Magnetic-field induced (inverse) catalysis for gluons through an improved interaction measure

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Magnetic fields and Quantum Chromodynamics

- early universe $\sqrt{eB} \simeq 2$ GeV
- RHIC/LHC 0.1..0.5 GeV QCD scale!
  non-central collisions
  charged spectators
  B perp. to reaction plane
- neutron stars, magnetars 1 MeV $B \simeq 10^{14}$ G

Magnetic fields and Quantum Chromodynamics

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- RHIC/LHC non-central collisions charged spectators B perp. to reaction plane	0.1 .. 0.5 GeV	QCD scale!
- neutron stars, magnetars	1 MeV	$B \simeq 10^{14}$ G
◦ cf. strongest field in lab		10^5 G (10^7 G unstable)
◦ refrigerator magnet		100 G
◦ earths magn. field		0.6 G

magn. fields as probes for our understanding of nonperturbative QCD

Setting

- quarks charged: $(q_u, q_d, q_s) = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})e$
  gluons neutral: indirect effects via strong coupling
- constant external magnetic field in equilibrium QCD (Euclidean)
  = idealized situation
- anisotropic $\Rightarrow$ talk by G. Endrődi
- free quarks: Landau orbits with min. energy zero and Landau '30
  degeneracy $\propto$ magn. flux: B induces many small eigenvalues

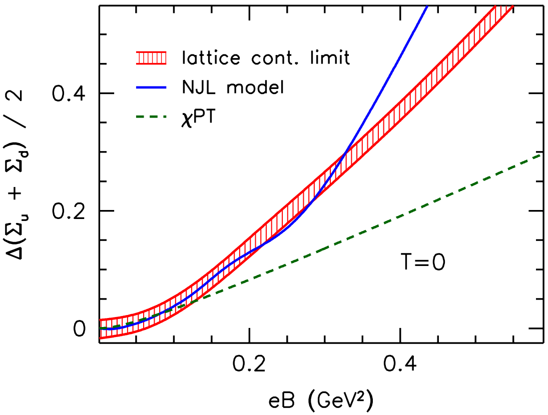
lattice:

- magnetic fields quantized and bounded, no sign problem
  state-of-the-art: $\sqrt{eB} = 0.1 \ldots 1$ GeV
- 2+1 staggered quarks: physical masses in continuum limit ▸ details
  $\Rightarrow$ new phenomena in contrast to other lattice simulations

D'Elia et al. '10, Ilgenfritz et al. '12

Magnetic catalysis of quarks

- change of light quark condensate with B (renormalized): Bali, FB et al. '12



magnetic catalyis: $\langle\bar{\psi}\psi\rangle(B) \nearrow$

Müller, Schramm[2] '92
Gusynin, Miransky, Shovkovy '96

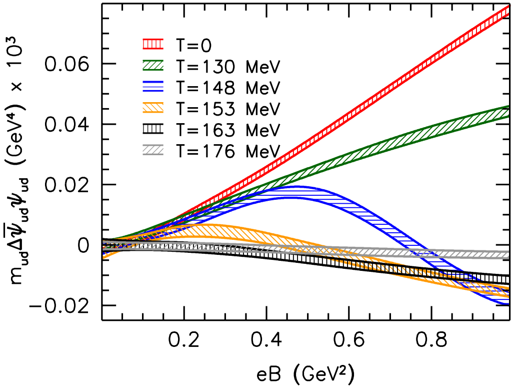
comparison to χPT and NJL

Cohen, McGady, Werbos '07; Andersen '12
Gatto, Ruggieri '10

robust effect: relying on Landau level degeneracy

Inverse magnetic catalysis of quarks

- change of light quark condensate with B at $T \simeq T_c$: Bali, FB et al. '12



non-monotonic ⇒ magn. catalysis turns into inverse magn. catalysis

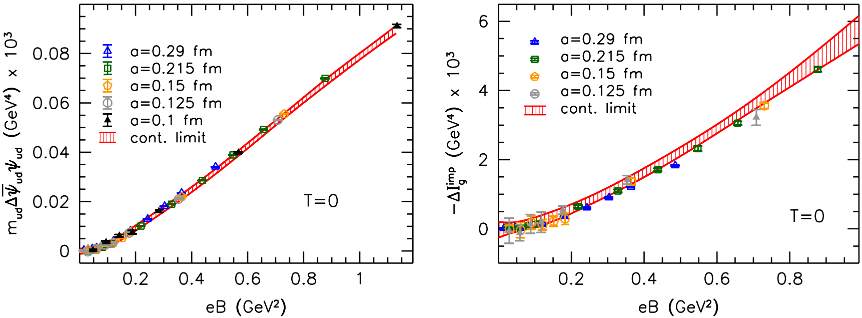
feedback of light quarks ⇒ talk by T. Kovács

