Nature of finite temperature and density phase transitions in many-flavor QCD

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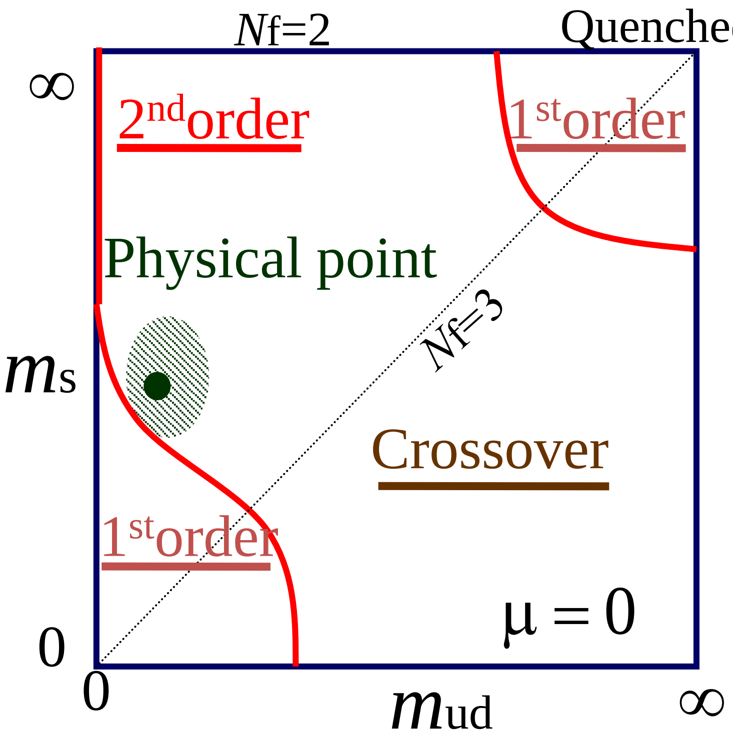
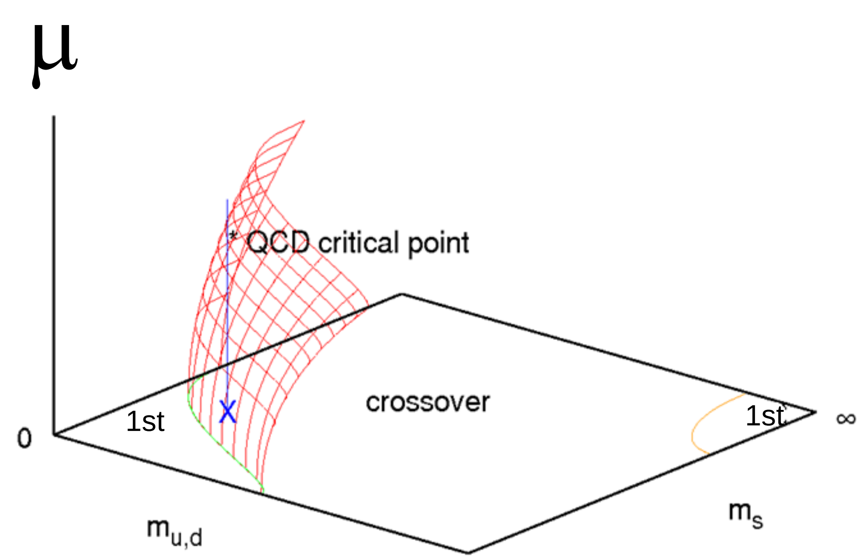
S. Ejiri & N. Yamada, Phys. Rev. Lett. 110, 172001 (2013) [arXiv:1212.5899]

S. Ejiri, Euro. Phys. J. A 49, 86 (2013) [arXiv:1306.0295] (mini-review)

Lattice 2013, Mainz, July 29 – August 3, 2013

Quark Mass dependence of QCD phase trantion





- Nature of QCD phase transition changes as a function of the mass.
- The determination of the boundary of 1st order region: important.
- On the line of physical mass, the crossover at low density ⇨ 1st order transition at high density.
- However, the 1st order region is very small, and simulations with very small quark mass are required. ⇨ Difficult to study.

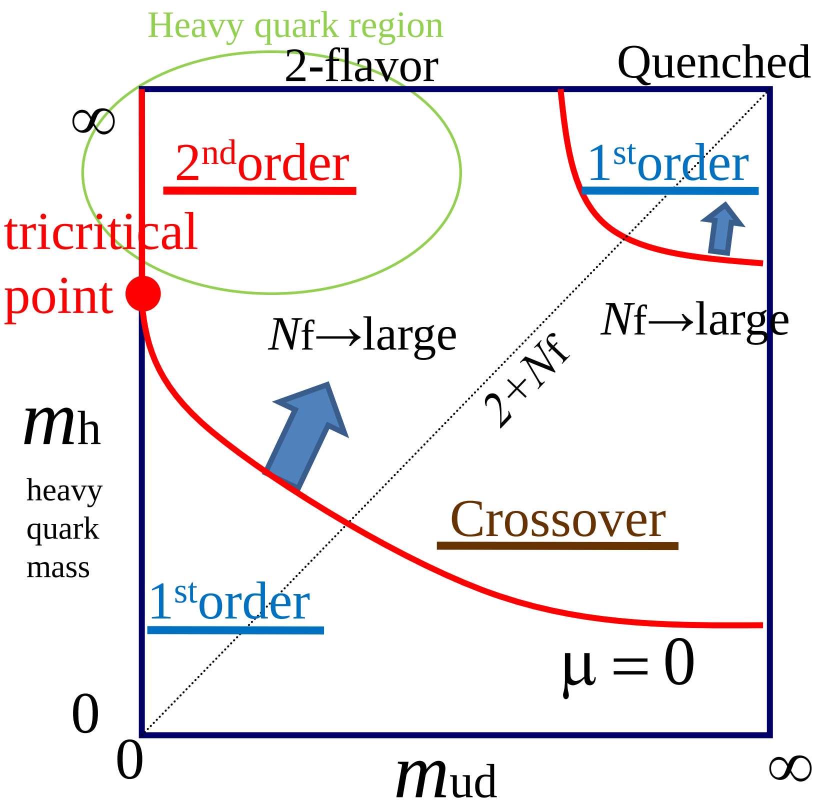
Finite T and μ phase transition in (2+many)-flavor QCD

