New actions for minimally doubled fermions and their counterterms

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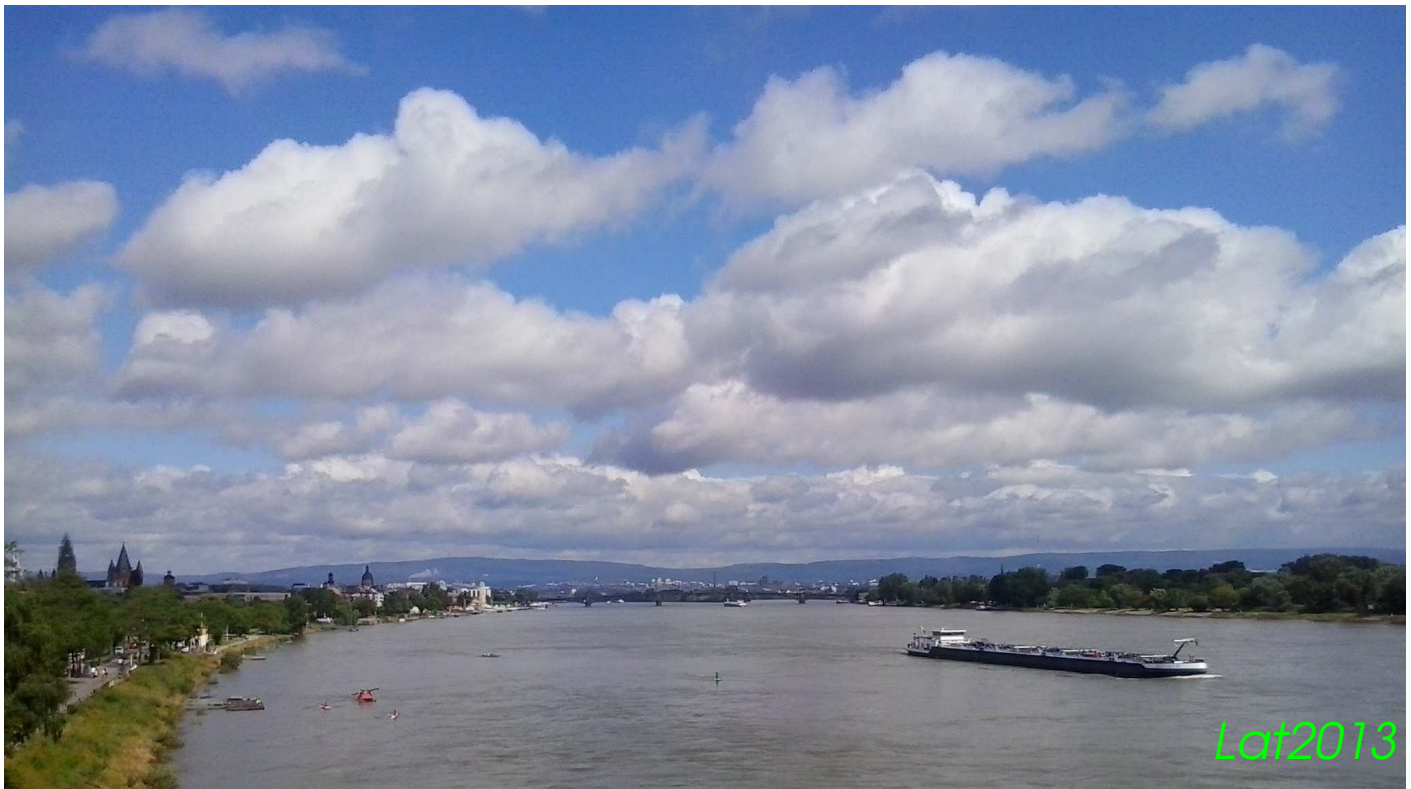

Minimally doubled fermions

Nielsen-Ninomiya theorem:

- using two fermion flavors one can maintain an exact chiral symmetry for any finite lattice spacing a, together with locality and unitarity

A chiral symmetry of the standard type *(not Ginsparg-Wilson)* – for a degenerate doublet of quarks

Minimally doubled fermions can still be kept ultralocal, like Wilson fermions

$\rightarrow$ *cheap for simulations*

no tuning of masses is required – chiral symmetry protects masses from additive renormalization

One can construct a conserved axial current, which has a simple expression, involving only nearest-neighbors sites

One of the very few lattice discretizations in which one can give a simple expression (and ultralocal) for a conserved axial current

`A convenient implementation of chiral symmetry at nonzero lattice spacing`

Minimally doubled fermions

Compared with staggered fermions:

- same kind of $U(1)$ chiral symmetry
- 2 flavors instead of 4

  $\Rightarrow$ no uncontrolled extrapolation to 2 physical light flavors
- no complicated intertwining of spin and flavor

Ideal for $N_f = 2$ simulations: no rooting needed!