consequence: $T_c(B)$ decreases ▸ details

missed in almost all non-lattice approaches

Magnetic catalysis of gluons

- change of condensate and gluonic action at $T = 0$: Bali, FB et al. '13



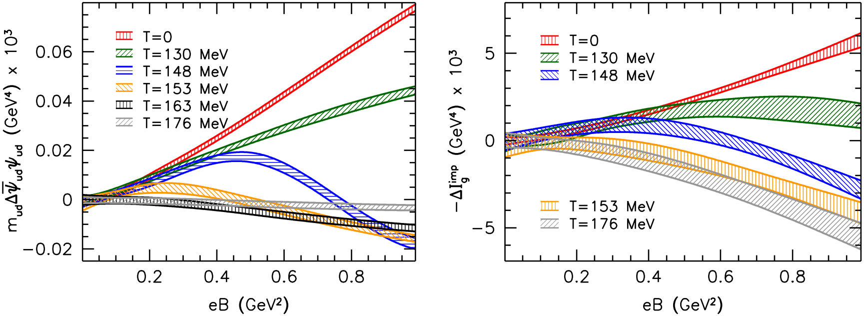
⇒ gluons inherit magnetic catalysis from quarks

since strongly coupled

magnitude $\mathcal{O}(100)$ larger for gluons, but $B = 0$ scale (= gluon condensate) already $\mathcal{O}(200)$ larger: relative effect larger on quarks

Inverse magnetic catalysis of gluons

- change of condensate and gluonic action, at $T \simeq T_c$: Bali, FB et al. '13



non-monotonic behaviour, similar shape for quarks and gluons

⇒ gluons inherit inverse magnetic catalysis from quarks, too

Details on the observable

choose your favorite representation and name:

$I = \epsilon - 3p$ ... interaction measure (free gas: $\epsilon = 3p$)

$= \langle \theta^\mu_\mu \rangle$ ... trace anomaly

energy-momentum-tensor: $\theta_{\mu\nu} = \frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} - 2g_{\mu\nu}\mathcal{L},\ \theta^\mu_\mu = 4\mathcal{L} - \frac{2\partial\mathcal{L}}{\partial \log g^{\mu\nu}}$

$= T^5 \frac{\partial}{\partial T} \frac{p}{T^4}$ (Stefan-Boltzmann: $\frac{p}{T^4} = \text{const.}$)

$= -\frac{T}{V} \sum_\mu \frac{\partial \log Z}{\partial \log L_\mu}$ ... scale anomaly

$\overset{\text{lattice}}{=} -\frac{T}{V} \frac{d \log Z}{d \log a}$

interaction measure (one flavor for simplicity):

$$-I = \frac{T}{V} \frac{d}{d \log a} \log Z(\beta_g; am) \qquad \beta_g = \frac{6}{g^2}$$

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$$= \langle s_g\rangle \underbrace{R_\beta}_{\frac{-6}{g^3}\beta_{\rm lat} = \beta_0 + \beta_1 g^2} + m\,\langle\bar\psi\psi\rangle \underbrace{(1+\gamma_{\rm lat})}_{1+\mathcal{O}(g^2)} \qquad \mathcal{O}(g^2) = \mathcal{O}\left(\frac{1}{\log a}\right)$$