(Cf. Kikukawa, Kohda and Yasuda,Phys.Rev.D77, 015014(2008))

- Technicolor model constructed by many-flavor QCD
- Chiral phase transition of QCD

  → Electroweak phase transition at finite temperature
- Nambu-Goldstone bosons
  - 3 bosons are absorbed into the gauge bosons. (3 massless bosons)
  - The other bosons have not observed yet. (The other bosons: heavy)
  - 2 techni-felmions are massless, and the others are heavy.
- Electro-weak baryogenesis
  - Strong first order transition: required.
  - From the analogy of 2+1-flavor QCD, 1st order at small mass; 2nd order or crossover at large mass.
- <u>It is important to determine the encpoint of the first order region in (2+many)-flavor QCD.</u>

Nature of phase transition of 2+N_f-flavor QCD



- Assumption: N_f-flavors are heavy.
  - Hopping parameter κ expansion

$$N_f \ln\left(\frac{\det M(\kappa,\mu)}{\det M(0,0)}\right) = \underbrace{288 N_{site} N_f \kappa^4 P}_{\text{(absoved into } S_g)} + 12\cdot 2^{N_t} N_s^3 \underline{N_f \kappa^{N_t}} \Omega_R + \cdots$$

(P: plaquette, ΩR: Polyakov loop)

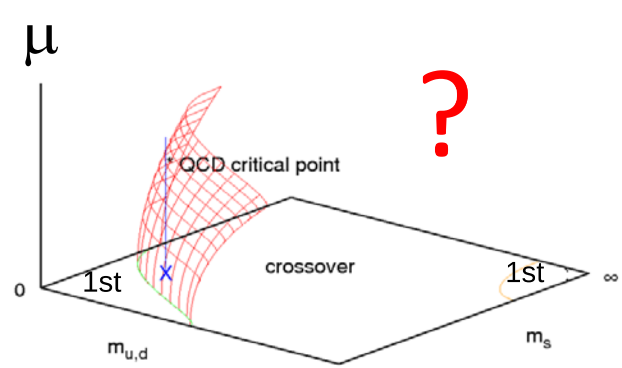
- Parameter: $\underline{N_f \kappa^{N_t}}$ ➡ $1/m_{h,ct} \sim \kappa_{ct} \propto N_f^{1/N_t}$(?)
- As increasing N_f, critical mass becomes larger (?)

- Tricritical scaling: the same as (2+1)-flavor QCD

Tricritical point m_E :	$m_{ud}^c \sim (m_E - m_h)^{5/2}$ $m_{ud}^c \sim \mu^5$

Good test ground

At finite density?



?

Histogram method

Distribution function & the effective potential

(S. Ejiri, Phys. Rev. D 77 (2008) 014508)

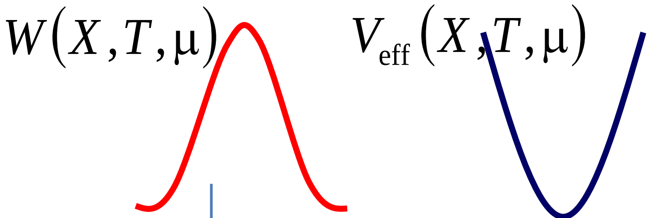
$$W(X;m,T,\mu) \equiv \int DU \delta\left(X - \hat{X}\right) \prod_{f=1}^{N_f} \det M\left(m_f, \mu_f\right) e^{-S_g}$$ (Histogram)

