NPR of bilinear operators with improved staggered quarks

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SWME Collaboration

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Introduction

- We present matching factors for the bilinear operators obtained using the non-perturbative renormalization method (NPR) for improved staggered fermions on the MILC asqtad lattices($N_f = 2 + 1$).
- We obtain the wave function renormalization factor Z_q from the conserved vector and axial currents. Also we obtain the mass renormalization factor Z_m from scalar and pseudo-scalar bilinear operators.
- We also calculate the renormalization factor of bilinear operators whose taste are scalar or pseudo-scalar.

Bilinear Operator Renormalization

- $\tilde{p}$ is the momentum in reduced Brillouin zone.

$$p \in (-\frac{\pi}{a}, \frac{\pi}{a}]^4, \qquad \tilde{p} \in (-\frac{\pi}{2a}, \frac{\pi}{2a}]^4, \qquad p = \tilde{p} + \pi_B$$

  where $\pi_B (\equiv \frac{\pi}{a} B)$ is cut-off momentum in hypercube.
- a : lattice spacing.
- B : vector in hypercube. Each element is 0 or 1
  ex) $B = (0, 0, 1, 1)$

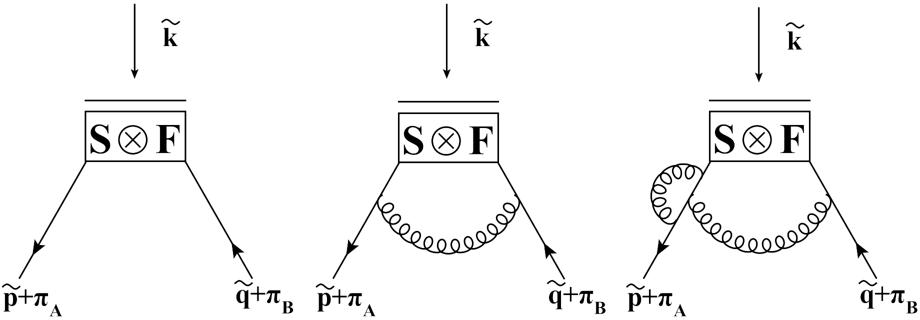
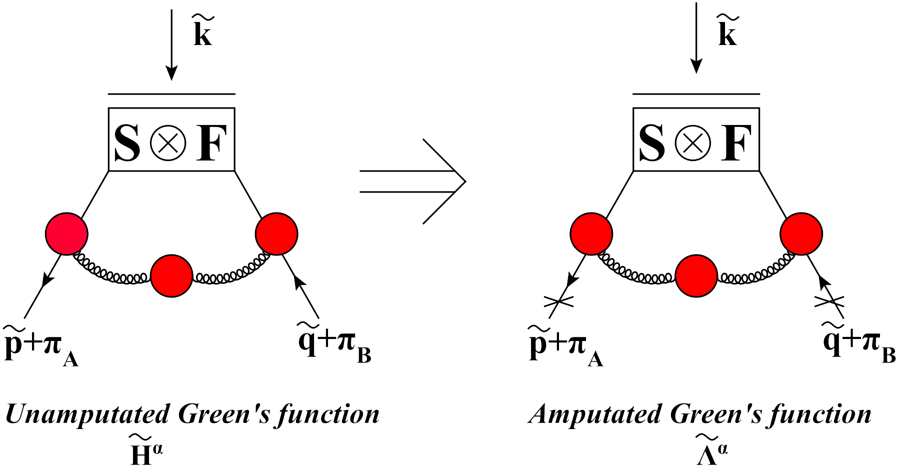
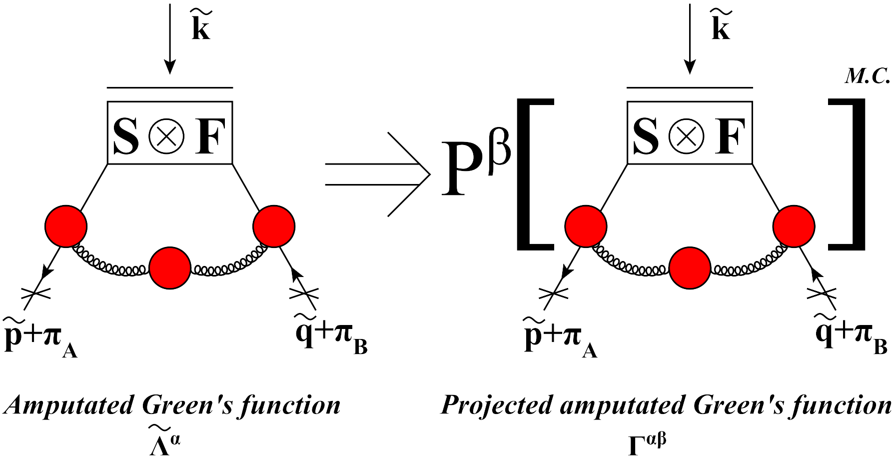


Figure : The Green's functions of bilinear operator : The diagrams that contribute to bilinear operator



Red Circle : 1PI Diagram.



