Perturbatively improving renormalization constants

M. Constantinou[1], M. Costa[1], M. Göckeler[2], R. Horsley[3], H. Panagopoulos[1], H. Perlt[4], P. E. L. Rakow[5], G. Schierholz[6] and A. Schiller[4]

[1]University of Cyprus, [2]Universität Regensburg, [3]University Edinburgh, [4]Universität Leipzig, [5]University Liverpool, [6]DESY

Lattice 2013, Mainz, Germany, July 29 - August 3, 2013

Outline

1. Introduction
2. Renormalization constants in RGI scheme
3. Subtraction of $O(a^2)$ lattice artifacts in one-loop
4. Summary

Outline

1. Introduction
2. Renormalization constants in RGI scheme
3. Subtraction of $O(a^2)$ lattice artifacts in one-loop
4. Summary

Outline

1. Introduction
2. Renormalization constants in RGI scheme
3. Subtraction of $O(a^2)$ lattice artifacts in one-loop
4. Summary

Outline

Motivation I

Talk is based on our recent paper
M. Constantinou et al. PR D87 (2013) 096019

- Connection between "lattice world" and "real world": renormalization constants Z
- Must know them as accurate as possible
- Perturbative approach: complicated, slow convergence, mixing problems, ...
- Nonperturbative approach: widely used scheme is RI-MOM scheme
  - Simple implementation
  - Gauge fixing required
- Simulations at finite lattice constant $a \to$ problem of lattices artifacts

Motivation I

Talk is based on our recent paper
M. Constantinou et al. PR D87 (2013) 096019

- Connection between "lattice world" and "real world": renormalization constants Z
- Must know them as accurate as possible
- Perturbative approach: complicated, slow convergence, mixing problems, ...
- Nonperturbative approach: widely used scheme is RI-MOM scheme
  - Simple implementation
  - Gauge fixing required
- Simulations at finite lattice constant $a \rightarrow$ problem of lattices artifacts

Motivation I

Talk is based on our recent paper
M. Constantinou et al. PR D87 (2013) 096019

- Connection between "lattice world" and "real world": renormalization constants Z
- Must know them as accurate as possible
- Perturbative approach: complicated, slow convergence, mixing problems, ...
- Nonperturbative approach: widely used scheme is RI-MOM scheme
  - Simple implementation
  - Gauge fixing required
- Simulations at finite lattice constant $a \rightarrow$ problem of lattices artifacts

Motivation I

Talk is based on our recent paper
M. Constantinou et al. PR D87 (2013) 096019

- Connection between "lattice world" and "real world": renormalization constants Z
- Must know them as accurate as possible
- Perturbative approach: complicated, slow convergence, mixing problems, ...
- Nonperturbative approach: widely used scheme is RI-MOM scheme
  - Simple implementation
  - Gauge fixing required
- Simulations at finite lattice constant $a \to$ problem of lattices artifacts

Motivation I

Talk is based on our recent paper
M. Constantinou et al. PR D87 (2013) 096019

- Connection between "lattice world" and "real world": renormalization constants Z
- Must know them as accurate as possible
- Perturbative approach: complicated, slow convergence, mixing problems, ...
- Nonperturbative approach: widely used scheme is RI-MOM scheme
  - Simple implementation
  - Gauge fixing required
- Simulations at finite lattice constant $a \rightarrow$ problem of lattices artifacts

Motivation I

Talk is based on our recent paper
M. Constantinou et al. PR D87 (2013) 096019

- Connection between "lattice world" and "real world": renormalization constants Z
- Must know them as accurate as possible
- Perturbative approach: complicated, slow convergence, mixing problems, ...
- Nonperturbative approach: widely used scheme is RI-MOM scheme
  - Simple implementation
  - Gauge fixing required
- Simulations at finite lattice constant $a \to$ problem of lattices artifacts