Much cheaper and simpler than Ginsparg-Wilson fermions
(overlap, domain-wall, fixed-point)

Two realizations of minimally doubled fermions:

- Boriçi-Creutz fermions
- Karsten-Wilczek fermions

The twisted-ordering method by Creutz and Misumi (2010) can also be useful for constructing other minimally doubled actions

Karsten-Wilczek fermions

Already in the Eighties: Karsten (1981) and then Wilczek (1987) proposed some particular kind of minimally doubled fermions

Unitary equivalent to each other, after phase redefinitions

Wilczek [PRL 59, 2397 (1987)] proposed a special choice of the function $P_\mu(p)$ which minimizes the numbers of doublers

The free Karsten-Wilczek Dirac operator

$$D(p) = i \sum_{\mu=1}^{4} \gamma_\mu \sin p_\mu + i\gamma_4 \sum_{k=1}^{\mathbf{3}} (1 - \cos p_k)$$

has zeros at $p_1 = (0,0,0,0)$ and $p_2 = (0,0,0,\pi)$

Drawback: it destroys the equivalence of the four directions under discrete permutations

$\rightarrow$ breaking of the hypercubic symmetry

Counterterms

The actions of minimally doubled fermions have two zeros

$\Rightarrow$ there is always a special direction in euclidean space
(given by the line that connects these two zeros)

Thus, these actions cannot maintain a full hypercubic symmetry

They are symmetric only under the **subgroup** of the hypercubic group which preserves (up to a sign) a **fixed direction**

For the Boriçi-Creutz action this is a major hypercube diagonal, while for other minimally doubled actions it may not be a diagonal – for example for the Karsten-Wilczek action is the x_4 axis

Each of these two bare actions does not contain all possible operators allowed by the respective symmetries (broken hypercubic group)

Radiative corrections generate new contributions whose form is not matched by any term in the original bare actions

Counterterms are then necessary for a consistent renormalized theory

This consistency requirement will uniquely determine their coefficients

Counterterms

Three counterterms required for massless Karsten-Wilczek fermions
(S. C., M. Creutz, J. Weber & H. Wittig (2010))

Dimension-4 fermionic counterterm: $d_4(g_0)\,\overline{\psi}\,\gamma_4 D_4\,\psi$

Dimension-3 fermionic counterterm: $\dfrac{i d_3(g_0)}{a}\;\overline{\psi}(x)\,\gamma_4\,\psi(x)$

It is not hard to imagine that in the case of Karsten-Wilczek fermions the temporal plaquettes will be renormalized differently from the other plaquettes

Indeed, the gluonic counterterm should compensate the asymmetry between these two kinds of plaquettes:

$$d_P(g_0)\sum_{\rho\lambda}\mathrm{tr}\,F_{\rho\lambda}(x)\,F_{\rho\lambda}(x)\,\delta_{\rho 4}$$

This is the only purely gluonic counterterm needed for this action, since introducing also a $\delta_{\lambda 4}$ in the above expression will produce a vanishing object

Counterterms

We can determine all these coefficients by requiring that the renormalized 1-loop propagators assume their standard forms

Perturbative calculation of the coefficients:

S. C., M. Creutz, J. Weber & H. Wittig (2010)

All counterterms remain of the same form at all orders of perturbation theory

Only the values of their coefficients depend on the number of loops

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The same counterterms appear at the nonperturbative level, and will be required for a consistent simulation of these fermions

Nonperturbative determination of the coefficients:

S. C., J. Weber & H. Wittig, *parallel talk at this conference*

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We also want to emphasize that counterterms not only provide additional Feynman rules for the calculation of loop amplitudes

They can modify Ward identities and hence, in particular, contribute additional terms to the conserved currents

Towards better actions

It would be of substantial interest to find minimally doubled actions that (like the above two cases) have the correct continuum limit, but that require fewer counterterms, or even none at all

In this work we have carried out some investigations to explore these issues

Can we have minimally doubled fermions which require fewer than three counterterms?

. . . *maybe even just one?*

. . . *and maybe even none?*

We introduce new nearest-neighbor minimally doubled actions which depend on 2 continuous parameters

For each counterterm there are curves in the parameter space on which its coefficient vanishes

$\Rightarrow$ renormalized actions with only 2 counterterms

Towards better actions

For all generalized actions that we introduce here, the 3 possible counterterms are the same of the standard Karsten-Wilczek action

This happens because both poles of the quark propagator still lie entirely on the temporal axis, and thus the temporal direction is always selected as the special one *(irrespective of the values of* α *and* λ*)*

Furthermore, the spinorial structure of all these actions is also the same

Thus, P is a symmetry, and also CT (Bedaque et al. , 2008), but T and C separately are violated *(unless the actions are properly renormalized)*

The values of the coefficients of the counterterms for which one obtains a consistent renormalized theory depend on the particular choices of α and λ

In the present study we investigate the effects of varying these parameters to see if one can remove some of the counterterms

The values of the coefficients of the counterterms for which the hypercubic symmetry is restored are continuous real functions of α and λ

So, in general there will be values of the parameters describing the actions for which some of these functions vanish

. . . so, let's begin our journey . . .