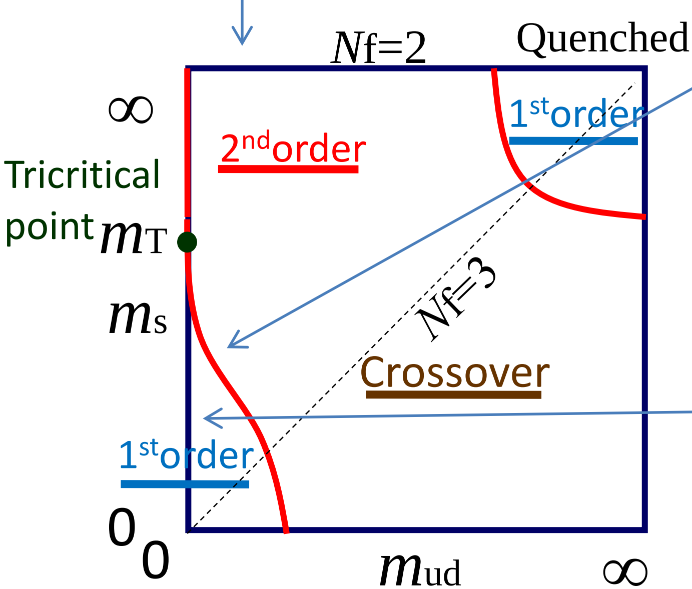
X: order parameters, total quark number, average plaquette, etc.

$$V_{\mathrm{eff}}(X) = -\ln W(X)$$

Crossover

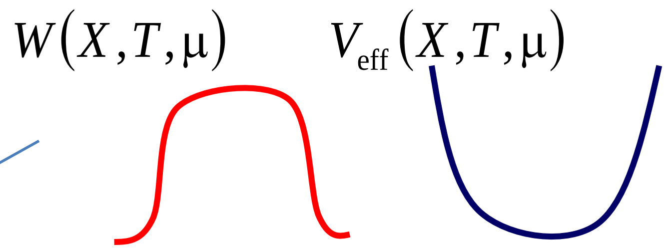
$W(X)$: Gaussian function
$V(X)$: Quadratic function



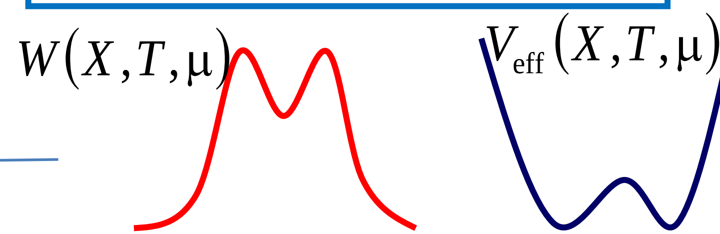


Critical point

W(X): Flat
V(X): Curvature: Zero



1^{st} order phase transition



W(X): Two phases coexist
V(X): Double well potential

Reweighting method for plaquette distribution function

$$W(P,\beta,m,\mu) \equiv \int DU\,\delta(\hat{P}-P)\prod_{f=1}^{N_{\mathrm{f}}} \mathrm{det}M(m_f,\mu_f)\,\mathrm{e}^{6N_{\mathrm{site}}\beta\hat{P}}$$

$$S_g = -6N_{\mathrm{site}}\beta\hat{P}$$

$$(\beta = 6/g^2)$$

plaquette P (1x1 Wilson loop for the standard action)

$$R(P,\beta,\beta_0 m,m_0,\mu) \equiv W(P,\beta,m,\mu)/W(P,\beta_0,m_0,0)$$

(Reweight factor)

$$R(P) = \frac{\left\langle \delta(\hat{P}-P)\,e^{6N_{\mathrm{site}}(\beta-\beta_0)\hat{P}}\prod_f \frac{\mathrm{det}M(m_f,\mu_f)}{\mathrm{det}M(m_0,0)}\right\rangle_{(\beta_0,\mu=0)}}{\left\langle \delta(\hat{P}-P)\right\rangle_{(\beta_0,\mu=0)}} \equiv \left\langle e^{6N_{\mathrm{site}}(\beta-\beta_0)\hat{P}}\prod_f \frac{\mathrm{det}M(m_f,\mu_f)}{\mathrm{det}M(m_0,0)}\right\rangle_{P:\mathrm{fixed}}$$

Effective potential:

$$V_{\mathrm{eff}}(P,\beta,m,\mu) = -\ln[W(P,\beta,m,\mu)] = V_{\mathrm{eff}}(P,\beta_0,m_0,0) - \ln R(P,\beta,\beta_0 m,m_0,\mu)$$

$$\ln R(P) = \underline{6N_{\mathrm{site}}(\beta-\beta_0)P} + \ln\left\langle \underline{\prod_f \frac{\mathrm{det}M(m_f,\mu_f)}{\mathrm{det}M(m_0,0)}}\right\rangle_{P:\mathrm{fixed}}$$

First order transition point: two phases coexist

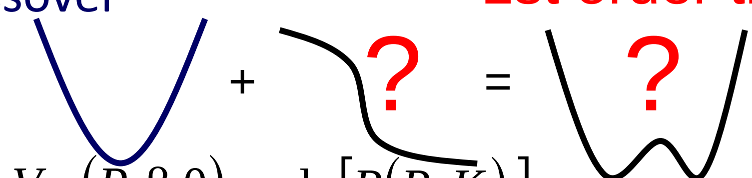
Plaquette distribution function

- Performing simulations of 2-flavor QCD,
- Dynamical effect of N_f-flavors are included by the reweighting.
- We assume N_f-flavors are heavy.
- Hopping parameter (κ) expansion（Wilson quark）

$$N_{\rm f} \ln\left(\frac{\det M(\kappa,\mu)}{\det M(0,0)} \right) = N_{\rm f} \left(288 N_{\rm site} \kappa^4 P + 12 \cdot 2^{N_t} N_s^3 \kappa^{N_t} \left(\cosh(\mu/T) \Omega_R + i \sinh(\mu/T) \Omega_I \right) + \cdots \right)$$

