Subtleties of Gauge Theories in Cold-Atomic Lattices

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1. **What kind of field theories can we simulate?**

There is new enthusiasm to build gauge magnets/quantum-link systems from laser-cooled atomic lattices. I'll discuss U(1) models, two states per link, and SU(2) X U(1) models, four states per link.

Taxonomy: **Brontosaurus** vs. **Apatosaurus**.

If these quantum systems can be realized, what do they really simulate?

Ideally we'd have a *Haldane argument* relating the Hamiltonian to a quantum field theory. The best scenario is that the latter is (or is like) QCD.

Integer spin chain $\rightarrow$ **gapped O(3) sigma model,** $\theta = 0$**,**
Half-integer s.c. $\rightarrow$ **ungapped O(3) sigma model,** $\theta = \pi$**.**

In B. Reznik's talk and the remainder of this parallel session, you can hear much more about simulation strategies.

Abelian models have relativistic magnons $\rightarrow$ compact QED. Simple SU(2) X U(1) model has non-relativistic magnons. Not Yang-Mills theory, *unless* quantum fluctuations locally disrupt the vacuum. This seems unlikely. :-(Spin-wave methods generally fail in the IR, but work in the UV. An important question is how breaking parity on links changes this with γ^5 (which makes spin-wave analyses harder).

The rest of the talk is about...

2. Ancient history
3. Origins of gauge invariance
4. Confinement
5. Spin waves
6. Some open problems

2. Here follows a **history of ideas about gauge-invariant magnetic systems,** because I feel like Bjarne Herjulfson glimpsing Vinland's coast, while remaining at sea.

David Horn first wrote down examples of Abelian and non-Abelian models. He informed Daniel Rohrlich, after Daniel told him (during lunch) about the paper we were writing. We also learned Banks and Zaks had studied the phase diagram.

Rokhsar, Kivelson, Fradkin and others realized that quantum dimer models of RVB wave functions have Abelian gauge symmetry. Quantum dimer models are Horn's U(1)-symmetric model, but with a matter field at sites.

Anderson discovered an SU(2) gauge invariance in the half-filled Hubbard model/antiferromagnet (for which the excitations are not gauge Bosons).

In 1990, neither Daniel nor I noticed the U(1) gauge invariance in the non-Abelian models. The gluon spin waves are non-relativistic in the simplest model. Daniel discovered the existence of some strange spin waves which acted like those of the Abelian model (but did not write it up).

We did find parity-violating non-Abelian models (without γ^5 terms) with relativistic spin waves.

We wanted confined/screened spin $\rightarrow$ electronic superconductivity! Hopping spins produce this kind of gauge theory, just as the half-filled Hubbard model produces AF Mott insulators (more on this later).

Sorry if I bored you with this. Now some physics...

3. Origins of gauge invariance

Schemes for making these systems are a significant part of this subject:

L. Tagliacozzo, A. Celi, A. Zamora and M. Lewenstein, Ann. of Phys. (2013) **330**, pp. 160-191, **arXiv:1205.0496** (U(1) gauge theories).