Nearest-neighbor minimally doubled actions

We study the class of (bare) nearest-neighbor fermionic actions

$$S^f(x;\alpha,\lambda) = a^4 \sum_x \Bigg[\frac{1}{2a} \sum_{\mu=1}^{4} \Big[\overline{\psi}(x) \left(\gamma_\mu - i\gamma_4 \left(\lambda + \delta_{\mu 4}(\cot\alpha - \lambda)\right)\right) U_\mu(x)\, \psi(x+a\widehat{\mu}) - \overline{\psi}(x+a\widehat{\mu}) \left(\gamma_\mu + i\gamma_4 \left(\lambda + \delta_{\mu 4}(\cot\alpha - \lambda)\right)\right) U_\mu^\dagger(x)\, \psi(x) \Big] + \overline{\psi}(x) \left(m_0 + \frac{i\gamma_4}{a} \left(3\lambda + \cot\alpha\right)\right) \psi(x) \Bigg]$$

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These Wilson-like minimally doubled fermions satisfy γ_5-hermiticity and have $\mu = 4$ as a special direction *(like in the standard Karsten-Wilczek action)*

They can be also expressed in the simple form

$$a^4 \sum_x \overline{\psi}(x) \left\{ \frac{1}{2} \sum_\mu \left[\gamma_\mu (\nabla_\mu + \nabla_\mu^\star) - ia\gamma_4 \left(\lambda + \delta_{\mu 4}(\cot\alpha - \lambda)\right) \nabla_\mu^\star \nabla_\mu \right] + m_0 \right\} \psi(x)$$

where the lattice discretizations of the covariant derivative are

$$\nabla_\mu \psi(x) = \frac{U_\mu(x)\psi(x+a\widehat{\mu}) - \psi(x)}{a}, \qquad \nabla_\mu^\star \psi(x) = \frac{\psi(x) - U_\mu^\dagger(x-a\widehat{\mu})\psi(x-a\widehat{\mu})}{a}$$

Nearest-neighbor minimally doubled actions

In momentum space the Dirac operators of the above minimally doubled fermions read, in the free case,

$$\mathcal{D}^f(p;\alpha,\lambda) = \frac{i}{a}\sum_{\mu=1}^{4}\gamma_\mu \sin ap_\mu + \frac{i\gamma_4}{a}\left[\lambda\sum_{k=1}^{3}(1-\cos ap_k)+\cot\alpha\,(1-\cos ap_4)\right]+m_0$$

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The two zeros, at $a\bar{p}_1 = (0,0,0,0)$ and $a\bar{p}_2 = (0,0,0,-2\alpha)$, describe two fermions of equal mass and opposite chirality

The range of α can be taken as $0 < \alpha < \pi$

For $\alpha = 0$ and $\alpha = \pi$ the action becomes singular (as $\cot\alpha = \infty$)

Although for the quark propagators corresponding to α and $\pi - \alpha$ the distance between the poles is the same, the actions corresponding to these two choices of α are not equivalent (even for the same value of λ)

Varying λ does not change the location of any of the zeros – this parameter has only the task of decoupling the 14 other fermions from the naive fermionic action giving them a mass of order $1/a$

It must also be $\lambda > (1-\cos\alpha)/(2\sin\alpha)$ to avoid the appearance of other doublers

Nearest-neighbor minimally doubled actions

All the actions considered here have the correct leading behavior for small p *(irrespective of the values of α and λ)*

All these actions still contain only nearest-neighbor interactions, that is they are Wilson-like with hopping terms of only one unit of lattice spacing

For this reason they are rather cheap to simulate – they are a little more expensive than Wilson fermions because the spinor matrices are slightly more complicated

The computational effort will be about a few times the one required for Wilson fermions

For $\lambda = 1/\sin\alpha$ our actions can be cast, after a redefinition of p_4, into the actions written by Creutz in Fourier space in 2010, which in the free massless case read

$$\mathcal{D}^C(p;\alpha) = \frac{i}{a}\sum_{k=1}^{3}\gamma_k \sin ap_k + \frac{i\gamma_4}{a\sin\alpha}\left(\cos\alpha + 3 - \sum_{\mu=1}^{4}\cos ap_\mu\right)$$

Furthermore, when this choice of λ is taken, the standard Karsten-Wilczek action can be then obtained as a special case by putting $\alpha = \pi/2$

Nearest-neighbor minimally doubled actions

P is a symmetry, and also CT, but T and C separately are violated unless the action is properly renormalized – like for the standard Karsten-Wilczek action

Then, the counterterms that must be added to these generalized actions are the same needed for the standard Karsten-Wilczek action

In quenched QCD only 2 of them are needed

Nearest-neighbor minimally doubled actions

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In quenched QCD only 2 of them are needed

One can construct a conserved axial current for all these actions, which only involves nearest-neighbor sites:

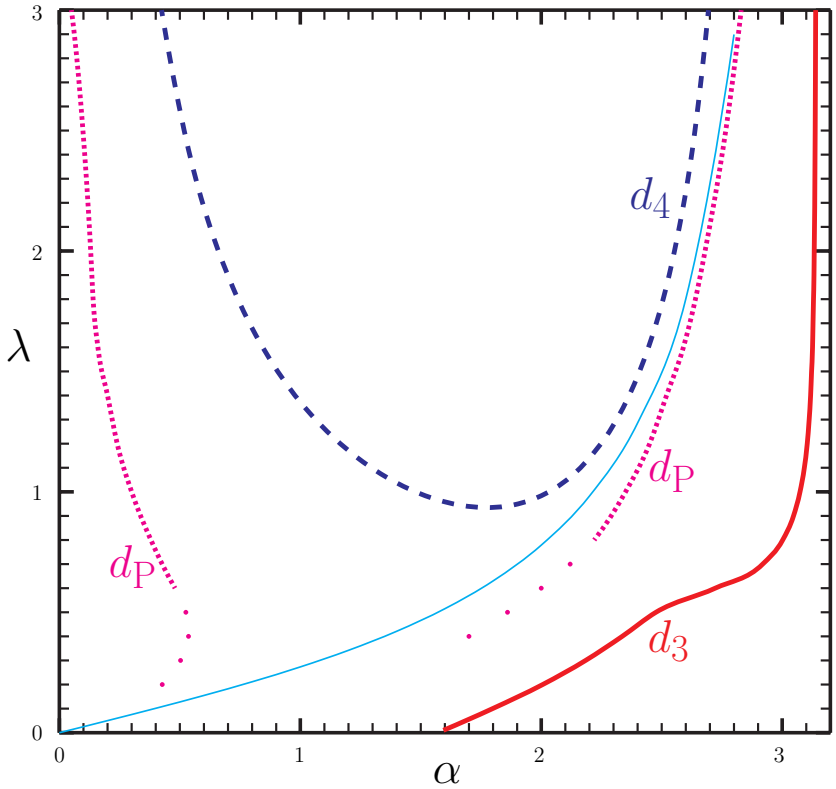
$$A_\mu^{cons}(x;\alpha,\lambda) = \frac{1}{2}\Big(\overline{\psi}(x)\,(\gamma_\mu - i\gamma_4\,(\lambda+\delta_{\mu 4}(\cot\alpha-\lambda)))\,\gamma_5\,U_\mu(x)\,\psi(x+a\widehat{\mu})$$

$$+\overline{\psi}(x+a\widehat{\mu})\,(\gamma_\mu + i\gamma_4\,(\lambda+\delta_{\mu 4}(\cot\alpha-\lambda)))\,\gamma_5\,U_\mu^\dagger(x)\,\psi(x)\Big)$$

$$+\frac{d_4(g_0)}{2}\Big(\overline{\psi}(x)\,\gamma_4\gamma_5\,U_4(x)\,\psi(x+a\widehat{4}) + \overline{\psi}(x+a\widehat{4})\,\gamma_4\gamma_5\,U_4^\dagger(x)\,\psi(x)\Big)$$

This is particularly important, as not many fermionic formulations exist for which a conserved axial current exists and is of such a simple form

1-loop results



Curves of zeros for the coefficients of the counterterms – interpolations of the points obtained from 1-loop calculations

Our calculations show that there are no intersections between these curves

The curve corresponding to a zero of d_4 is not symmetric with respect to the reflection $\alpha \to \pi/2 - \alpha$

The distance between the 2 poles of the quark propagator does not change when $\alpha \to \pi/2 - \alpha$, but these values of α correspond to different actions

The purpose here is not the computation of all zeros with a high precision, but rather to show that such curves of zeros exist and see what shape they have

Going nonperturbative

It is likely that also in numerical simulations the removal of some counterterms can be accomplished for appropriate choices of the parameters α and λ

It is worthwhile in any case to check if the qualitative pattern of the curves found in this work is also reproduced nonperturbatively

The dependence of the coefficients of the counterterms on the parameters of the action appears to be rather smooth

It will then probably be not too expensive to perform first a quick rough tuning of the parameters around the curves of zeros that we have found perturbatively

Subsequently one can calculate with more precision the positions of the nonperturbative zeros using a much finer tuning

It could also turn out that the locations of these zeros do not differ too much from the perturbative results – and so one could take the perturbative results as a good starting guess

It could happen that **intersection points** appear at the nonperturbative level

This would make possible to simulate renormalized minimally doubled actions with not more than one counterterm

Still more actions?

Even when it is not possible to remove all counterterms, it is good to have been able to accomplish a reduction in the dimensionality of the parameter space of their coefficients – it makes their numerical determination easier

In particular, if there is only one counterterm left, it is much simpler to carry out the determination of its coefficient, because one has to deal with just a one-dimensional space instead of a multi-parameter one

It is always useful to possess many different minimally doubled actions – some of them could turn out to have better theoretical or practical properties

The effective amount of the mass difference between the $\pi^{\pm}$ and the π^{0} can turn out to be small for a few of these actions and not for the other ones

Similarly for the mass splittings that can arise as a peculiar feature of minimally doubled fermions

In general it can be convenient to have minimally doubled actions where the distance between the two poles of the quark propagator can be arbitrarily varied

Special values of this distance could also provide actions which are more advantageous for Monte Carlo simulations

. . . so, let's go a bit farther . . .

Next-to-nearest-neighbor actions

We would like to have actions for which intersections between the curves of zeros exist, so that 2 or even more of the possible counterterms can then be removed

One can think of widening the pool by considering also couplings between next-to-nearest-neighbor lattice sites

In the quest for minimally doubled actions without counterterms, investigating such kind of actions could turn out at the end to be rewarding

We do not know in fact whether there could be theoretical impediments in principle to countertermless minimally doubled actions when one only considers nearest-neighbor interactions

It is conceivable that introducing interactions also at distance $2a$ or larger could allow actions with different kinds of properties

The hope is that at the end some of these actions will not require any counterterms to be properly renormalized

We find then useful to propose here a first example of a class of minimally doubled actions with next-to-nearest-neighbor interactions:

Next-to-nearest-neighbor actions

$$
\begin{aligned}
S^f_{ntn}(x;\alpha,\lambda,\lambda',\rho) \;=\; a^4 \sum_x \Bigg[& \frac{1}{2a} \sum_{\mu=1}^{4} \Big[\overline{\psi}(x)\,(\gamma_\mu - i\gamma_4\, f^{(1)}_\mu)\, U_\mu(x)\, \psi(x+a\widehat{\mu}) \\
& - \overline{\psi}(x+a\widehat{\mu})\,(\gamma_\mu + i\gamma_4\, f^{(1)}_\mu)\, U^\dagger_\mu(x)\, \psi(x) \Big] \\
& + \frac{i}{4a} \sum_{\mu=1}^{4} f^{(2)}_\mu \cdot \Big[\overline{\psi}(x)\,\gamma_4\, U_\mu(x)\, U_\mu(x+a\widehat{\mu})\, \psi(x+2a\widehat{\mu}) \\
& + \overline{\psi}(x+2a\widehat{\mu})\,\gamma_4\, U^\dagger_\mu(x+a\widehat{\mu})\, U^\dagger_\mu(x)\, \psi(x) \Big] \\
& + \overline{\psi}(x) \left(m_0 + \frac{i\gamma_4}{a}\, f^{(0)} \right) \psi(x) \Bigg]
\end{aligned}
$$

where

$$
\begin{aligned}
f^{(0)}(\alpha,\lambda,\lambda',\rho) &= 3\lambda + \frac{9}{2}\lambda' + \left(\rho + \frac{3}{4}\frac{1-\rho}{\sin^2\alpha} \right) \cot\alpha \\
f^{(1)}_\mu(\alpha,\lambda,\lambda',\rho) &= \lambda + 2\lambda' + \delta_{\mu 4} \left(\left(\rho + \frac{1-\rho}{\sin^2\alpha} \right) \cot\alpha - \lambda - 2\lambda' \right) \\
f^{(2)}_\mu(\alpha,\lambda',\rho) &= \lambda' + \delta_{\mu 4} \left(\frac{1-\rho}{2\sin^2\alpha} \cot\alpha - \lambda' \right)
\end{aligned}
$$

are functions diagonal in spinor and color space

Next-to-nearest-neighbor actions

There are simple relations between these functions, and if one defines

$$f_\mu^{(h)}(\alpha,\lambda,\rho) = \lambda + \delta_{\mu 4}\left(\rho\cot\alpha - \lambda\right)$$

then knowing $f_\mu^{(1)}$ one can obtain

$$\begin{aligned} f_\mu^{(2)} &= \frac{1}{2}\left(f_\mu^{(1)} - f_\mu^{(h)}\right) \\ f^{(0)} &= \sum_{\mu=1}^{4}\left(\frac{3}{4}\,f_\mu^{(1)} + f_\mu^{(h)}\right) = \sum_{\mu=1}^{4}\left(f_\mu^{(h)} + \frac{3}{2}\,f_\mu^{(2)}\right) \end{aligned}$$

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The corresponding momentum-space actions are given in the free case by

$$\frac{i}{a}\sum_{\mu=1}^{4}\gamma_\mu \sin ap_\mu + \frac{i\gamma_4}{a}\left\{\sum_{k=1}^{3}\left(\lambda\left(1-\cos ap_k\right) + \lambda'\left(1-\cos ap_k\right)^2\right)\right.$$
$$\left. +\cot\alpha\left(\rho\left(1-\cos ap_4\right) + \frac{1-\rho}{2\sin^2\alpha}\left(1-\cos ap_4\right)^2\right)\right\} + m_0$$

For $\lambda' = 0$ & $\rho = 1$ one falls back to the case of the nearest-neighbor actions

Next-to-nearest-neighbor actions

These actions satisfy γ_5 -hermiticity, and the temporal direction is again the special one which is selected and which then breaks hypercubic symmetry

Same symmetries of the Karsten-Wilczek action: P is a symmetry but T and C separately are violated, unless the action is properly renormalized

So, the counterterms that must be added to these generalized actions are again the same needed for the standard Karsten-Wilczek action

The parameter α regulates the distance between the two zeros, which are at the same positions $a\bar{p}_1 = (0,0,0,0)$ and $a\bar{p}_2 = (0,0,0,-2\alpha)$ as in the nearest-neighbor actions

That there are only two zeros is certain if $-3 \leq \rho \leq 1$ and $-\pi/2 < \alpha < \pi/2$

