Quantum Simulation of Non-Abelian Lattice Gauge Theories

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D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, U.-J. Wiese and P. Zoller,
Phys. Rev. Lett. **110** (2013) 125303 [arXiv:1211.2242].

Why and How to Quantum Simulate Gauge Theories?

- Address dynamical questions
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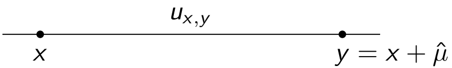
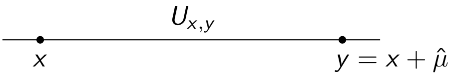
- The simple quantum link model shares features like
  - confinement
  - chiral symmetry breaking and restoration



Wilson Theory

- Infinite dimensional Hilbert space already on a single link
- Link $u \in SU(N), U(N)$
- Requiring gauge invariance of the Hamiltonian $[G_x^a, H] = [G_x, H] = 0$



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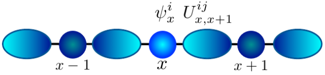
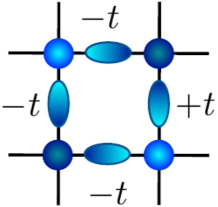
Quantum Link Models

- Finite dimensional Hilbert space
- Quantum link $U \sim$ matrix of operators, $\mathrm{Re}(U^{ij}), \mathrm{Im}(U^{ij}) \in su(2N)$
- Requiring gauge invariance of the Hamiltonian $[G_x^a, H] = [G_x, H] = 0$

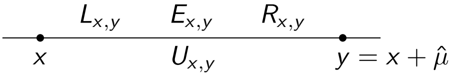
The Hamiltonian of a Simple $(1+1)$-d $U(N)$ Quantum Link Model with staggered fermions



$$H = -t \sum_x \left(\psi_x^{i\dagger} U_{x,x+1}^{ij} \psi_{x+1}^j + \text{h.c.} \right) + m \sum_x (-1)^x \psi_x^{i\dagger} \psi_x^i$$



Building blocks of a simple quantum link model



- $\psi_x^i, \psi_x^{i\dagger}$: Fermion operators
- Generators of $su(2N)$ algebra:
  - $U_{xy}^{ij}, U_{xy}^{ij\dagger}$: $2N^2$ generators
  - L_{xy}^a, R_{xy}^a: $2(N^2-1)$ generators
  - E_{xy}: 1 generator
  - Total: $4N^2 - 1$ generators
- $i \in \{1, \ldots, N\}$ – color index
- $a \in \{1, \ldots, N^2 - 1\}$

$$[E, U^{ij}] = U^{ij}, \quad [E, U^{ij\dagger}] = -U^{ij\dagger}, \quad [U^{ij}, U^{kl\dagger}] \neq 0,$$
$$[L^a, L^b] = 2if^{abc} L^c, \quad [R^a, R^b] = 2if^{abc} R^c, \quad [L^a, U^{ij\dagger}] = \lambda_{ik}^a U^{kj}, \ldots$$

Symmetries

- Local $SU(N)$ or $U(N)$ gauge symmetry
- Translation / charge conjugation / parity symmetry
- Global baryon number symmetry (in the $SU(N)$ case)
- $\mathbb{Z}_2$ chiral symmetry

Gauge Symmetry

Generators of the $SU(N)$ or $U(N)$ gauge transformations

$$\begin{aligned} G_x^a &= \psi_x^{i\dagger}\lambda_{ij}^a\psi_x^j + \sum_\mu \left(L^a_{x,x+\hat{\mu}} - R^a_{x-\hat{\mu},x}\right) \\ G_x &= \psi_x^{i\dagger}\psi_x^i - \sum_\mu \left(E_{x,x+\hat{\mu}} - E_{x-\hat{\mu},x}\right) \end{aligned}$$

λ^a generators of $su(N)$.

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Gauge transformation: $W = \prod_x \exp(i\alpha_x^a G_x^a)$. Transformation rules:

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U_{xy} &\rightarrow W\ U_{xy}\ W^\dagger = \exp(i\alpha_x^a \lambda^a)\ U_{xy}\ \exp(-i\alpha_y^a \lambda^a) \\
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Requiring physical states to be gauge invariant: $G_x^a|\psi_x\rangle = 0$
$\longrightarrow$ Reduction of the Hilbert space of quantum link models

Chiral Symmetry

The chiral symmetry is broken explicitly by the mass term in the Hamiltonian

$$m \sum_x (-1)^x \psi_x^{i\dagger} \psi_x^i$$

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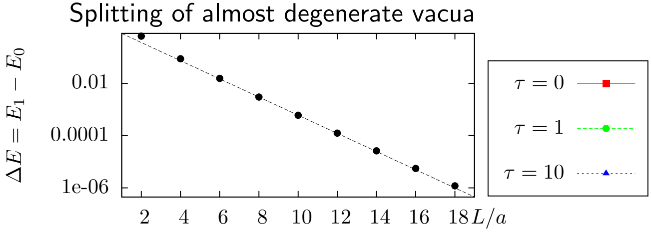
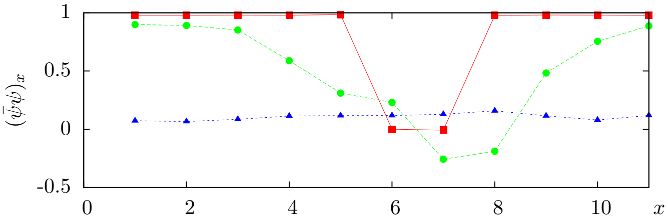
In $(1+1)$d with staggered fermions