L. Tagliacozzo, A. Celi, M. Lewenstein and me, **arXiv:1211.2704** (SU(2) gauge theories).

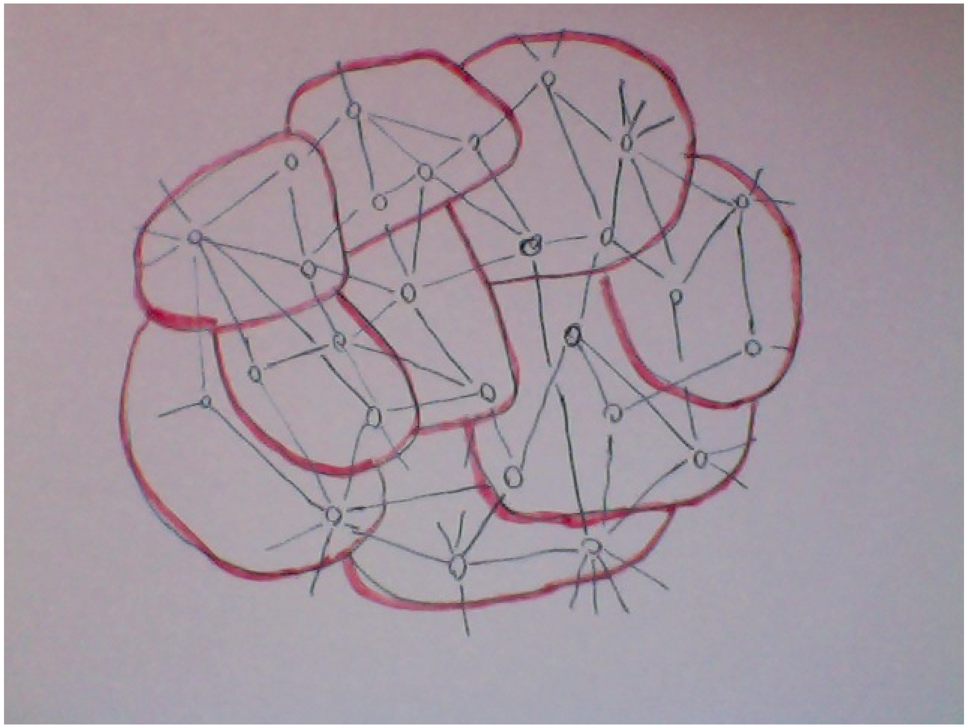
These use mesoscopic Rydberg gates:

M. Müller, I. Lesanovsky, H. Weimer, H. P. Büchler, and P. Zoller, Phys. Rev. Lett. **102**, 170502 (2009).

H. Weimer, M. Müller, I. Lesanovsky, P. Zoller, and H. P. Büchler, Nat. Phys. **6**, 382 (2010).

On the next few pages is another approach.

Consider the lattice of sites $i, j, \ldots$. Particles, with quantum numbers (spin or color) $S^a_j = c^\dagger_j S^a c_j$ can sit or hop. Sites are in bags/femto-universes (Bjorken), $F, G, \ldots$.



$H = -t\sum_{\langle i,j\rangle} c^\dagger_i c_j + U\sum_F V_F$, where the g. s. of V_F is a singlet and the first excited states are a multiplet.

Hopping-parameter expansion

Theorem: The effective Hamiltonian obtained in perturbation theory in t, is a gauge theory, where the Gauss law operator is

$$G_F^a = \sum_{j\in F} S_j^a - P_F \left(\sum_{j\in F} S_j^a \right) P_F \sim (\vec{D}\cdot\vec{E})^a - \rho^a,$$

where P_F projects onto the multiplet of f. e. states of F. This operator annihilates the lowest energy states of the effective theory. The excited state is a color charge. Note: Gauss' law is tautological (it is true by definition). The effective Hamiltonian will have terms of order t^m/U^{m-1} on polygons (plaquettes) of with m sides. A gauge-invariant matter coupling appears at order t^2/U.

Sometimes the effective theory is gauge-invariant by virtue of having no dynamics. Alesio Celi, Luca Taglicozzo and I found an example which had this unfortunate feature. So start from the right place.

An old example of this idea is a model of links with two vacancies, 8 sites around a plaquette, as done in :
SU(2) gauge invariance in Hubbard models and superconductivity, NBI-HE-90-29, June 1990, which can be found at the KEK link:

http://ccdb5fs.kek.jp/cgi-bin/img/allpdf?200031897

reissued as **arXiv:1207.0455**, with a few typos fixed and some preliminary remarks.

The figures are on the next page:

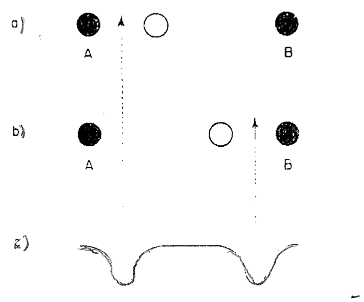


Fig. 1

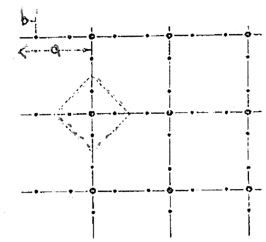


Fig. 2

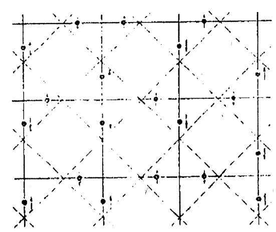


Fig. 3

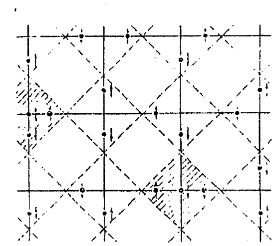
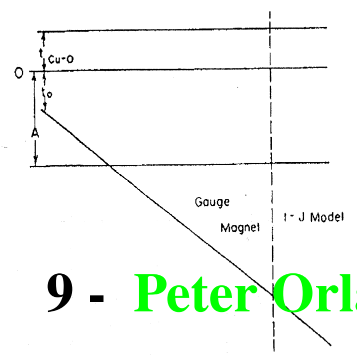


Fig. 4



9 - Peter Orland, Lattice 2013

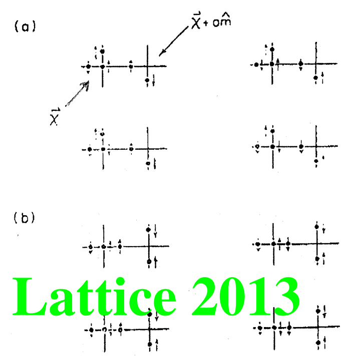


Fig. 5

Fig. 6

3. Hamiltonians (not in full generality)

U(1) **gauge theory:** A 2×2 matrix-valued lattice link operator

$$U_j(x)_{bc} = \frac{1}{2}[\sigma^x_j(x)\tau^x_{bc} + \sigma^y_j(x)\tau^y_{bc}] = \sum_{\pm}\sigma^{\pm}_j(x)\tau^{\pm}_{bc}, \text{ or } U_j(x) = \sum_{\pm}\sigma^{\pm}_j(x)\otimes\tau^{\pm}$$

joins site x to site $x+\hat{j}a$, where j is the link direction, a is the lattice spacing, and both $\sigma^{x,y,z}$ and $\tau^{x,y,z}$ are Pauli matrices.

$$\begin{aligned} H &= \sum_{x,\, j\neq k} \mathrm{Tr}\, U_j(x)U_k(x+\hat{j}a)U_j(x+\hat{k}a)^{\dagger}U_k(x)^{\dagger} \\ &= \sum_{x,\, j\neq k} \sigma^{\pm}_j(x)\sigma^{\mp}_k(x+\hat{j}a)\sigma^{\pm}_j(x+\hat{k}a)\sigma^{\mp}_k(x). \end{aligned}$$

The trace is on the τ's. **Gauss' law operator or ice/6-vertex rule:**

$$G(x) = \sum_{j=1}^{d}[\sigma^z_j(x) - \sigma^z_j(x-\hat{j}a)], \quad d = \text{ lattice dimension.}$$

$\mathrm{SU}(2)\times\mathrm{U}(1)$ **gauge theory:** 2×2 **matrix of 4-d operators**

$$V_j(x) = U_j(x) + \alpha_j(x) U^5_j(x), \text{ where } \alpha_j(x)\in\mathbb{C}$$
$$U_j(x) = \gamma^0_j(x)\otimes 1\!\!1 - \mathrm{i}\vec{\gamma}_j(x)\cdot\otimes\vec{\tau},\ \ U^5_j(x) = \rho^0_j(x)\otimes 1\!\!1 - \mathrm{i}\vec{\rho}_j(x)\cdot\otimes\vec{\tau},$$

and 4×4 operators on (x,j) are $[\gamma^\mu,\gamma^\nu]_+ = 2\delta^{\mu\nu}$, $\gamma^5 = -\mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$, $\rho^\mu = -\mathrm{i}\gamma^5\gamma^\mu$. Local spin operators: $\sigma^{\mu\nu} = -\frac{\mathrm{i}}{4}[\gamma^\mu,\gamma^\nu]_-$, $\Sigma^{b\,\pm} = \frac{1}{4}\varepsilon^{bcf}\sigma^{cf}\pm\sigma^{0b}$. Hamiltonian and Gauss' law ops, respectively:

$$H = J\sum_{x,\ j\neq k}\mathrm{Tr}\, V_j(x)V_k(x+\hat{j}a)V_j(x+\hat{k}a)^\dagger V_k(x)^\dagger \pm K\sum_{x,j}\gamma^5_j(x),$$