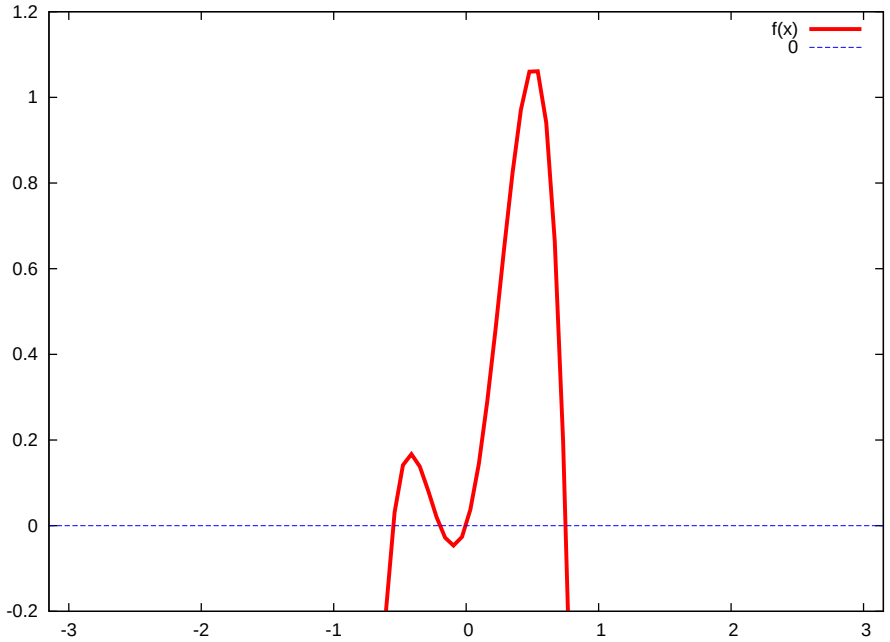
For choices of ρ outside of this range, additional zeros can in general appear, and one can still get minimally doubled actions but only for a restricted domain of α (whose extension depends on the value of ρ)

One must also take, to ensure that there are no more than two fermions,

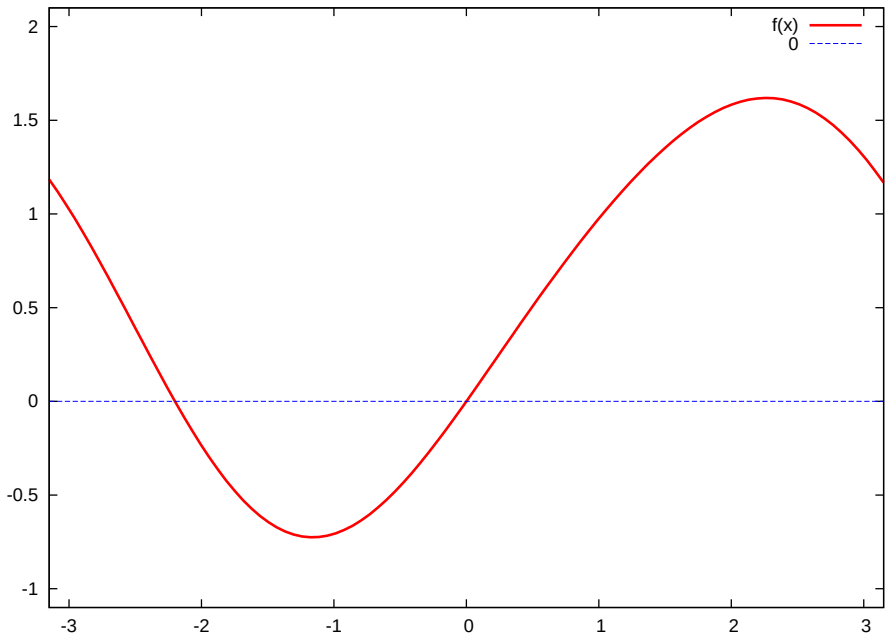
$$\lambda + 2\lambda' > -\min\left\{\sin x + \cot\alpha\left(\rho\left(1-\cos x\right) + \left(1-\rho\right)\left(1-\cos x\right)^2/(2\sin^2\alpha)\right)\right\}/2$$

Next-to-nearest-neighbor actions

Obtaining minimally doubled actions is not trivial: profile of the action *(proportional to γ_4)* vs. p_4 *(for $\vec{p} = (0,0,0)$)* in the case $(\alpha, \rho) = (0.1, 1.1)$



Everything's ok for $(\alpha, \rho) = (1.1, 0.4)$:



Next-to-nearest-neighbor actions

It is worth noting that the above actions in position space can also be written more concisely in the simple form

$$a^4 \sum_x \overline{\psi}(x) \left\{ \sum_\mu \left[\frac{1}{2}\,\gamma_\mu(\nabla_\mu+\nabla^\star_\mu) - ia\gamma_4 \left\{ \frac{1}{2}\, f^{(1)}_\mu\, \nabla^\star_\mu \nabla_\mu - f^{(2)}_\mu\, \widetilde{\nabla}^\star_\mu \widetilde{\nabla}_\mu \right\} \right] + m_0 \right\} \psi(x)$$

where in addition to the standard ∇_μ and $\nabla^\star_\mu$ one has also introduced another discretization for the lattice covariant derivative, extending this time over two lattice sites:

$$\widetilde{\nabla}_\mu\, \psi(x) \;=\; \frac{U_\mu(x)\, U_\mu(x+a\widehat{\mu})\, \psi(x+2a\widehat{\mu}) - \psi(x)}{2a}$$

$$\widetilde{\nabla}^\star_\mu\, \psi(x) \;=\; \frac{\psi(x) - U^\dagger_\mu(x-a\widehat{\mu})\, U^\dagger_\mu(x-2a\widehat{\mu})\, \psi(x-2a\widehat{\mu})}{2a}$$

Note that in this concise notation it is apparent that there is no mass term left if one sets $m_0 = 0$

This was also true for the nearest-neighbor actions

Terms like $i\overline{\psi}(x)\gamma_4\psi(x)/a$ are in fact part of the various Laplacians

Next-to-nearest-neighbor actions

Our primary motivation for introducing these next-to-nearest-neighbor actions is that for special choices of the parameters one could hit on renormalized actions which do not require any counterterms