- Effective potential

$$V_{\rm eff}(P,\beta,\kappa) = -\ln[R(P,\kappa)W(P,\beta,0)] =$$



$$\ln R(P) = \ln \left\langle e^{6N_{\rm site}(\beta-\beta_0)\hat{P}} \prod_f \frac{\det M(m_f,\mu_f)}{\det M(m_0,0)} \right\rangle_{P:\text{fixed}} \approx \ln \left\langle \exp\left(6hN_s^3 \hat{\Omega}_R\right) \right\rangle_{P:\text{fixed}}$$ +(linear term of P)

(degenerate mass case at μ=0)

Curvature of the effective potential

$$V_{\rm eff}(P,\beta,h,\mu) = V_{\rm eff}(P,\beta_0,0,0) - \ln \bar{R}(P,h,\mu) + \text{ (linear term of } P\text{)}$$

$$\bar{R}(P) = \left\langle \exp\left(6N_s^3 h \Omega_R\right)\right\rangle_{P:\text{fixed}} \qquad \text{(for the case of } \mu=0\text{)}$$

Wilson quark

$$h = 2N_{\rm f}(2\kappa_{\rm h})^{N_t}$$

Staggered quark

$$h = N_{\rm f} \Big/ \left(4(2m_{\rm h})^{N_t}\right)$$

- Linear term of P is irrelevant to the curvature
- β-dependence is only in the linear term.
- The curvature is independent of β.

χ_P: plaquette susceptibility

$$\frac{d^2 V_{\rm eff}(0)}{dP^2} \approx \frac{6N_{\rm site}}{\chi_P}$$

$$\frac{d^2 V_{\rm eff}}{dP^2}(P,h,\mu) = \frac{d^2 V_{\rm eff}}{dP^2}(P,0,0) - \frac{d^2 \ln \bar{R}}{dP^2}(P,h,\mu)$$

2-flavor

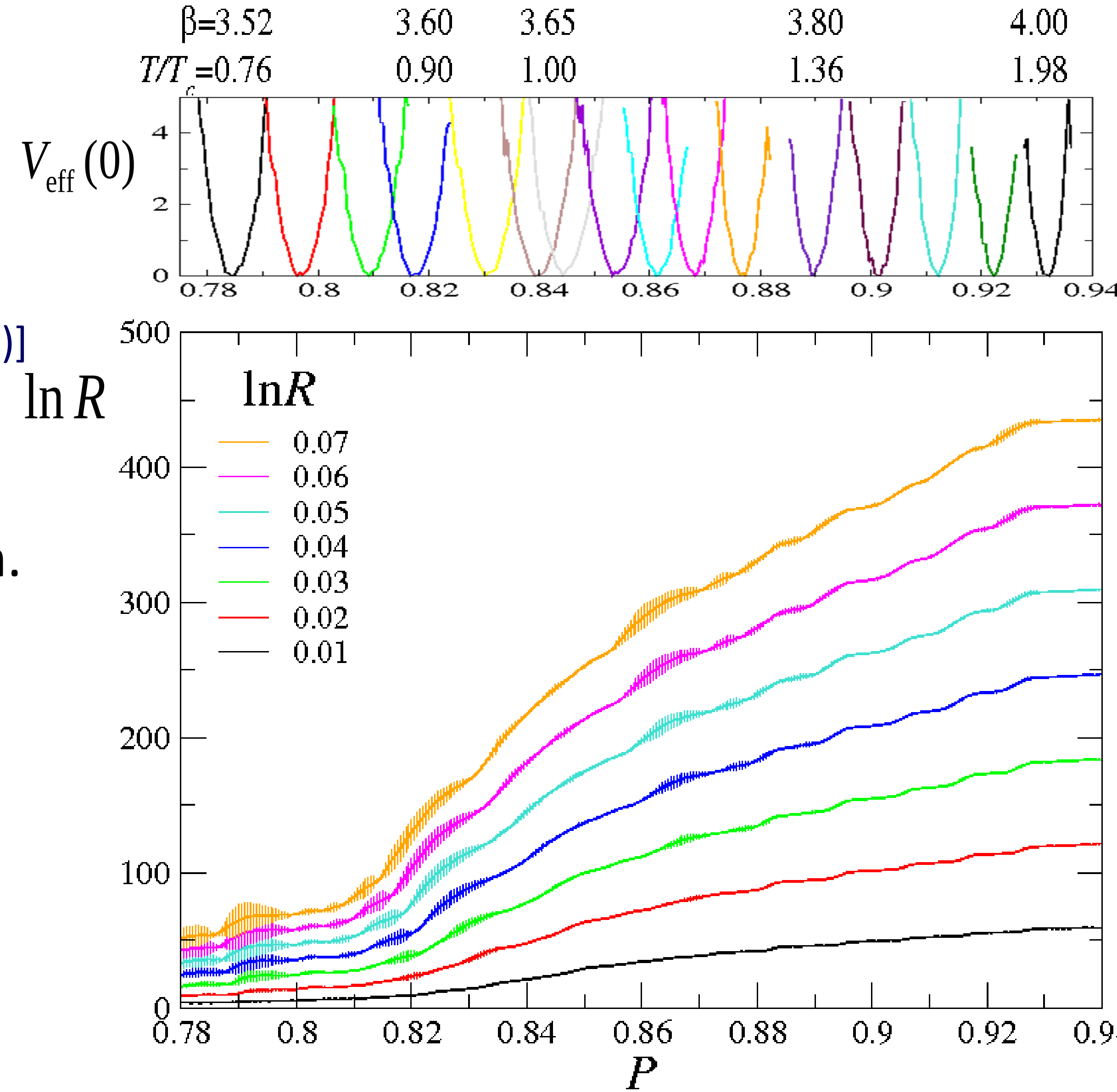
- If there exists the negative curvature region,