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- changes $\Delta I \equiv I(B) - I(0)$ and $m\,\Delta\langle\bar\psi\psi\rangle$ are finite (add. div. removed) with lattice artifacts $O(a^2)$ $\Rightarrow$

unimproved

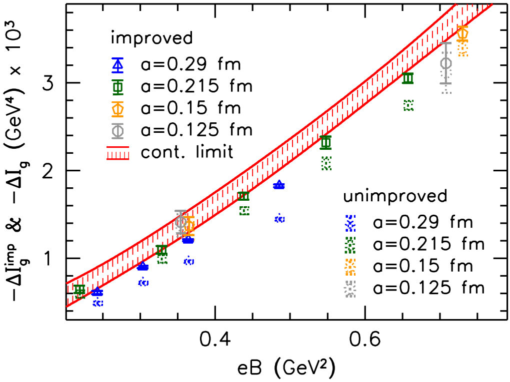
$$-\Delta I = \overbrace{\Delta\langle s_g\rangle R_\beta}^{\text{finite}+\mathcal{O}\left(\frac{1}{\log a}\right)} + \overbrace{m\Delta\langle\bar\psi\psi\rangle\gamma_{\rm lat}}^{\text{finite}+\mathcal{O}\left(\frac{1}{\log a}\right)} + m\Delta\langle\bar\psi\psi\rangle$$

improved

$$\underbrace{\Delta\langle s_g\rangle R_\beta + m\Delta\langle\bar\psi\psi\rangle\gamma_{\rm lat}}_{\text{finite}+\mathcal{O}(a^2)} \quad \underbrace{m\Delta\langle\bar\psi\psi\rangle}_{\text{finite}+\mathcal{O}(a^2)}$$

Effect of improvement

$\Delta I_g \to \Delta I_g^{\text{imp}}$ incl. quark contribution:



better continuum limit: $\mathcal{O}\left(\frac{1}{\log a}\right) \to \mathcal{O}(a^2)$

I_g^{imp} perturbatively RG-scale invariant

Tarrach '81; Grinstein, Randall '89

Summary

- gluons inherit from the quarks:
    - magnetic catalysis at $T = 0$
    - inverse magnetic catalysis at $T \simeq T_c$
      in part. non-monotonic behavior ($\risingdotseq$ decrease of T_c)
- interaction measure:
    - gluonic action density (with β_{lat}) $+$ condensate (with γ_{lat})
      $\Rightarrow$ improved scaling with a
    - gluonic and quark (inverse) magnetic catalysis add up in I

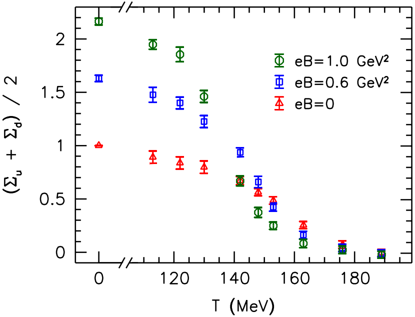
Backup: Simulation details

as for transition studies at $B = 0$ — Budapest-Wuppertal

- tree-level improved gauge action
- stout smeared staggered fermions, rooting trick
- 2 light quarks + strange quark, charges $(q_u, q_d, q_s) = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})e$
- lattice spacing set at $T = 0, B = 0$
  physical pion masses
  set by f_K, f_K/m_π and f_K/m_K
- $T = 0$: $24^3 \times 32$, $32^3 \times 48$ and $40^3 \times 48$ lattices
- $T > 0$: $N_t = 6, 8, 10$ meaning $a = 0.2, 0.15, 0.12$ fm
  $N_s = 16, 24, 32$ for finite volumes
- condensates from stochastic estimator method with 40 vectors
- magn. flux quanta: $N_B \leq 70 < \frac{N_x N_y}{4} = 144$

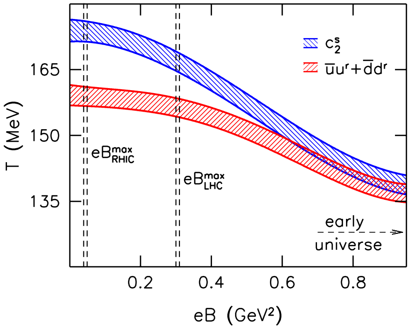
QCD phase diagram

- condensate as a function of *T* for different *B*'s:



inflection points

$\Rightarrow$ phase diagram with *B*:



$\Rightarrow$ T_c decreases by O(10) MeV

relevant for LHC??