Motivation I

Talk is based on our recent paper
M. Constantinou et al. PR D87 (2013) 096019

- Connection between "lattice world" and "real world": renormalization constants Z
- Must know them as accurate as possible
- Perturbative approach: complicated, slow convergence, mixing problems, ...
- Nonperturbative approach: widely used scheme is RI-MOM scheme
  - Simple implementation
  - Gauge fixing required
- Simulations at finite lattice constant $a \to$ problem of lattices artifacts

Motivation I

Talk is based on our recent paper
M. Constantinou et al. PR D87 (2013) 096019

- Connection between "lattice world" and "real world": renormalization constants Z
- Must know them as accurate as possible
- Perturbative approach: complicated, slow convergence, mixing problems, ...
- Nonperturbative approach: widely used scheme is RI-MOM scheme
  - Simple implementation
  - Gauge fixing required
- Simulations at finite lattice constant $a \rightarrow$ problem of lattices artifacts

RGI scheme

$$\mathcal{O}^{\mathrm{RGI}} = \Delta Z^{\mathcal{S}}(M)\, \mathcal{O}^{\mathcal{S}}(M) = \Delta Z^{\mathcal{S}}(M)\, Z^{\mathcal{S}}_{\mathrm{bare}}(M,a)\mathcal{O}_{\mathrm{bare}} = Z^{\mathrm{RGI}}(a)\, \mathcal{O}_{\mathrm{bare}}$$

- $\mathcal{S} = \mathrm{RI}' - \mathrm{MOM}$ is not covariant for most operators $\to$ not suitable for computing anomalous dimensions (needed in $\Delta Z^{\mathcal{S}}(M)$)
- Intermediate scheme $\mathcal{S}$: $\mathrm{RI}' - \mathrm{MOM} \to \mathcal{S} \to$ RGI
- $Z^{\mathrm{RGI}}(a) = \Delta Z^{\widetilde{MOMgg}}(M = \mu_p)\, Z^{\widetilde{MOMgg}}_{\mathrm{RI}'-\mathrm{MOM}}(M = \mu_p)\, Z^{\mathrm{RI}'-\mathrm{MOM}}_{\mathrm{bare}}(\mu_p, a)$
- $\Delta Z^{\widetilde{MOMgg}}$ and $Z^{\widetilde{MOMgg}}_{\mathrm{RI}'-\mathrm{MOM}}(M = \mu_p)$ computed in continuum PT
- $Z^{\mathrm{RI}'-\mathrm{MOM}}_{\mathrm{bare}}(\mu, a)$ is the nonperturbatively measured Z-factor
- In MC simulations $(a\mu)$ is not small $\to$ lattice artifacts!
- Artifacts under control $\to$ determination of Z^{RGI} with better accuray!

RGI scheme

$$\mathcal{O}^{\rm RGI} = \Delta Z^{\mathcal{S}}(M)\, \mathcal{O}^{\mathcal{S}}(M) = \Delta Z^{\mathcal{S}}(M)\, Z^{\mathcal{S}}_{\rm bare}(M,a)\mathcal{O}_{\rm bare} = Z^{\rm RGI}(a)\, \mathcal{O}_{\rm bare}$$

- $\mathcal{S} = \mathrm{RI}' - \mathrm{MOM}$ is not covariant for most operators $\rightarrow$ not suitable for computing anomalous dimensions (needed in $\Delta Z^{\mathcal{S}}(M)$)
- Intermediate scheme $\mathcal{S}$: $\mathrm{RI}' - \mathrm{MOM} \rightarrow \mathcal{S} \rightarrow$ RGI
- $Z^{\rm RGI}(a) = \Delta Z^{\widetilde{MOMgg}}(M = \mu_p)\, Z^{\widetilde{MOMgg}}_{\mathrm{RI}'-\mathrm{MOM}}(M = \mu_p)\, Z^{\mathrm{RI}'-\mathrm{MOM}}_{\rm bare}(\mu_p, a)$
- $\Delta Z^{\widetilde{MOMgg}}$ and $Z^{\widetilde{MOMgg}}_{\mathrm{RI}'-\mathrm{MOM}}(M = \mu_p)$ computed in continuum PT
- $Z^{\mathrm{RI}'-\mathrm{MOM}}_{\rm bare}(\mu, a)$ is the nonperturbatively measured Z-factor
- In MC simulations $(a\mu)$ is not small $\rightarrow$ lattice artifacts!
- Artifacts under control $\rightarrow$ determination of $Z^{\rm RGI}$ with better accuray!