⇒ First order transition (double-well potential)

Effective potential at $h \neq 0$

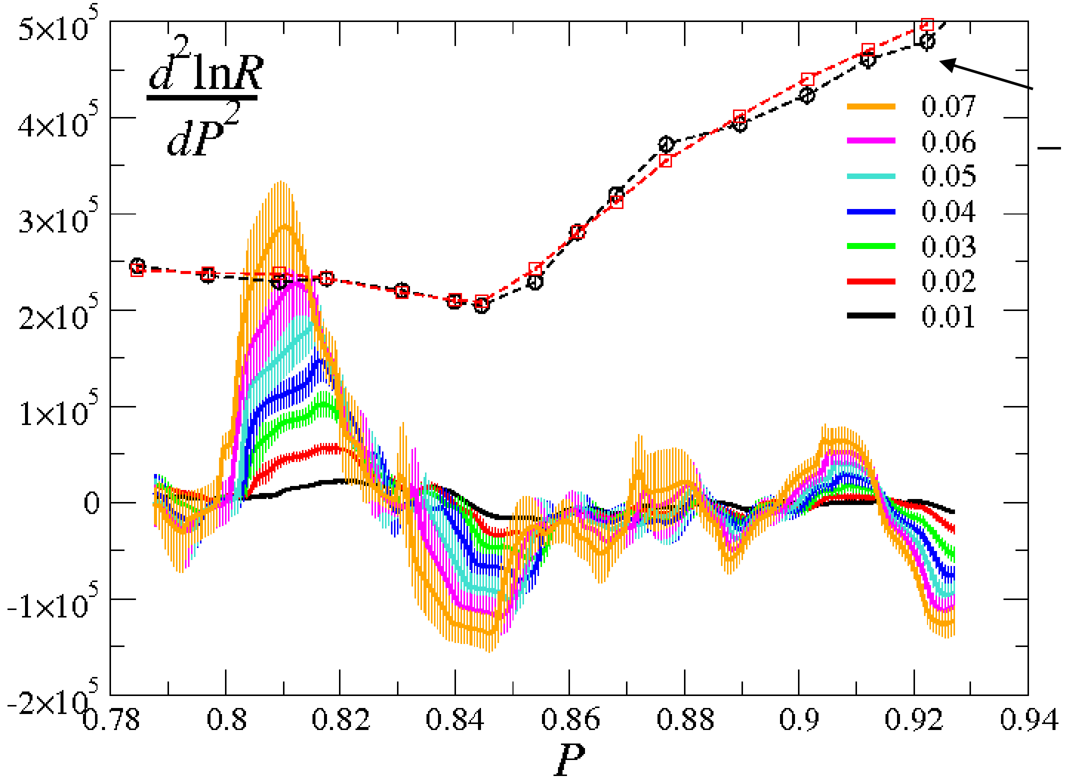
$$V_{\text{eff}}(P,\beta,h) = V_{\text{eff}}(P,\beta,0) - \ln R(P,h)$$

N_f=2 p4-staggared,
$m_\pi/m_\rho \approx 0.7$
[data: Beilefeld-Swansea Collab.,PRD71,054508(2005)]



- det$M$: hopping parameter expansion.
- lnR increases as increasing h.
- The curvature increases with h.

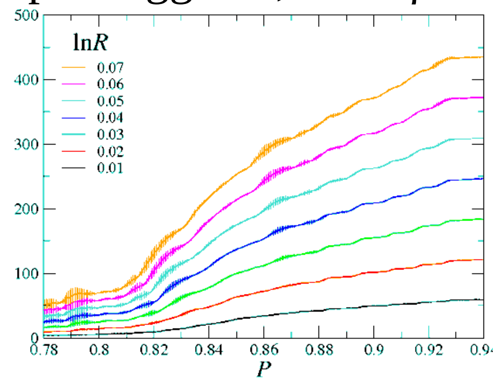
Curvature of the effective potential

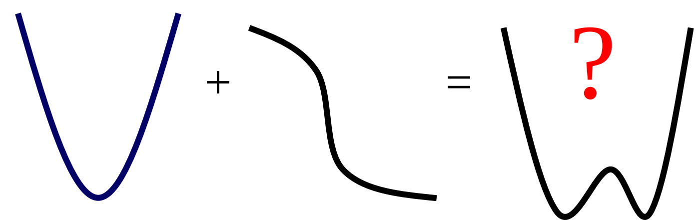


$-\frac{d^2 \ln W}{dP^2}$

at h=0

Nf=2 p4-staggared, $m\pi/m\rho\approx0.7$





Critical point: $$\frac{d^2 V_{\mathrm{eff}}(P,\beta,h)}{dP^2} = \frac{d^2 V_{\mathrm{eff}}(P,\beta,0)}{dP^2} - \frac{d^2 \ln \overline{R}(P,h)}{dP^2} = 0$$

$$h = 2N_{\mathrm{f}}(2\kappa_{\mathrm{h}})^{N_t}$$

(Wilson quarks)

