The quark-gluon plasma in an external magnetic field

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July 31, 2013 Session 5B

Motivation

- The quark gluon plasma can appear together with strong (electromagnetic) magnetic fields $O(10^{14-15})$ T
- (Units $0.2\ \mathrm{GeV}^2/e \approx 3 \times 10^{15}$ T.)
  - LHC/RHIC: noncentral heavy ion collision
  - Early universe
- Would it modify T_c, the order of the transition, the EOS, etc.?

Previous studies

- Work so far has focused on chiral properties, *e.g.* $\langle\bar{\psi}\psi\rangle$ and the effect on T_c, and required generating ensembles with an external B field.
  - D'Elia *et al.* [arXiv:1005.5365,1103.2080]
  - See also plenary talk by Szabó.
  - Bali *et al.* [arXiv:1111.4956,1111.5155,1301.5826] , take continuum limit.
  - Ilgenfritz *et al.* [arXiv:1203.3360]
- Nice study of the equation of state in the hadron resonance gas model.
  - Endrödi [arXiv:1301.1307]
- Our exploratory study aims to calculate the EOS in lattice QCD.

Taylor expansion

- Taylor expansion of the thermodynamic potential to get pressure:
- Eliminates the need to generate ensembles with different B.

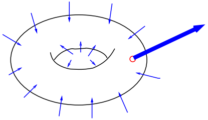
$$\frac{p(T)}{T^4} = \frac{\ln Z(B)}{T^3 V} = \sum_{n=0}^{\infty} C_n(T)(|e|B/T^2)^n$$
$$C_n(T) = \frac{L_t^3}{L_s^3}\frac{1}{n!}\left.\frac{\partial^n \ln Z(B)}{\partial(|e|B/T^2)^n}\right|_{B=0},$$

- Only even terms are nonzero? (yes, CP symmetry)
- Need $T = 0$ subtraction? (partly, for renormalization – see later)
- Convergence? D'Elia *et al.* [arXiv:1005.5365,1103.2080] calculate $\langle\bar{\psi}\psi\rangle$: for $eB \lesssim 0.7$ GeV^2; the $O(B^4)$ correction is small (coarse lattice, heavy quarks).

Magnetic field on a lattice torus - conventional approach

- Take $\vec{B} = B\hat{z}$ constant. Torus quantization (quark charge $|q| = |e|/3$):

$$|q|B = 2\pi b/(L_x L_y a^2), \qquad 0 < b < L_x L_y/2.$$



- Continuum vector potential, for example

$$A_y = Bx, \qquad A_\mu = 0 \quad \text{for} \quad \mu = x, z, t.$$

- Lattice $U(1)$ links choice.

$$u_y(B, q, X) = e^{ia^2 qBx}$$

$$u_{x,z,t}(B, q, X) = \begin{cases} 1 & \text{for } x \in [0, L_x - 2] \\ e^{-ia^2 qBL_x y} & \text{for } x = L_x - 1 \end{cases}$$

- But quantization is unsuitable for a Taylor expansion.

Half-and-half solution

- Half the lattice, positive flux, half negative.

$$B_z(x) = \begin{cases} +B & \text{for } x < L_x/2 \\ -B & \text{for } x \geq L_x/2 \end{cases}$$

- In effect, an extra boundary condition. Should be OK for large volumes.
- Must check for finite volume effects. Probably $O(1/L)$.

The Dirac operator with magnetic field

- Partition function $2+1$ flavors

$$Z(B) = \int dU e^{-S_g} e^{\frac{1}{4}\ln\det M_u(B,q_u)}\, e^{\frac{1}{4}\ln\det M_d(B,q_d)}\, e^{\frac{1}{4}\ln\det M_s(B,q_s)}.$$

- HISQ/asqtad fermion matrix for flavor f:

$$M^f_{X,Y}(B,q_f) = am_f\delta_{X,Y} + D^{z,t,x}_{X,Y} + D^{y}_{X,Y}(B,q_f),$$

  where the B-independent term $D^{z,t,x}_{X,Y}$ is a sum of the Dirac operators in the x, z and t directions at all points.

- The third term includes the $U(1)$ field ($x' = x - L_x/4$ for $x \le L_x/2$, and $x' = 3L_x/4 - x$ when $x > L_x/2$)

$$\begin{aligned} D^{\hat{y}}_{X,Y}(B,q_f) &= \frac{1}{2}\eta_y(X)\Big[U^{(F)}_y(X)\mathrm{e}^{iq_f a^2 B x'}\delta_{X+\hat{y},Y} \\ &\quad + U^{(L)}_y(X)\mathrm{e}^{3iq_f a^2 B x'}\delta_{X+3\hat{y},Y} - h.c.\Big]. \end{aligned}$$

Some calculational details

- We need derivatives
  $\partial^n \ln \det M_f / (\partial a^2 B)^n$
  $\partial^n \mathrm{Tr} M_f^{-1} / (\partial a^2 B)^n$ (for the interaction measure)
- They are computed in terms of derivatives of the fermion matrix:

$$\left. \frac{\partial^n M^f(B, q_f)}{\partial a^2 B^n} \right|_{B=0} = \frac{1}{2} \eta_y(X) \Big[(i q_f x')^n U_y^{(F)}(X) \delta_{X+\hat{y},Y} + (3 i q_f x')^n U_y^{(L)}(X) \delta_{X+3\hat{y},Y} - h.c. \Big] .$$

- Traces of these terms are calculated using stochastic estimators.
- Taylor coefficients are assembled "off line."