- α, β : the indices to represent different operators.
  ex) $\alpha = (\gamma_\mu \otimes 1)$, $\beta = (1 \otimes 1)$
- M.C. : momentum conservation condition. $\tilde{p} = \tilde{q} + \tilde{k}$

The projection operator is

$$\hat{\mathbb{P}}^{\beta}_{BA;c_2c_1} = \frac{1}{48}\overline{\overline{(\gamma^{\dagger}_{S'} \otimes \xi^{\dagger}_{F'})}}_{BA}\delta_{c_2c_1}$$

The renormalization of $\Gamma(\tilde{p}, \tilde{q})$ is

$$\Gamma^{\alpha\sigma}_{R}(\tilde{p}, \tilde{q}) = \sum_{\beta} Z_q^{-1} Z_O^{\alpha\beta} \Gamma^{\beta\sigma}_{0}(\tilde{p}, \tilde{q})$$

- A, B: hypercube index
- c : color index
- α, β, σ: the indices to represent different operators.
- Γ_0 : bare projected amputated Green's function
- Γ_R : renormalized projected amputated Green's function
- Z_q : the wave function renormalization factor for quark fields
- $Z_O^{\alpha\beta}$: the renormalization factor of operator.

The RI-MOM scheme prescription is

$$\Gamma_R^{\alpha\sigma}(\tilde{p},\tilde{p}) = \Gamma_{tree}^{\alpha\sigma}(\tilde{p},\tilde{p}) = \delta^{\alpha\sigma} \,,$$

where $\Gamma_{tree}^{\alpha\sigma}(\tilde{p},\tilde{p})$ is tree level projected amputated Green's function.

Therefore

$$Z_q \cdot (Z_O^{-1})^{\alpha\beta} = \Gamma_0^{\alpha\beta}(\tilde{p},\tilde{p})$$

Simulation Detail

- $20^3 \times 64$ MILC asqtad lattice ($a \approx 0.12fm,\ am_\ell/am_s = 0.01/0.05$).
- HYP smearing
- The number of configurations is 30.
- 5 valence quark masses (0.01, 0.02, 0.03, 0.04, 0.05)
- 14 external momenta in the units of $(\frac{2\pi}{L_s}, \frac{2\pi}{L_s}, \frac{2\pi}{L_s}, \frac{2\pi}{L_t})$.

n(x, y, z, t)	$a\tilde{p}$	GeV	n(x, y, z, t)	$a\tilde{p}$	GeV
$(1,0,1,3)$	0.5330	0.8835	$(1,2,2,4)$	1.0210	1.6922
$(1,1,1,2)$	0.5785	0.9588	$(2,1,2,6)$	1.1114	1.8420
$(1,1,1,3)$	0.6187	1.0254	$(2,2,2,7)$	1.2871	2.1332
$(1,1,1,4)$	0.6710	1.1122	$(2,2,2,8)$	1.3421	2.2243
$(1,1,1,5)$	0.7328	1.2146	$(2,2,2,9)$	1.4018	2.3233
$(1,1,1,6)$	0.8019	1.3291	$(2,3,2,7)$	1.4663	2.4302
$(1,2,1,5)$	0.9128	1.5128	$(3,3,3,9)$	1.8562	3.0764

Conserved Vector Current Analysis

For the conserved vector current, the renormalization factor $Z_O^{\alpha\beta} = 1$.

Therefore

$$Z_q = \Gamma_0^{\alpha\beta}(\tilde{p}, \tilde{p}) \,,$$

where $\alpha = \beta = (\gamma_\mu \otimes 1)$.

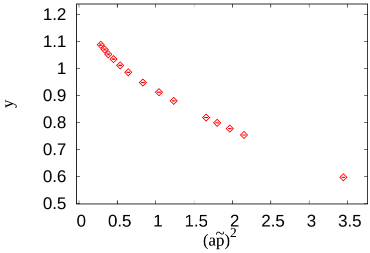
RI-MOM scheme to SI scheme

We divide the RI-MOM scheme data by the RG running factor to calculate the scale-invariant(SI) quantity.