- First order transition for h > 0.6

Critical value: $h_c = 0.0614(69)$

N_f–dependence of the critical mass

$$h_c = 0.0614(69)$$

- Critical mass increases as N_f increases.

$$h = 2N_f\left(2\kappa_h\right)^{N_t} \Rightarrow \kappa_h^c = \frac{1}{2}\left(\frac{h_c}{2N_f}\right)^{1/N_t}$$

  - When N_f is large, κ is small. Then, the hopping parameter (κ) expansion is good.
  - On the hand, when N_f is small, the κ-expansion is bad.

- In a quenched simulation with N_t=4, the first and second terms becomes comparable around κ=0.18.
- For N_f=10, N_t=4, $h_c = 0.0614(69)$ $\Rightarrow$ $\kappa_h^c \approx 0.118$
  - It may be applicable for N_f~10.

Curvature of the effective potential at finite μ

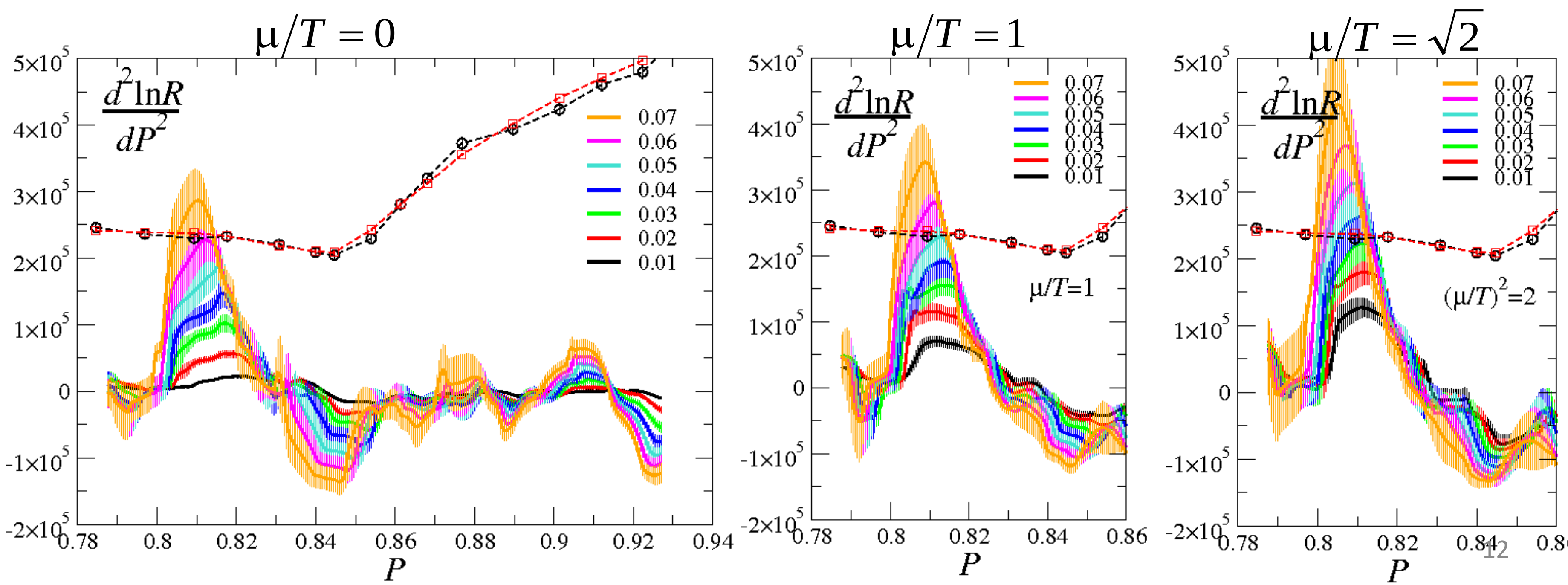
$$\frac{d^2V_{\text{eff}}}{dP^2}(P,h,\mu)=\frac{d^2V_{\text{eff}}}{dP^2}(P,0,0)-\frac{d^2\ln\overline{R}}{dP^2}(P,h,\mu)$$

$$h=2N_{\mathrm{f}}\left(2\kappa_{\mathrm{h}}\right)^{N_t} \qquad \text{for Wilson quarks}$$

$$\ln R(P)=\ln\left\langle \underbrace{\left(\frac{\det M(m,\mu)}{\det M(m,0)}\right)^2}\underbrace{\left(\frac{\det M(h,\mu_h)}{\det M(0,0)}\right)^{N_{\mathrm{f}}}}\right\rangle_{P:\text{fixed}}$$

- Calculations of detM: Taylor expansion up to O(μ^6)
- Distribution function of the complex phase of detM: approximated by a Gaussian function