RGI scheme

$$\mathcal{O}^{\rm RGI} = \Delta Z^{\mathcal{S}}(M)\, \mathcal{O}^{\mathcal{S}}(M) = \Delta Z^{\mathcal{S}}(M)\, Z^{\mathcal{S}}_{\rm bare}(M,a)\mathcal{O}_{\rm bare} = Z^{\rm RGI}(a)\, \mathcal{O}_{\rm bare}$$

- $\mathcal{S} = \mathrm{RI'} - \mathrm{MOM}$ is not covariant for most operators $\to$ not suitable for computing anomalous dimensions (needed in $\Delta Z^{\mathcal{S}}(M)$)
- Intermediate scheme $\mathcal{S}$: $\mathrm{RI'} - \mathrm{MOM} \to \mathcal{S} \to$ RGI
- $Z^{\rm RGI}(a) = \Delta Z^{\widetilde{MOMgg}}(M = \mu_p)\, Z^{\widetilde{MOMgg}}_{\rm RI'-MOM}(M = \mu_p)\, Z^{\rm RI'-MOM}_{\rm bare}(\mu_p, a)$
- $\Delta Z^{\widetilde{MOMgg}}$ and $Z^{\widetilde{MOMgg}}_{\rm RI'-MOM}(M = \mu_p)$ computed in continuum PT
- $Z^{\rm RI'-MOM}_{\rm bare}(\mu, a)$ is the nonperturbatively measured Z-factor
- In MC simulations $(a\mu)$ is not small $\to$ lattice artifacts!
- Artifacts under control $\to$ determination of $Z^{\rm RGI}$ with better accuray!

RGI scheme

$$\mathcal{O}^{\mathrm{RGI}} = \Delta Z^{\mathcal{S}}(M)\,\mathcal{O}^{\mathcal{S}}(M) = \Delta Z^{\mathcal{S}}(M)\,Z^{\mathcal{S}}_{\mathrm{bare}}(M,a)\mathcal{O}_{\mathrm{bare}} = Z^{\mathrm{RGI}}(a)\,\mathcal{O}_{\mathrm{bare}}$$

- $\mathcal{S} = \mathrm{RI}' - \mathrm{MOM}$ is not covariant for most operators $\rightarrow$ not suitable for computing anomalous dimensions (needed in $\Delta Z^{\mathcal{S}}(M)$)
- Intermediate scheme $\mathcal{S}$: $\mathrm{RI}' - \mathrm{MOM} \rightarrow \mathcal{S} \rightarrow$ RGI
- $Z^{\mathrm{RGI}}(a) = \Delta Z^{\widetilde{MOMgg}}(M = \mu_p)\,Z^{\widetilde{MOMgg}}_{\mathrm{RI}'-\mathrm{MOM}}(M = \mu_p)\,Z^{\mathrm{RI}'-\mathrm{MOM}}_{\mathrm{bare}}(\mu_p, a)$
- $\Delta Z^{\widetilde{MOMgg}}$ and $Z^{\widetilde{MOMgg}}_{\mathrm{RI}'-\mathrm{MOM}}(M = \mu_p)$ computed in continuum PT
- $Z^{\mathrm{RI}'-\mathrm{MOM}}_{\mathrm{bare}}(\mu, a)$ is the nonperturbatively measured Z-factor
- In MC simulations $(a\mu)$ is not small $\rightarrow$ lattice artifacts!
- Artifacts under control $\rightarrow$ determination of Z^{RGI} with better accuray!

