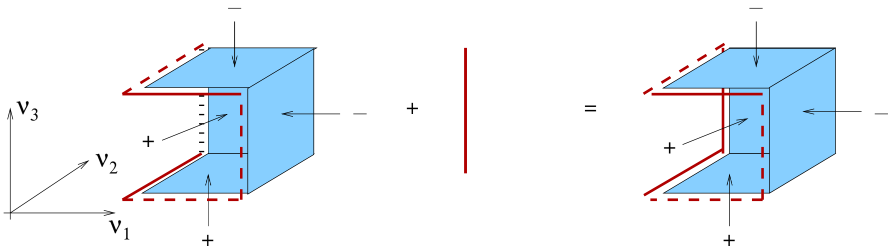


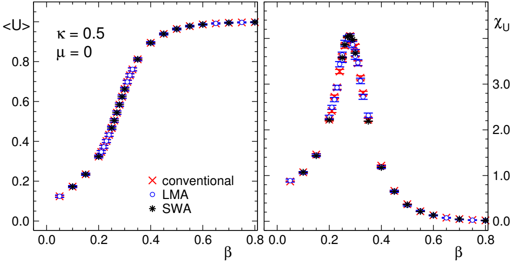
Updating scheme

1. One link is inserted at a random position of the lattice L_0.
2. The worm may insert a new segment at L_v, healing the constraints at this position and then move to one of other three links of the segment.
3. The worm ends modifying the link occupation number at L_v.



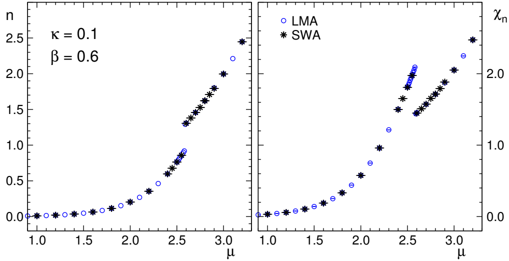
Checking correctness of the SWA...

$\mu = 0$ case



$\mu \neq 0$ case

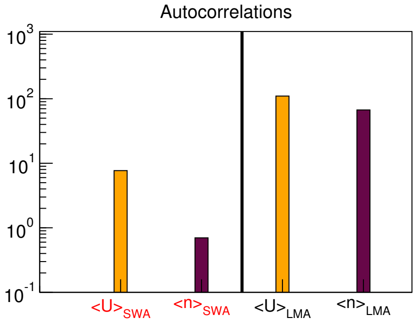
sign problem solved!



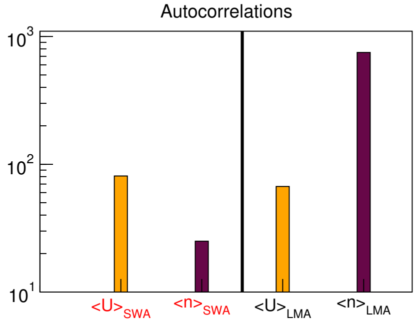
SWA vs. LMA

- $\langle U \rangle$ function of plaquettes.
- n function of temporal links.

Close to crossover.
$\kappa = 0.5, \beta = 0.28, \mu = 0$



Links are expensive.
$\kappa = 0.1, \beta = 0.8, \mu = 1.6$



The U(1) gauge-Higgs model

- Conventional representation

$$S_G = -\frac{\beta}{2}\sum_x \sum_{\nu<\rho} \left[U_{x,\nu\rho} + U^*_{x,\nu\rho}\right] ,$$

$$S_H = +\sum_x \left[\kappa|\phi_x|^2 + \lambda|\phi_x|^4\right] - \sum_{x,\nu}\left[\phi^*_x\, U_{x,\nu}\, \phi_{x+\hat{\nu}} + \phi^*_x\, U^{\,*}_{x-\hat{\nu},\nu}\, \phi_{x-\hat{\nu}}\right] ,$$

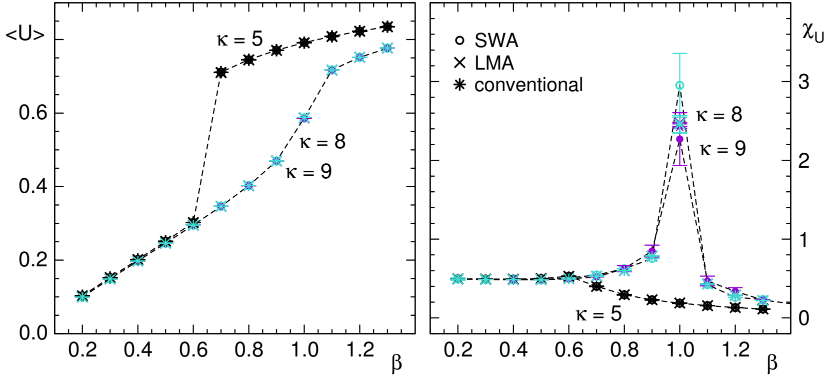
$$U_{x,\nu} \in U(1) \quad ; \quad \phi_x \in \mathbb{C}.$$