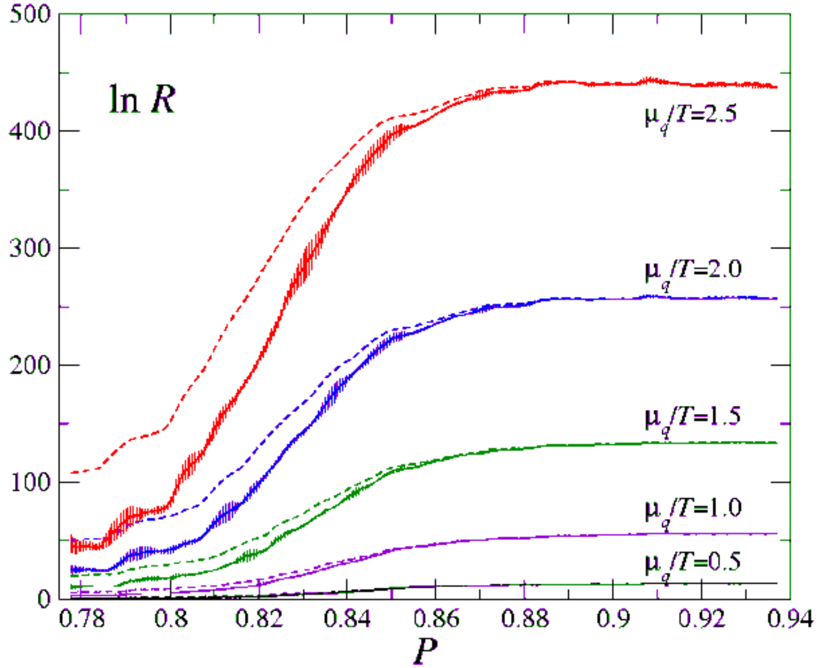
$$\mu_h/T=0$$



Reweighting factors at $h\neq 0$ $\mu\neq 0$

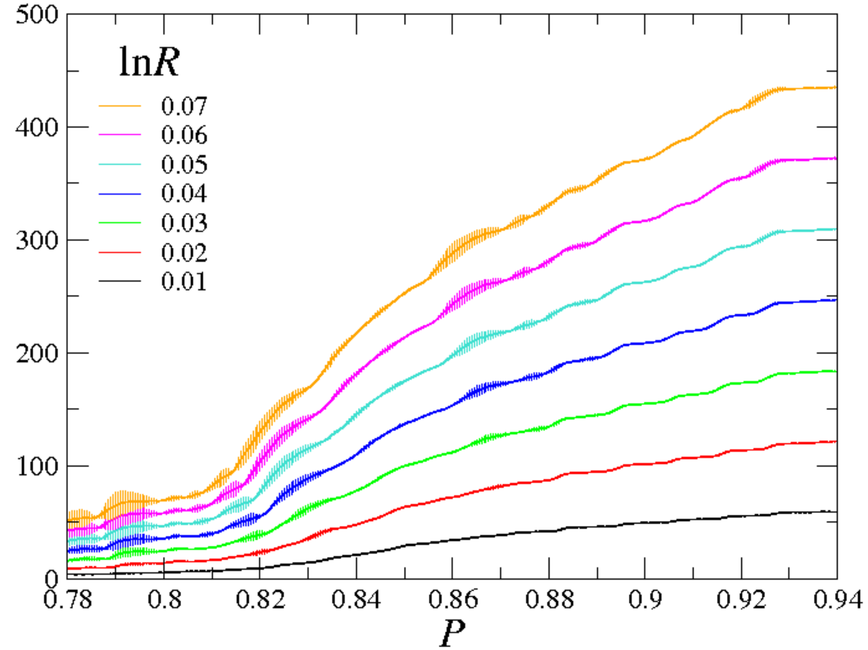
$$\ln R(P;h,\mu)=\ln\left\langle\left(\frac{\det M(m,\mu)}{\det M(m,0)}\right)^2\left(\frac{\det M(h,0)}{\det M(0,0)}\right)^{N_f}\right\rangle_{P:\text{fixed}}\approx \ln R(P;0,\mu)+\ln R(P;h,0)$$

$\ln R(P;0,\mu)$



(S. Ejiri, Phys. Rev. D 77 (2008) 014508)

$\ln R(P;h,0)$



N_f=2 p4-staggared, $m_\pi/m_\rho\approx 0.7$ [data in PRD71,054508(2005)]

- The curvatures of lnR(P;μ,0) and lnR(P;0,h) are large at the same P.

→ The curvature of lnR(P;μ,h) is enhanced.

