Strange and Charm Spin in the nucleon from Anomalous Ward Identity with Overlap Fermion

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χ QCD Collaboration

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Outline

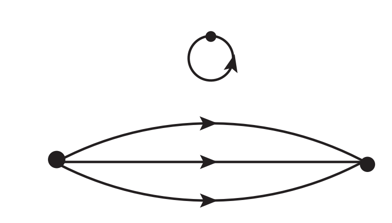
Introduction

"Spin crisis"

$$\Delta\Sigma = \sum_q \Delta q \sim 0.2 - 0.3$$

- Quark spin?
- Quark orbital angular momentum?
- Glue spin?
- Glue orbital angular momentum?

Lattice approach to the disconnected insertion contribution to the quark spin



- The nucleon two-point functions
- The quark loops

The direct method

The forward matrix element of the axial current

$$s_\mu \Delta q = \langle p, s | \bar{q} i \gamma_\mu \gamma_5 q | p, s \rangle$$

- The recent Δs result from QCDSF is $-0.020(10)(4)$.
- The axial loop is not saturated by the low modes and gain little benefit from the LMA technique[a]. More inversions should be done to get satisfactory statistical signals.
- A flavor-singlet renormalization factor should be taken care of.

[a] M. Gong et al., arXiv:1304.1194

The method with the anomalous Ward identity

The three-point function with pseudo-scalar and anomaly currents

$$\langle p,s|A_\mu|p,s\rangle\, s_\mu = \lim_{q\to 0}\frac{i|s|}{q.s}\left\langle p',s\left|2\sum_{f=1}^{N_f} m_f\bar{q}_f i\gamma_5 q_f + N_f\frac{i}{8\pi^2}G^{\alpha}_{\mu\nu}\tilde{G}^{\alpha\mu\nu}(q)\right|p,s\right\rangle$$

- R. Gupta and J. Mandula first tried this method with quenched configurations.
- The statistical signal is rather poor by involving the gauge links.

The local topological charge density operator

$$D_{ov} = 1 + \gamma_5 \epsilon(\gamma_5 D_w)$$

$$q(x) = tr\gamma_5(1 - \frac{1}{2} D_{ov}) = \frac{1}{16\pi^2} G^{\alpha}_{\mu\nu} \tilde{G}^{\alpha\mu\nu}$$

$$\langle p, s | A_\mu | p, s \rangle s_\mu = \lim_{q \to 0} \frac{i|s|}{q.s} \left\langle p', s \left| 2 \sum_{f=1}^{N_f} m_f \bar{q}_f i\gamma_5 q_f + 2iN_f q \right| p, s \right\rangle$$

- The low-mode and the high-mode parts are separated and can be improved with different techniques.
- The loop part is renormalization group invariant with overlap fermion.
- The anomaly part constructed with overlap Dirac operator is also renormalization group invariant.
- The contribution from anomaly and from each flavor can be investigated separately.

Techniques adopted

- Deflated overlap inverter with HYP-smeared DWF configurations.
  - The inversion is sped up by more than 50 times[a].
- Low-mode substitution is adopted to construct the nucleon correlation function with smeared Z_3 noise grid sources.
  - The error bars of nucleon mass are reduced to $1/7$[b].
- Low-mode average is adopted to construct the quark loop.
  - The pseudo-scalar loop is well saturated by low modes.
- The topological charge density is calculated with diluted Z_4 noise sources.
  - The dilution scheme is $(2, 2, 2, 2)$ with even-odd dilution. $32 \times 12 \times 8 = 3072$ Dirac matrix multiplications per configuration.

[a] A. Li et al., Phys.Rev.D82:114501,2010
[b] M. Gong et al., arXiv:1304.1194

Extraction the form factor from the 3pt and 2pt functions

The ratio of 3pt and 2pt functions

$$R(t', t) = \frac{iE_p}{m_N p} e^{(E_p - m_N)(t-t')} \frac{\left\langle C_N^{pol}(t, p)(m_q L_{PS}(t', p) + q(t, p) - V.A.)\right\rangle}{\left\langle C_N^{unpol}(t, 0)\right\rangle}$$