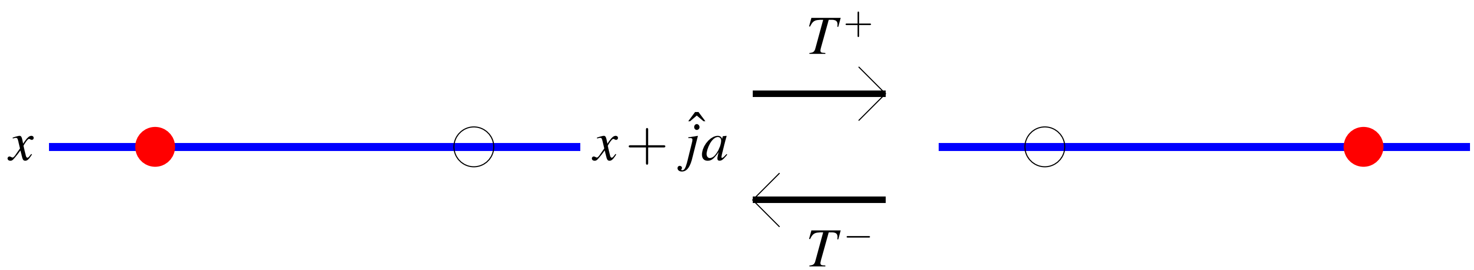
$$G^b(x) = \sum_{j=1}^{d}\left[\Sigma^{b\,+}_j(x) - \Sigma^{b\,-}_j(x-\hat{j}a)\right],\ \mathrm{SU}(2),$$

$$G^5(x) = \sum_{j=1}^{d}\left[\gamma^5_j(x) - \gamma^5_j(x-\hat{j}a)\right],\ \mathrm{U}(1).$$

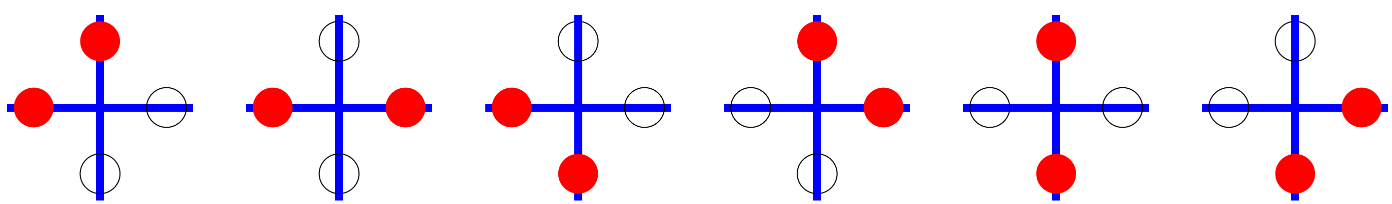
A particular representation of the operators on links can be realized by writing:

$$\gamma^0 = (T^+ + T^-) \otimes \mathbb{1}\,, \quad \vec{\gamma} = \mathrm{i}(T^+ - T^-) \otimes 2\vec{S},$$

where $\vec{S}$ is the spin of a particle which can fill one of two vacancies on a link, and $T^{\pm}$ moves the particle between the two vacancies:



U(1) gauge inv. (ice rule):



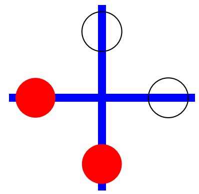
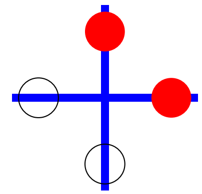
SU(2) Gauss law is the the statement that the spins of the particles neighboring a vertex are in a singlet state. We have an ice rule again (with spin).

Allowing four or zero spins near a site weakens the ice rule to allow for eight vertices, breaking the U(1) gauge invariance to a $\mathbb{Z}_2$ gauge invariance.

Color sources are introduced by allowing the spin states to be non-singlets. Electric U(1) sources are introduced by changing the ice rule to allow different numbers of particles near a vertex. But quarks and antiquarks have electric charge, as well as color.