RGI scheme

$$\mathcal{O}^{\mathrm{RGI}} = \Delta Z^{\mathcal{S}}(M)\,\mathcal{O}^{\mathcal{S}}(M) = \Delta Z^{\mathcal{S}}(M)\,Z^{\mathcal{S}}_{\mathrm{bare}}(M,a)\mathcal{O}_{\mathrm{bare}} = Z^{\mathrm{RGI}}(a)\,\mathcal{O}_{\mathrm{bare}}$$

- $\mathcal{S}$ = $\mathrm{RI}' - \mathrm{MOM}$ is not covariant for most operators $\rightarrow$ not suitable for computing anomalous dimensions (needed in $\Delta Z^{\mathcal{S}}(M)$)
- Intermediate scheme $\mathcal{S}$: $\mathrm{RI}' - \mathrm{MOM} \rightarrow \mathcal{S} \rightarrow$ RGI
- $Z^{\mathrm{RGI}}(a) = \Delta Z^{\widetilde{MOMgg}}(M=\mu_p)\,Z^{\widetilde{MOMgg}}_{\mathrm{RI}'-\mathrm{MOM}}(M=\mu_p)\,Z^{\mathrm{RI}'-\mathrm{MOM}}_{\mathrm{bare}}(\mu_p,a)$
- $\Delta Z^{\widetilde{MOMgg}}$ and $Z^{\widetilde{MOMgg}}_{\mathrm{RI}'-\mathrm{MOM}}(M=\mu_p)$ computed in continuum PT
- $Z^{\mathrm{RI}'-\mathrm{MOM}}_{\mathrm{bare}}(\mu,a)$ is the nonperturbatively measured Z-factor
- In MC simulations $(a\mu)$ is not small $\rightarrow$ lattice artifacts!
- Artifacts under control $\rightarrow$ determination of Z^{RGI} with better accuray!

RGI scheme

$$\mathcal{O}^{\mathrm{RGI}} = \Delta Z^{\mathcal{S}}(M)\,\mathcal{O}^{\mathcal{S}}(M) = \Delta Z^{\mathcal{S}}(M)\,Z^{\mathcal{S}}_{\mathrm{bare}}(M,a)\mathcal{O}_{\mathrm{bare}} = Z^{\mathrm{RGI}}(a)\,\mathcal{O}_{\mathrm{bare}}$$

- $\mathcal{S} = \mathrm{RI}' - \mathrm{MOM}$ is not covariant for most operators $\rightarrow$ not suitable for computing anomalous dimensions (needed in $\Delta Z^{\mathcal{S}}(M)$)
- Intermediate scheme $\mathcal{S}$: $\mathrm{RI}' - \mathrm{MOM} \rightarrow \mathcal{S} \rightarrow$ RGI
- $Z^{\mathrm{RGI}}(a) = \Delta Z^{\widetilde{MOMgg}}(M = \mu_p)\, Z^{\widetilde{MOMgg}}_{\mathrm{RI}'-\mathrm{MOM}}(M = \mu_p)\, Z^{\mathrm{RI}'-\mathrm{MOM}}_{\mathrm{bare}}(\mu_p, a)$
- $\Delta Z^{\widetilde{MOMgg}}$ and $Z^{\widetilde{MOMgg}}_{\mathrm{RI}'-\mathrm{MOM}}(M = \mu_p)$ computed in continuum PT
- $Z^{\mathrm{RI}'-\mathrm{MOM}}_{\mathrm{bare}}(\mu, a)$ is the nonperturbatively measured Z-factor
- In MC simulations $(a\mu)$ is not small $\rightarrow$ lattice artifacts!
- Artifacts under control $\rightarrow$ determination of Z^{RGI} with better accuray!