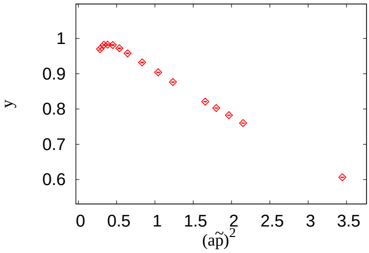
$$Z_q^{\mathrm{SI}} = \frac{c(\alpha_s(\mu_0))}{c(\alpha_s(\mu))} Z_q^{\text{RI-MOM}}(\mu), \qquad (\mu_0 = 2\mathrm{GeV}, \quad \mu^2 = \tilde{p}^2)$$

This Wilson coefficient $c(x)$ is calculated using four-loop running formula.

The data point is $y = \dfrac{Z_q}{Z_{V\otimes S}}$.



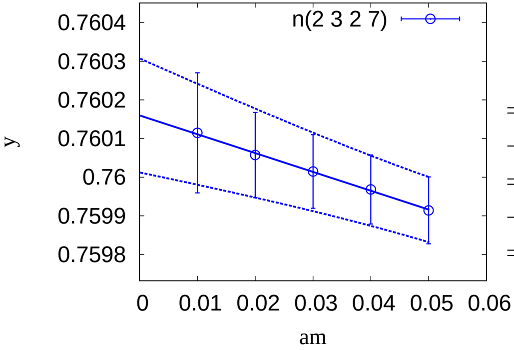
(a) RI-MOM scheme



(b) SI scheme

Mass Fitting ($y = \frac{Z_q}{Z_{V\otimes S}}$)

We use linear fit for mass fitting and take chiral limit value for each momentum.



$$f(m, a, \tilde{p}) = c_1 + c_2(am)$$

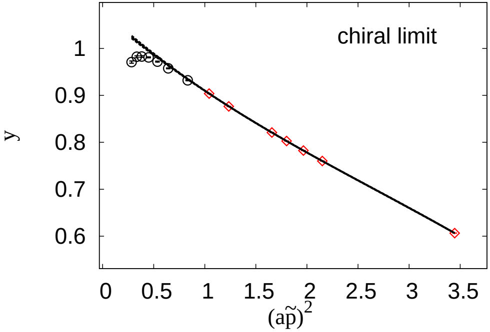
c_1	c_2
0.76016(15)	-0.0049(21)
χ^2/d.o.f	
0.0024(62)	

Momentum Fitting ($y = \frac{Z_q}{Z_{V\otimes S}}$)

We fit using $(a\tilde{p})^2 > 1$ data to avoid non-perturbative effects at small $(a\tilde{p})^2$. The fitting function is a power series of $(a\tilde{p})^2$.

$$f(m, a, \tilde{p}) = c_1 + c_2(a\tilde{p})^2 + c_3((a\tilde{p})^2)^2 + c_4((a\tilde{p})^2)^3$$

To remove the discretization error, we take the $(a\tilde{p})^2 \to 0$ limit after fitting.



c_1	c_2
1.0764(44)	-0.1908(69)
c_3	c_4
0.0279(33)	-0.00350(49)
χ^2/d.o.f	
0.06(16)	

Scalar Bilinear Operator $(1 \otimes 1)$ Analysis

By the Ward-Takahashi identity,

$$Z_m = \frac{1}{Z_S}$$

Therefore

$$\frac{Z_q}{Z_O^{\alpha\beta}} = \Gamma_0^{\alpha\beta}(\tilde{p}, \tilde{p}) = Z_q \cdot Z_m \,,$$

where $\alpha = \beta = (1 \otimes 1)$.

RI-MOM scheme to SI scheme

We divide the RI-MOM scheme data by the RG running factor to calculate the scale-invariant(SI) quantity.

$$Z_q^{\text{SI}} \cdot Z_m^{\text{SI}} = \frac{c(\alpha_s(\mu_0))}{c(\alpha_s(\mu))} Z_q^{\text{RI-MOM}}(\mu) \cdot \frac{c'(\alpha_s(\mu_0))}{c'(\alpha_s(\mu))} Z_m^{\text{RI-MOM}}(\mu)$$

$$(\mu_0 = 2\text{GeV}, \quad \mu^2 = \tilde{p}^2)$$

Mass Fitting

From the Ward-Takahashi identity, the amputated Green's function is

$$\Lambda_S = \frac{1}{48} \frac{\partial Tr[S^{-1}(\tilde{p})]}{\partial m}$$

where $S(\tilde{p})$ is quark propagator. and the propagator is

$$\frac{1}{48} Tr[S^{-1}(\tilde{p})] = C_1 \frac{Z_q \langle \overline{\chi} \chi \rangle}{\tilde{p}^2} + Z_q Z_m m + \ldots$$

When we consider the effect of zero-modes on chiral condensate term,

$$\langle \overline{\chi} \chi \rangle \sim \frac{n_0}{mV} ,$$

where n_0 is the number of zero-modes, V is the four dimensional space-time volume.