- the chiral symmetry is a $\mathbb{Z}_2$ symmetry
- corresponds to a translation by 1 lattice spacing: ${}^{\chi}U^{ij}_{x,y} = U^{ij}_{x+1,y+1}$, ${}^{\chi}\psi^a_x = \psi^a_{x+1}, \dots$
- is broken spontaneously

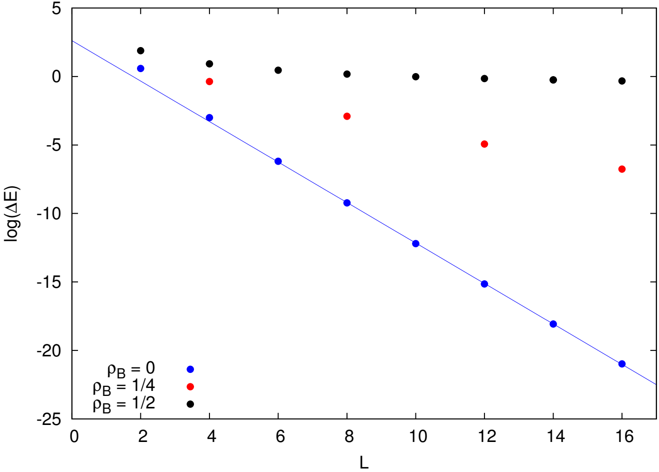
Real-time Evolution of a Chirally Symmetric Region





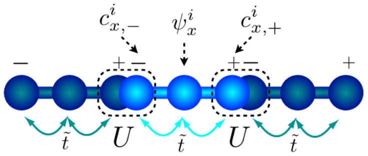
Chiral order parameter $(\bar{\psi}\psi)_x = (-1)^x \langle \psi_x^{i\dagger}\psi_x^i - \frac{N}{2} \rangle$ at different times τ

Chiral Symmetry Breaking and Restoration



Splitting of the almost degenerate vacua ΔE as a function of the system size L at different baryon densities ρ_B in the $SO(3)$ QLM

Rishons



There exists a rishon representation:

- Introduce two fermions per link $c^i_\pm, c^{i\dagger}_\pm$ ($\sim$ rishons)
- Rewrite gauge link: $U^{ij} = c^i_+ c^{j\dagger}_-$
- Fixed number of rishons per link $\mathcal{N} = c^{i\dagger}_- c^i_- + c^{i\dagger}_+ c^i_+$

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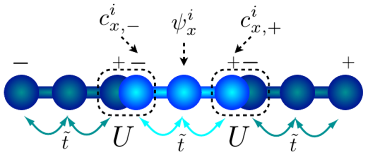

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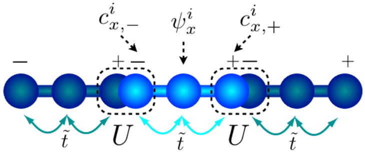
$$\begin{aligned} H &= -t \sum_x \left(\psi^{i\dagger}_x U^{ij}_{x,x+1} \psi^j_{x+1} + \text{h.c.} \right) + m \sum_x (-1)^x \psi^{i\dagger}_x \psi^i_x \\ &= -t \sum_x \left(\psi^{i\dagger}_x c^i_{x,+} \; c^{j\dagger}_{x+1,-} \psi^j_{x+1} + \text{h.c.} \right) + m \sum_x (-1)^x \psi^{i\dagger}_x \psi^i_x \end{aligned}$$

$\longrightarrow$ Hubbard like Hamiltonian, hopping of fermions

Microscopic atomic Hamiltonian



Encoding in an optical lattice:

- degrees of freedom $(c_\pm, \psi)$ are represented by alkaline earth atoms (e.g. ^{87}Sr or ^{173}Yb) in an optical lattice
- encode color in the Zeeman levels of the atoms (nuclear spin $I \Longrightarrow SU(2I+1)$ symmetry)
- interaction of fermions–rishons $\longleftrightarrow$ hopping on the optical lattice
- introducing the microscopic atomic Hamiltonian $(t = \tilde{t}^2/U)$:

$$\widetilde{H} = U\sum_x (\mathcal{N}_{x,x+1} - n)^2 - \tilde{t}\sum_x \left(\psi^{i\dagger}_x c^i_{x,+} + \psi^{j\dagger}_x c^j_{x,-} + \text{h.c.}\right)$$
$$+ m\sum_x (-1)^x \psi^{i\dagger}_x \psi^i_x$$

Other implementations: E. Zohar, J. I. Cirac and B. Reznik, Phys. Rev. Lett. **110** (2013) 125304

Conclusion and Outlook

- Quantum simulation construction of $U(N)$ or $SU(N)$ gauge theories in an optical lattice setup

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- Later: Simulation of baryon superfluidity, color superconductivity at high densities and "nuclear collisions"
- Long term goal: quantum simulate QCD (requires dimensional reduction of a $(4+1)$d quantum link model)
  - simulating at large baryon density
  - real time evolution, e.g. heavy-ion collision