Critical line at finite density

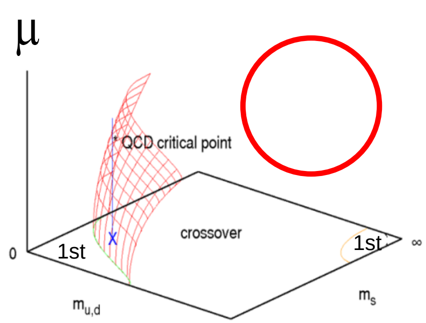
$$h = 2N_{\mathrm{f}} \left(2\kappa_{\mathrm{h}}\right)^{N_t}$$

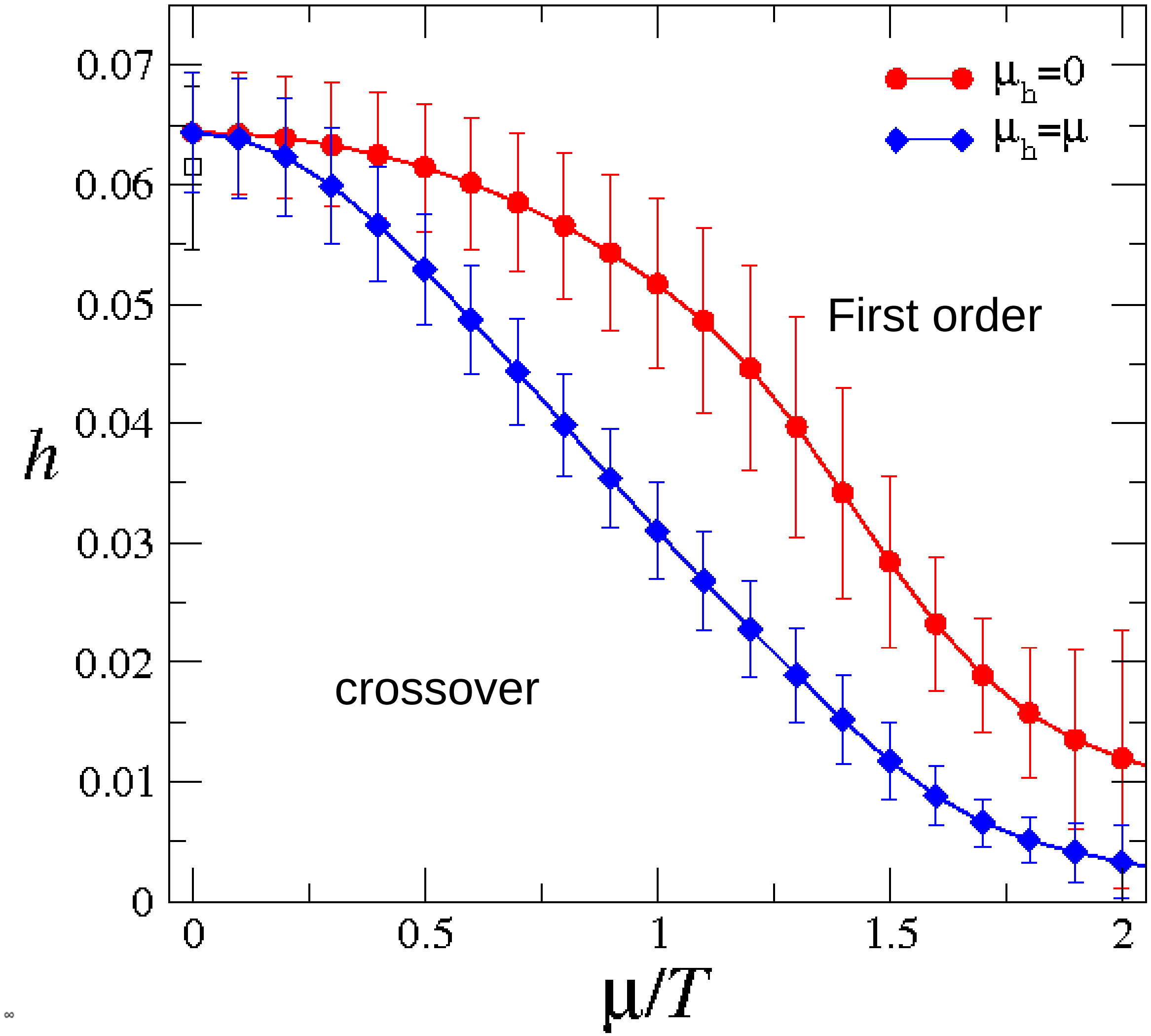
for Wilson quarks

$$h = N_{\mathrm{f}} \Big/ \left(4\left(2m_{\mathrm{h}}\right)^{N_t}\right)$$

for staggered quarks

- Calculations of detM: Taylor expansion up to O(μ^6)
- Distribution function of the complex phase of detM: approximated by a Gaussian function

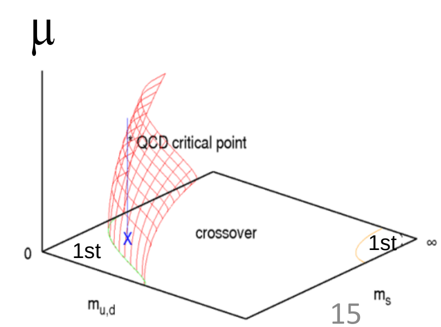




Summary

- We investigated the phase structure of (2+Nf)-flavor QCD.
  - This model is interesting for the feasibility study of the electroweak baryogenesis in the technicolor scenario.
  - An appearance of a first order phase transition at finite temperature is required for the baryogenesis.
- Applying the reweighting method, we determine the critical mass of heavy flavors terminating the first order region.
  - The critical mass becomes larger with N_f.
  - The first order region becomes wider as increasing μ.
- This may be a good test for the determination of boundary of the first order region in (2+1)-flavor QCD at finite density.
- Study by an improved Wilson quark action

  → N. Yamada's Talk (Wed)



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