Some technical details

- Polyakov loop choice $A_y = Bx$ implies $\int A_y dy = BL_y x$ along a loop at fixed x that closes at the y boundary. So it is gauge invariant. Other choices, such as $A_y = B(x - x_0)$ give a different Polyakov loop but same B field. This is a boundary condition that leads to potential finite volume effects. True in the continuum also.
- Renormalization of the field (vacuum). Endrödi: [arXiv:1301.1307]. The vacuum pressure depends on B. The zero temperature (vacuum) $O(B^2)$ term (divergent when $a \to 0$) renormalizes the electric charge, so we need at least to subtract it from the divergent nonzero temperature $O(B^2)$ term. We go further and just calculate the thermal contribution to the pressure change, which removes the vacuum pressure entirely.

$$\begin{aligned} \Delta p(B,T) &= p(B,T) - p(0,T) - p(B,0) + p(0,0) \\ &= C_2^r(T)(eB)^2 + C_4^r(T)(eB)^4/T^4 + \dots \end{aligned}$$

where $C_n^r(T) = C_n(T) - C_n(0)$.

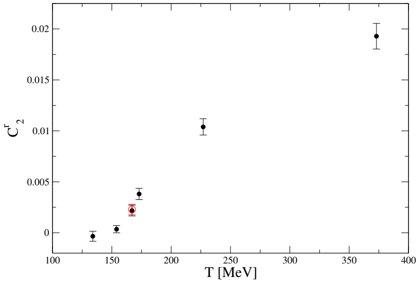
Exploratory calculation

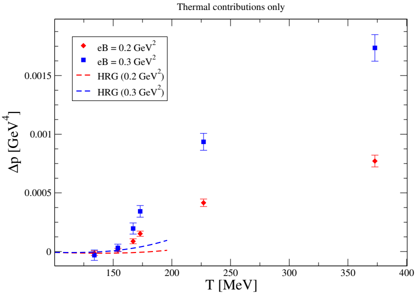
- $2+1$ flavor HISQ plus tree-level Symanzik gauge action. (HotQCD project)
- Follow the $m_l = 0.05 m_s$ line of constant physics at fixed $N_t = 8$.
- So far we have calculated terms in the presssure expansion only up to $O(B^2)$ (susceptibility).
- Only $50 - 70$ gauge configurations in each case.

T [MeV]	β	m_l/m_s	$V_{T\neq 0}$	$V_{T=0}$	Random sources		$C_1^r \times 10^{-4}$	$C_2^r \times 10^{-3}$
					$T \neq 0$	$T = 0$		
134	6.195	0.00440/0.0880	$32^3 \times 8$	$32^3 \times 32$	2400	400	$-1(7)$	$-0.3(5)$
154	6.341	0.00370/0.0740	$32^3 \times 8$	$32^3 \times 32$	2400	500	3(7)	0.4(4)
167	6.423	0.00335/0.0670	$32^3 \times 8$	$32^3 \times 32$	1200	200	$-4(7)$	2.2(5)
167	6.423	0.00335/0.0670	$48^3 \times 8$	$48^3 \times 48$	1200	400	$-9(4)$	2.3(5)
173	6.460	0.00320/0.0640	$32^3 \times 8$	$32^3 \times 64$	1200	200	$-6(8)$	3.8(6)
227	6.740	0.00238/0.0476	$32^3 \times 8$	$48^3 \times 48$	1200	200	$-4(3)$	10.4(8)
373	7.280	0.00142/0.0284	$32^3 \times 8$	$48^3 \times 64$	1200	40	0(7)	19.3(1.3)

- Cost: About 30K GPU-hours.

Results and conclusions





- Lattice scale, HotQCD [arXiv:1111.1710]
- (left) Red square shows larger volume result.
- (right) Dashed lines show HRG values [Endrödi arXiv:1301.1307].
- At very high T expect $\Delta p \to 0$.
- Δp is rather small ($\lesssim$1% of the zero field pressure) for fields up to $O(10^{15}\,\mathrm{T}) \approx 0.1$ GeV$^2/e$ relevant for heavy-ion collision experiments.
- For fields as high as $O(10^{16}\,\mathrm{T}) \approx 1$ GeV$^2/e$, 20% or more.