Since there are 4 parameters, and not just 2 as in the nearest-neighbor case, there should be many more "curves" on which the counterterms become zero and, above all, more chances for intersections among these curves

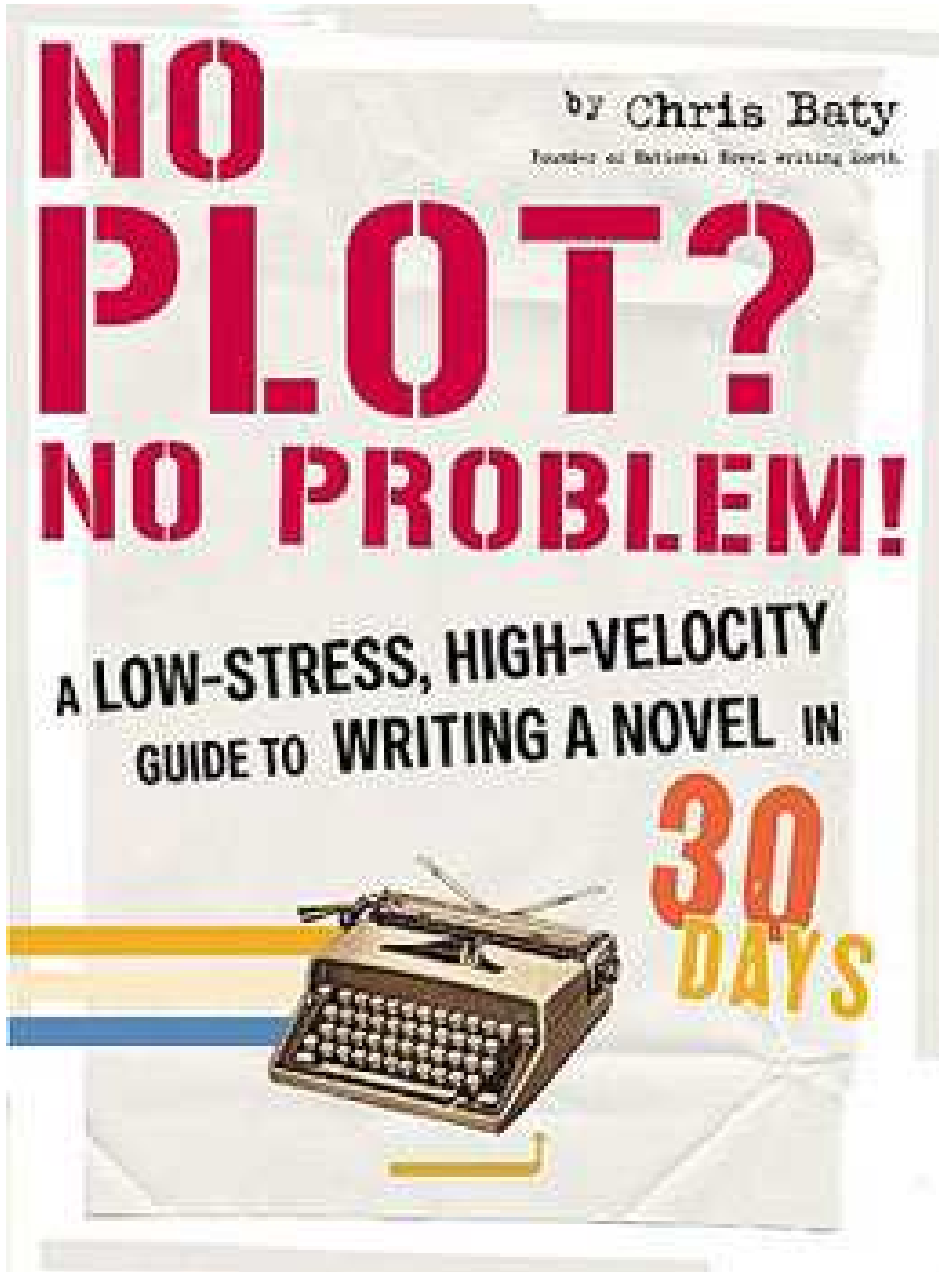
(Actually, the "curves" are likely to be 3-dimensional manifolds)

It could then happen that there are some values of the parameters for which one ends up with just one counterterm, or none at all

Of course to explore adequately this larger parameter space will be more expensive than for the nearest-neighbor actions

It is probably not too difficult to go one step further and construct minimally doubled fermions with hopping terms extending to 3 (or more) lattice spacings

This will enlarge even further the space in which to search for actions which do not require counterterms – although incrementing the range of the couplings renders such actions increasingly less convenient for simulations


NO PLOT? NO PROBLEM!
by Chris Baty
A LOW-STRESS, HIGH-VELOCITY GUIDE TO WRITING A NOVEL IN 30 DAYS

More minimally doubled actions?

It is possible that still cleverer minimally doubled actions can be constructed, which accomplish an even greater reduction of the number of counterterms

This could include the optimal situation where a maximal reduction can be accomplished, that is no counterterms at all are needed

In this case one will be able to obtain [consistent] physical results from Monte Carlo simulations using just the bare tree-level actions

Simulations of minimally doubled actions without counterterms will be cheaper than the cases in which one needs to add counterterms to the bare actions – and cheaper than the already convenient standard Karsten-Wilczek fermions

Thus, this work can also be considered as an inspiration to undertake further searches for new minimally doubled actions which possess a reduced number of counterterms, and possibly (in the best of cases) none at all

In any case, the next-to-nearest-neighbor actions (depending on 4 parameters) that we have introduced could also be taken as a starting point for a special direction in this undertaking

The Boriçi-Creutz action appears to be much more constrained in its form and to leave little room for changes

ENJOY YOUR STAY IN MAINZ !