- Dual representation

$$Z \propto \sum_{\{p,kl\}} \mathcal{W}[p,k,l]\; \mathcal{C}_S[k]\; \mathcal{C}_L[p,k] ,$$

$$p_{x,\nu\rho} \in (-\infty,+\infty) \;,\; k_{x,\nu} \in (-\infty,+\infty) \;,\; l_{x,\nu} \in [0,+\infty)$$

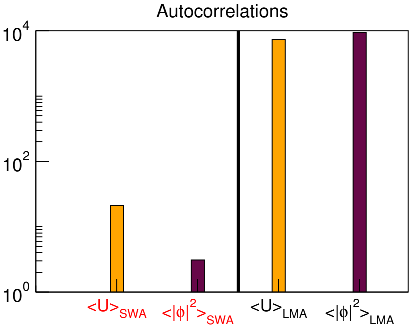
The SWA also works for the U(1) model



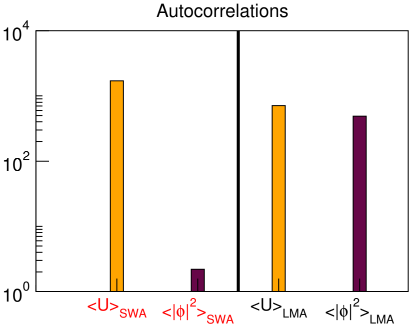
SWA vs. LMA

- $\langle U \rangle$ function of plaquettes.
- $\langle |\phi|^2 \rangle$ function of links.

Close to the 1st order transition.
$\kappa = 5, \beta = 0.65, \lambda = 1$



Links are expensive.
$\kappa = 8, \beta = 1.1, \lambda = 1$



SWA and the 2 flavor Abelian-Higgs model

- Action on the lattice (see talk by A. Schmidt and plenary talk by C. Gattringer)

$$S_G = -\frac{\beta}{2}\sum_x \sum_{\nu<\rho}\left[U_{x,\nu\rho} + U^*_{x,\nu\rho}\right] \qquad \phi^i_x \in \mathbb{C}\,; U_{x,\nu} = e^{iA_\nu} \in U(1)$$

$$S^1_H = \sum_x \left[M^2|\phi^1_x|^2 + \lambda|\phi^1_x|^4\right] - \sum_{x,\nu}\left[e^{-\mu\delta_{\nu 4}}\phi^{1*}_x U_{x,\nu}\phi^1_{x+\hat\nu} + e^{\mu\delta_{\nu 4}}\phi^{1*}_x U^*_{x-\hat\nu,\nu}\phi^1_{x+\hat\nu}\right]$$

$$S^2_H = \sum_x \left[M^2|\phi^2_x|^2 + \lambda|\phi^2_x|^4\right] - \sum_{x,\nu}\left[e^{-\mu\delta_{\nu 4}}\phi^{2*}_x U^*_{x,\nu}\phi^2_{x+\hat\nu} + e^{\mu\delta_{\nu 4}}\phi^{2*}_x U_{x-\hat\nu,\nu}\phi^2_{x+\hat\nu}\right]$$

- Dual representation (YD, C. Gattringer, A. Schmidt, 1307.6120)

$$Z \propto \sum_{\{p,k^1,k^2,l^1,l^2\}} \mathcal{W}[p,k,l]\;\mathcal{C}_S[k^1]\,\mathcal{C}_S[k^2]\,\mathcal{C}_L[p,k^1,k^2]$$

- Update of constrained variables:
  - SWA for each variable.
  - Sweep of winding loops made of both constrained variables.

Summary

- Considerable progress was made towards rewriting several systems in the dual representation, where the sign problem is solved.
- We are able to explore the complete phase diagram of the U(1) gauge-Higgs model.
- We developed an efficient algorithm for the update of the dual variables.
- Outlook:
  - Phase diagram at finite density.
  - Application to condensed matter physics.
  - Dual representation of non-abelian theories??

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Thank you for your attention!