The summed ratio

$$R'(t) = \sum_{t'=t_0+1}^{t-1} R(t', t)$$

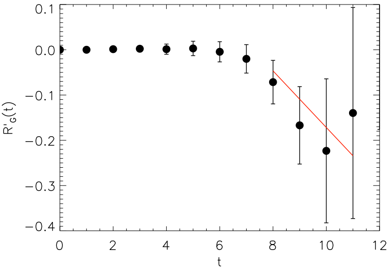
The slope technique

$$R'(t) \sim \Delta q^{disc} t + C$$

The simulation parameters

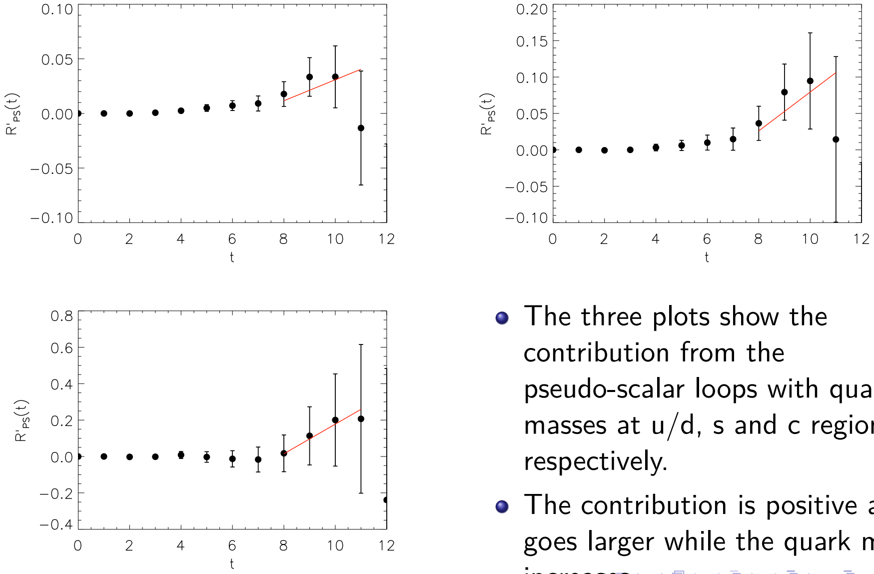
Fermion actions	Valence overlap on DWF configurations from RBC
Lattice size	$24^3 \times 64$
Lattice spacing	$1/a = 1.73(3)$ GeV
$m_{ud}^{(sea)}$	0.005
$m_s^{(sea)}$	0.04
$m_\pi^{(sea)}$	≈ 331 MeV
Number of configurations	91
Momenta	$(\pm\frac{2\pi}{L}, 0, 0)$, $(0, \pm\frac{2\pi}{L}, 0)$, $(0, 0, \pm\frac{2\pi}{L})$

The plot of the anomaly part



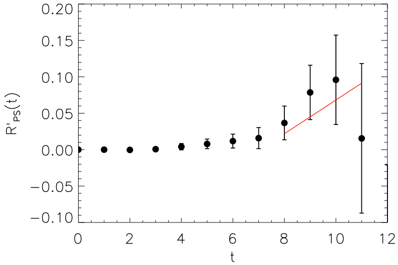
- The left plot shows the contribution from the topological charge density operator.
- The anomaly contribution is negative.
- A preliminary fitting gives $-0.063(40)$ for each flavor.

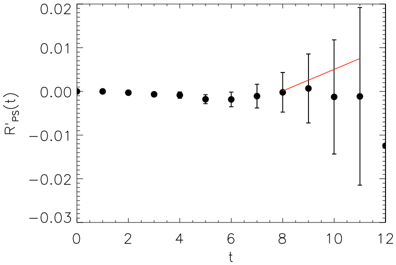
The plot of the pseudo-scalar part



- The three plots show the contribution from the pseudo-scalar loops with quark masses at u/d, s and c region respectively.
- The contribution is positive and goes larger while the quark mass increases.

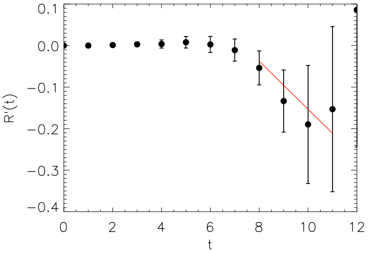
The low/high modes for the pseudo-scalar part

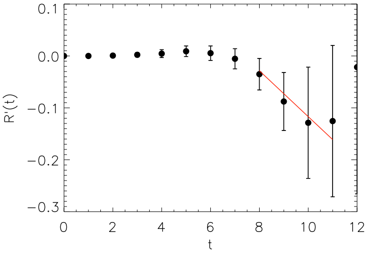


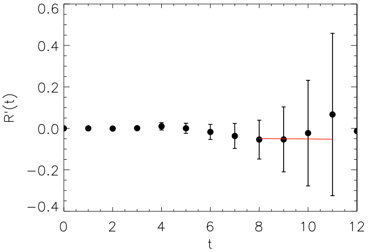


- The left plot shows the low-mode contribution and the right plot shows the high-mode contribution. The quark mass is at the strange region.
- The pseudo-scalar part is well saturated by the low modes.