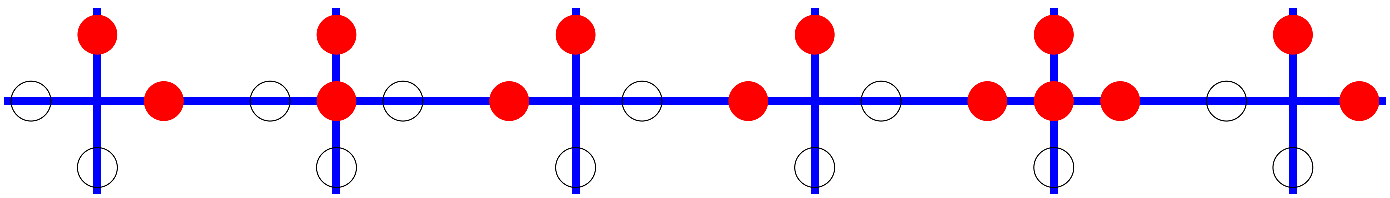
4. Confinement

A. First example. Keep $\pm K \sum_{x,j} \gamma_j^5(x)$, $K \gg J$. Favors one of the vertices:

$\pm = -$  $\pm = +$ 

It is naive to interpret this as an E-squared term.

This breaks (link) parity. If $K \gg J$, vacuum is frozen, a large gap of order K. A $q\bar{q}$-pair produces a line of defects:

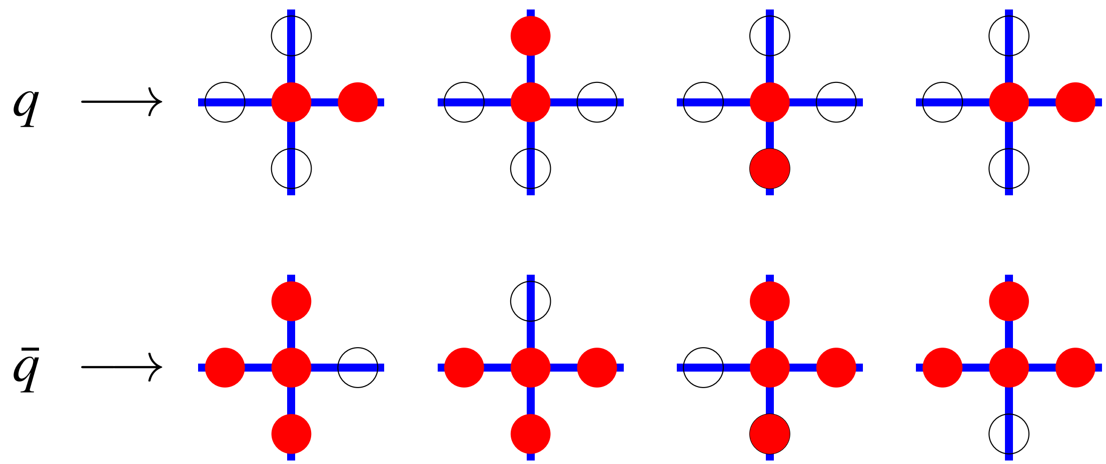


Defect string (here of length $3a$), with string tension K/a.

B. Second example. Set $K=0$ and $\alpha_j(x)=0$:

$$H = J \sum_{x,\, j\neq k} \mathrm{Tr}\, U_j(x) U_k(x+\hat{j}a) U_j(x+\hat{k}a)^{\dagger} U_k(x)^{\dagger}.$$

Also confines! Quark or antiquark on a vertex modifies configs. to:



Except for spins, same as U(1). A $q\bar{q}$-pair produces a string or cuts a piece of string in a background of transversely oscillating hard strings (me, early 1990's, but with wrong wave functions).

Confinement in both ranges of parameters is similar to the U(1) gauge theory with a term proportional to $\sigma_j^z(x)$. All I really showed you was electric charge confinement. Maybe someone in the audience has thought about this more.

Color is confined, however, as in the confinement phase of the Georgi-Glashow model.

Conclusion: The Banks-Zaks transition in K/J (if it exists) is not a deconfining transition.