RGI scheme

$$\mathcal{O}^{\rm RGI} = \Delta Z^{\mathcal{S}}(M)\, \mathcal{O}^{\mathcal{S}}(M) = \Delta Z^{\mathcal{S}}(M)\, Z^{\mathcal{S}}_{\rm bare}(M,a)\mathcal{O}_{\rm bare} = Z^{\rm RGI}(a)\, \mathcal{O}_{\rm bare}$$

- $\mathcal{S} = \mathrm{RI}' - \mathrm{MOM}$ is not covariant for most operators $\to$ not suitable for computing anomalous dimensions (needed in $\Delta Z^{\mathcal{S}}(M)$)
- Intermediate scheme $\mathcal{S}$: $\mathrm{RI}' - \mathrm{MOM} \to \mathcal{S} \to$ RGI
- $Z^{\rm RGI}(a) = \Delta Z^{\widetilde{MOMgg}}(M = \mu_p)\, Z^{\widetilde{MOMgg}}_{\mathrm{RI}'-\mathrm{MOM}}(M = \mu_p)\, Z^{\mathrm{RI}'-\mathrm{MOM}}_{\rm bare}(\mu_p, a)$
- $\Delta Z^{\widetilde{MOMgg}}$ and $Z^{\widetilde{MOMgg}}_{\mathrm{RI}'-\mathrm{MOM}}(M = \mu_p)$ computed in continuum PT
- $Z^{\mathrm{RI}'-\mathrm{MOM}}_{\rm bare}(\mu, a)$ is the nonperturbatively measured Z-factor
- In MC simulations $(a\mu)$ is not small $\to$ lattice artifacts!
- Artifacts under control $\to$ determination of $Z^{\rm RGI}$ with better accuray!

RGI scheme

$$\mathcal{O}^{\mathrm{RGI}} = \Delta Z^{\mathcal{S}}(M)\,\mathcal{O}^{\mathcal{S}}(M) = \Delta Z^{\mathcal{S}}(M)\,Z^{\mathcal{S}}_{\mathrm{bare}}(M,a)\mathcal{O}_{\mathrm{bare}} = Z^{\mathrm{RGI}}(a)\,\mathcal{O}_{\mathrm{bare}}$$

- $\mathcal{S} = \mathrm{RI}' - \mathrm{MOM}$ is not covariant for most operators $\rightarrow$ not suitable for computing anomalous dimensions (needed in $\Delta Z^{\mathcal{S}}(M)$)
- Intermediate scheme $\mathcal{S}$: $\mathrm{RI}' - \mathrm{MOM} \rightarrow \mathcal{S} \rightarrow$ RGI
- $Z^{\mathrm{RGI}}(a) = \Delta Z^{\widetilde{MOMgg}}(M=\mu_p)\,Z^{\widetilde{MOMgg}}_{\mathrm{RI}'-\mathrm{MOM}}(M=\mu_p)\,Z^{\mathrm{RI}'-\mathrm{MOM}}_{\mathrm{bare}}(\mu_p,a)$
- $\Delta Z^{\widetilde{MOMgg}}$ and $Z^{\widetilde{MOMgg}}_{\mathrm{RI}'-\mathrm{MOM}}(M=\mu_p)$ computed in continuum PT
- $Z^{\mathrm{RI}'-\mathrm{MOM}}_{\mathrm{bare}}(\mu,a)$ is the nonperturbatively measured Z-factor
- In MC simulations $(a\mu)$ is not small $\rightarrow$ lattice artifacts!
- Artifacts under control $\rightarrow$ determination of Z^{RGI} with better accuray!

Settings, parameters

Following settings are used for the discussed examples

- Operators: $\mathcal{O}^{\rm S}$, $\mathcal{O}^{\rm V}_\mu$, $\mathcal{O}^{\rm A}_\mu$, $\mathcal{O}^{\rm T}_{\mu\nu}$, $\mathcal{O}_{v_{2,a}}$, $\mathcal{O}_{v_{2,b}}$
- Clover improved Wilson fermions + plaquette gauge action, $N_f = 2$
- Landau gauge
- $\beta = (5.20, 5.25, 5.29, 5.40)$
- $r_0\,\Lambda_{\overline{\rm MS}} = (0.700, 0.789) \to g^{\mathcal{S}} \to \Delta\, Z^{\mathcal{S}}(M)$
- $r_0 = 0.501\mathrm{fm}$, $r_0/a = 6.050(\beta = 5.20), 6.603(\beta = 5.25), 7.004(\beta = 5.29), 8.285(\beta = 5.40)$