Combine the two parts

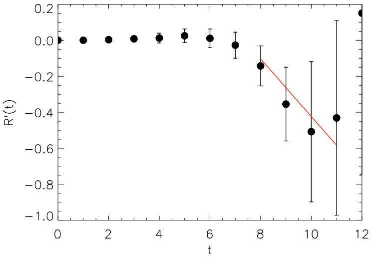






- The plots show the disconnected contribution from theses flavors.
- The disconnected insertion contribution is negative and eliminated at heavy quark limit.
- Preliminary fitting gives -0.058(34), -0.044(25) and -0.001(73) for these flavors

Sum over u, d, s flavors



- The plot shows $\Delta u^{disc} + \Delta d^{disc} + \Delta s$
- A preliminary fitting gives -0.160(92).

Conclusion

Summary

- The disconnected quark contribution to the nucleon spin is calculated from the anomalous Ward identity with overlap fermions.
- This method has advantages and can be improved with many techniques.
  - The low-mode and the high-mode parts are separated and can be improved with different techniques.
  - Both the loop part and the anomaly part are renormalization group invariant.
  - The contribution from anomaly and from each flavor can be investigated separately.
- The anomaly contribution is negative. For heavy quarks, the anomaly part and the pseudo-scalar part cancel out.
- The disconnected insertion contributions of the u/d/s quarks are negative.

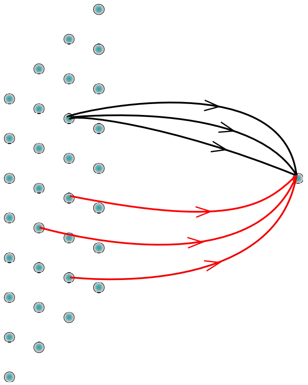
Conclusion

Future plans

- More data with different momenta is being analyzed and we will extrapolate the results at $p \to 0$
- We will have more nucleon correlators from different source time slices and will fold the data from different parity channels.
- Data on different lattices and sea quark masses will be done and we will do the chiral and continuum extrapolations.
- Connected insertion will also be calculated.

Thank you !

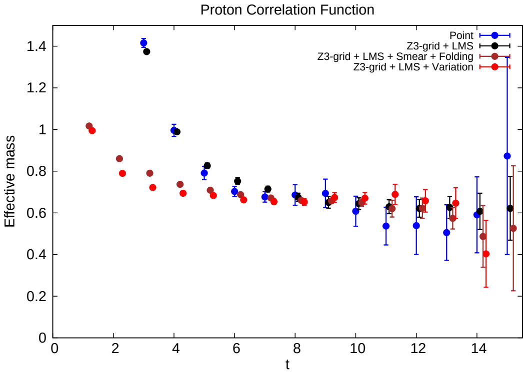
Backup pages



Backup pages

Lattice settings

Lattice size : $24^3 \times 64$, $m_{ud}^{(sea)} = 0.005$, $m_s^{(sea)} = 0.04$, $m_\pi \approx 305\text{MeV}$
47 configurations are used



Point source

$m_{proton} = 1.13(14)\text{GeV}$

Grid with LMS

$m_{proton} = 1.08(5)\text{GeV}$

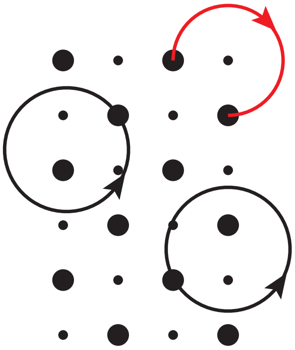
Smeared Grid with LMS

$m_{proton} = 1.14(2)\text{GeV}$

Variation

$m_{proton} = 1.12(1)\text{GeV}$

Backup pages

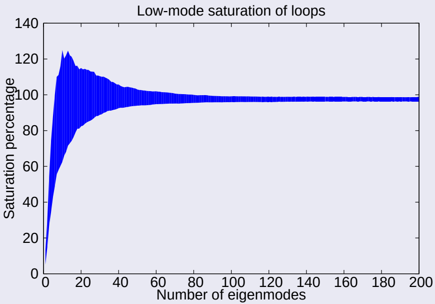


Backup pages

Lattice settings

Lattice size : $24^3 \times 64$, $m_{ud}^{(sea)} = 0.005$, $m_s^{(sea)} = 0.04$, $m_\pi \approx 305\text{MeV}$
11 configurations are used
The lowest 200 pair eigenmodes of each configuration are extracted

Saturation of pseudo-scalar loops



Saturation of scalar loops