T. Banks and A. Zaks, Nucl. Phys. **B196** (1982) 189.

5. Spin waves (all done in the 1990 paper with Rohrlich).

A. $K = 0, \alpha_j(x) = 0$. 2nd example already discussed (confines).

Generalize $s = 1/2$ to large s. Action formulation: $\gamma^\mu \to n^\mu$, $\rho^\mu \to m^\mu$, with $n^2 = m^2 = 1$, $m \cdot n = 0$, $\Sigma^{b\,\pm} \to N^{\,b\pm} = n^0 m^b - m^0 n^b \pm \varepsilon^{abc} n^b m^c$. Gauge field is $U_j(x) = n^0 \otimes \mathbb{1} - \mathrm{i}\vec{n} \cdot \otimes \vec{\tau}$.

$$S = \sum_{x,j} s \int dt \int_0^\infty du\ \varepsilon_{\alpha\beta\mu\nu}\, n_j^\alpha m_j^\beta \left(\frac{\partial n_j^\mu}{\partial t} \frac{\partial n_j^\nu}{\partial u} + \frac{\partial m_j^\mu}{\partial t} \frac{\partial m_j^\nu}{\partial u} \right) \quad \text{(a WZNW term)}$$

$$-J \sum_{x,\, j \neq k} \mathrm{Tr}\, U_j(x,t) U_k(x + \hat{j}a, t) U_j(x + \hat{k}a, t)^\dagger U_k(x,t)^\dagger$$

$$-\sum_x A_0^b(x,t) \sum_{j=1}^{d} [N_j^{b\,+}(x,t) - N_j^{b\,-}(x - \hat{j}a, t)].$$

Non-relativistic spin waves: $|E| = 4Jp^2$. SSB of time-reversal symmetry, like ferromagnets. Kinetic term is first-order in $\partial/\partial t$.

Embarrassing that $\sum_{x,i} K\gamma^5(x,i)$ can't be implemented in the semiclassical SO(4) formalism. Related to the fact that both $\vec{N}^{\pm}$ are unit vectors. An SU(4) formalism can deal with γ^5, but then γ^μ can't be thought of as a unit vector.

B. A model with relativistic spin waves. In d=2+1

$$H = J_1 \sum_{x^1+x^2 \text{ even}} \mathrm{Tr}\, UUUU + J_2 \sum_{x^1+x^2 \text{ odd}} \mathrm{Tr}\, U^5U^5U^5U^5,$$

There is an (obvious) version in d=3+1 too.

This has spin waves similar to those of the 1-dim. chain (Aharonov-Casher-Susskind):

$$H_{ACS} = J_1 \sum_{x \text{ even}} \gamma(x)\cdot\gamma(x+1) + J_2 \sum_{x \text{ odd}} \rho(x)\cdot\rho(x+1).$$

This describes relativistic Fermions, with speed of light and mass gap,

$$c = 8\sqrt{|J_1 J_2|},\;\; m = \frac{|J_1 - J_2|}{8|J_1 J_2|},\;\; \text{respectively.}$$

6. Problems I am trying to solve (in no particular order):

A. **Find a better approach to spin waves. Find a Holstein-Primakoff rep. (write the operators in terms of harmonic-oscillator a's and $a^\dagger$'s).**

B. HALDANE! **The non-Abelian model with relativistic spin waves can be rewritten as $1+1$-dim left- and right-handed gauged $\mathrm{SU}_1(2)$ WZNW models coupled together. I think this can be used to find an effective continuum action. Suspicion: Chern-Simons in d=2+1, $\theta=\pi$ in d=3+1.**

C. **There should be Rokhsar-Kivelson points, where non-Abelian models are critical. This would give some insight into the effective QFT's, which is ESSENTIAL.**

D. **Confinement in SU(N) theories without U(1) invariance.**

E. Make the entire standard model SU(3) X SU(2) X U(1) from hopping particles. Probably isn't how nature does it. Different excitations will have different relations between momentum and energy, *e.g.*, different speeds of light. But fun!

I once read something interesting about professional tennis:
You must be smart enough to play it, but just dumb enough to believe it matters.

In the context of a reductionist theory of nature, the corresponding aphorism is:
You must be smart enough to investigate it, but not dumb enough to believe it.

THANK YOU!