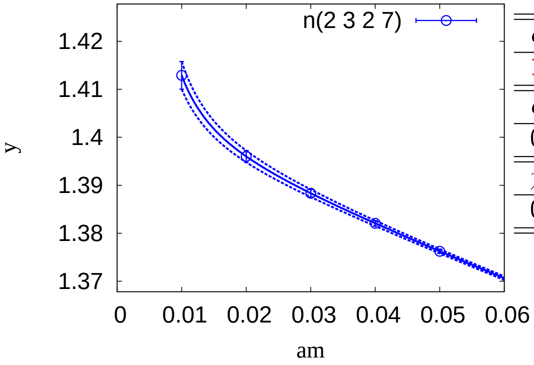
So we take derivative about mass on chiral condensate term, we can obtain $\frac{1}{(am)^2}$ term.

The fitting function is

$$f(m, a, \tilde{p}) = c_1 + c_2(am) + c_3(am)^2 + c_4 \frac{1}{(am)^2} \,,$$

We take c_1 as a chiral limit value.

The data point is $y = Z_q \cdot Z_m = \dfrac{Z_q}{Z_{S\otimes S}}$.



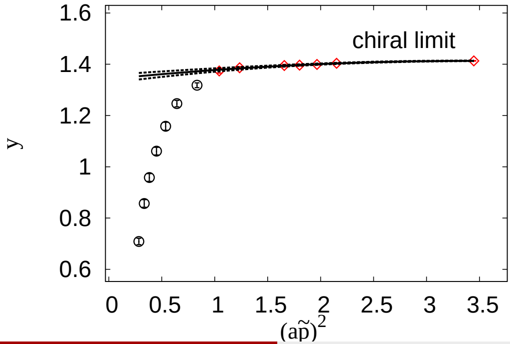
c_1	c_2
1.4036(22)	-0.573(72)
c_3	c_4
0.28(67)	0.000001
χ^2/d.o.f	
0.00008(51)	

Momentum Fitting ($y = Z_q \cdot Z_m = \frac{Z_q}{Z_{S\otimes S}}$)

We fit using $(a\tilde{p})^2 > 1$ data to avoid non-perturbative effects at small $(a\tilde{p})^2$. The fitting function is a power series of $(a\tilde{p})^2$.

$$f(m, a, \tilde{p}) = c_1 + c_2(a\tilde{p})^2 + c_3((a\tilde{p})^2)^2$$

To remove the discretization error, we take the $(a\tilde{p})^2 \to 0$ limit after fitting.



c_1	c_2
1.342(16)	0.041(12)
c_3	χ^2/d.o.f
-0.0060(21)	0.18(28)

Preliminary Result

Z_q	NPR	χ^2/dof (m_q fit)	χ^2/dof ($\tilde{p}$ fit)
$(\gamma_\mu \otimes 1)$	1.0764(44)	0.0024(62)	0.06(16)
$(\gamma_{\mu 5} \otimes \gamma_5)$	1.075(32)	0.0003(27)	0.12(28)

Z_m	NPR	χ^2/dof (m_q fit)	χ^2/dof ($\tilde{p}$ fit)
$(1 \otimes 1)$	1.246(15)	0.00008(51)	0.18(28)
$(\gamma_5 \otimes \gamma_5)$	1.255(18)	0.0000008(36)	0.06(19)

$Z_O^{\alpha\alpha}$	NPR	χ^2/dof (m_q fit)	χ^2/dof ($\tilde{p}$ fit)
$(\gamma_5 \otimes 1)$	1.113(20)	0.0397(22)	0.40(35)
$(\gamma_{\mu 5} \otimes 1)$	1.131(23)	0.0001(12)	0.11(41)
$(\gamma_{\mu\nu} \otimes 1)$	1.125(21)	0.0003(24)	0.76(51)
$(1 \otimes \gamma_5)$	1.079(18)	0.00004(23)	0.19(48)
$(\gamma_\mu \otimes \gamma_5)$	1.157(10)	0.000004(23)	0.40(60)
$(\gamma_{\mu\nu} \otimes \gamma_5)$	1.154(23)	0.0008(34)	0.56(47)

Conclusion

- We obtain the wave function renormalization factor Z_q from conserved vector and axial current and mass renormalization factor Z_m from scalar and pseudo-scalar bilinear operators.
- Also we calculate the renormalization factor of bilinear operators which has scalar or pseudo-scalar taste.
- We plan to analyse other bilinear operators which have V, A, T tastes in near future.