ENJOY YOUR STAY IN MAINZ !

. . . AND HAVE A NICE TRIP BACK HOME . . .

BACKUP SLIDES

Boriçi-Creutz fermions

Boriçi and Creutz: fermionic action with the free Dirac operator (in momentum space)

$$D(p) = i \sum_\mu (\gamma_\mu \sin p_\mu + \gamma'_\mu \cos p_\mu) - 2i\Gamma + m_0$$

where

$$\Gamma = \frac{1}{2}(\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4) \qquad (\Gamma^2 = 1)$$

and

$$\gamma'_\mu = \Gamma \gamma_\mu \Gamma = \Gamma - \gamma_\mu$$

Useful relations:

$$\sum_\mu \gamma_\mu = \sum_\mu \gamma'_\mu = 2\Gamma, \quad \{\Gamma, \gamma_\mu\} = 1, \quad \{\Gamma, \gamma'_\mu\} = 1$$

The action vanishes at $p_1 = (0,0,0,0)$ and $p_2 = (\pi/2, \pi/2, \pi/2, \pi/2)$

A linear combination of two (physically equivalent) naive fermions , corresponding to the first two terms in the action

$\Gamma = \frac{1}{2}(\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4)$ selects a special direction $\rightarrow$ hypercubic breaking

Counterterms

Three counterterms required for massless Boriçi-Creutz fermions
(S. C., M. Creutz, J. Weber & H. Wittig (2010))

Dimension-4 fermionic counterterm: $c_4(g_0)\,\overline{\psi}\,\Gamma\sum_\mu D_\mu\psi$

Dimension-3 fermionic counterterm: $\dfrac{ic_3(g_0)}{a}\;\overline{\psi}(x)\,\Gamma\,\psi(x)$

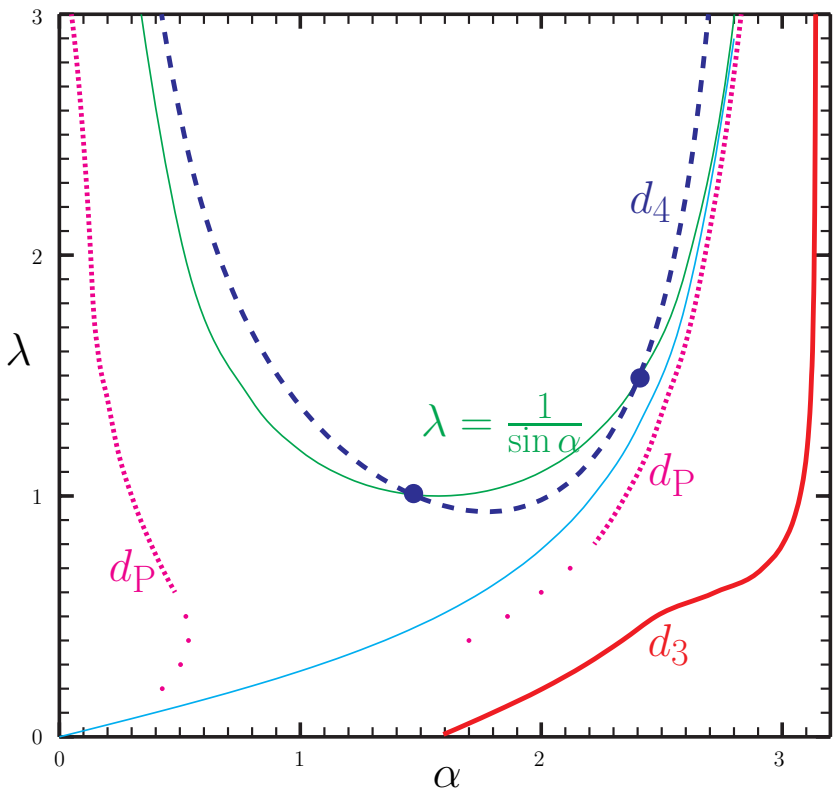
There are counterterms also for the pure gauge part

Although at the bare level the breaking of hypercubic symmetry is a feature of the fermionic actions only, in the renormalized theory it propagates (*via the interactions between quarks and gluons*) also to the pure gauge sector

Purely gluonic counterterm for the Boriçi-Creutz action:

$$c_P(g_0)\sum_{\lambda\rho\tau}\mathrm{tr}\,F_{\lambda\rho}(x)\,F_{\rho\tau}(x)$$

1-loop results



It is interesting that there are two points for which the curve $\lambda = 1/\sin\alpha$ intersects the curve of zeros of d_4

Then, the action proposed by Creutz, which in general requires 3 counterterms, needs only 2 two of them when either of the following two choices of α is made:

$$(\alpha, \lambda) = (1.47, 1.01)$$

or

$$(\alpha, \lambda) = (2.41, 1.49)$$

In both cases it is the fermionic counterterm of dimension 4 which is eliminated