Settings, parameters

Following settings are used for the discussed examples

- Operators: $\mathcal{O}^{\mathrm{S}}$, $\mathcal{O}^{\mathrm{V}}_{\mu}$, $\mathcal{O}^{\mathrm{A}}_{\mu}$, $\mathcal{O}^{\mathrm{T}}_{\mu\nu}$, $\mathcal{O}_{v_{2,a}}$, $\mathcal{O}_{v_{2,b}}$
- Clover improved Wilson fermions + plaquette gauge action, $N_f = 2$
- Landau gauge
- $\beta = (5.20, 5.25, 5.29, 5.40)$
- $r_0 \Lambda_{\overline{\mathrm{MS}}} = (0.700, 0.789) \to g^{\mathcal{S}} \to \Delta Z^{\mathcal{S}}(M)$
- $r_0 = 0.501\mathrm{fm}$, $r_0/a = 6.050(\beta = 5.20), 6.603(\beta = 5.25), 7.004(\beta = 5.29), 8.285(\beta = 5.40)$

Settings, parameters

Following settings are used for the discussed examples

- Operators: $\mathcal{O}^{\rm S}$, $\mathcal{O}^{\rm V}_\mu$, $\mathcal{O}^{\rm A}_\mu$, $\mathcal{O}^{\rm T}_{\mu\nu}$, $\mathcal{O}_{v_{2,a}}$, $\mathcal{O}_{v_{2,b}}$
- Clover improved Wilson fermions + plaquette gauge action, $N_f = 2$
- Landau gauge
- $\beta = (5.20, 5.25, 5.29, 5.40)$
- $r_0\,\Lambda_{\overline{\rm MS}} = (0.700, 0.789) \to g^{\mathcal{S}} \to \Delta\, Z^{\mathcal{S}}(M)$
- $r_0 = 0.501{\rm fm}$, $r_0/a = 6.050(\beta = 5.20), 6.603(\beta = 5.25), 7.004(\beta = 5.29), 8.285(\beta = 5.40)$

Settings, parameters

Following settings are used for the discussed examples

- Operators: $\mathcal{O}^{\mathrm{S}}$, $\mathcal{O}^{\mathrm{V}}_{\mu}$, $\mathcal{O}^{\mathrm{A}}_{\mu}$, $\mathcal{O}^{\mathrm{T}}_{\mu\nu}$, $\mathcal{O}_{v_{2,a}}$, $\mathcal{O}_{v_{2,b}}$
- Clover improved Wilson fermions + plaquette gauge action, $N_f = 2$
- Landau gauge
- $\beta = (5.20, 5.25, 5.29, 5.40)$
- $r_0\,\Lambda_{\overline{\mathrm{MS}}} = (0.700, 0.789) \rightarrow g^{\mathcal{S}} \rightarrow \Delta\, Z^{\mathcal{S}}(M)$
- $r_0 = 0.501\mathrm{fm}$, $r_0/a = 6.050(\beta = 5.20), 6.603(\beta = 5.25), 7.004(\beta = 5.29), 8.285(\beta = 5.40)$

Settings, parameters

Following settings are used for the discussed examples

- Operators: $\mathcal{O}^{\mathrm{S}}$, $\mathcal{O}^{\mathrm{V}}_{\mu}$, $\mathcal{O}^{\mathrm{A}}_{\mu}$, $\mathcal{O}^{\mathrm{T}}_{\mu\nu}$, $\mathcal{O}_{v_{2,a}}$, $\mathcal{O}_{v_{2,b}}$
- Clover improved Wilson fermions + plaquette gauge action, $N_f = 2$
- Landau gauge
- $\beta = (5.20, 5.25, 5.29, 5.40)$
- $r_0\,\Lambda_{\overline{\mathrm{MS}}} = (0.700, 0.789) \to g^{\mathcal{S}} \to \Delta\, Z^{\mathcal{S}}(M)$
- $r_0 = 0.501\mathrm{fm}$, $r_0/a = 6.050(\beta = 5.20), 6.603(\beta = 5.25), 7.004(\beta = 5.29), 8.285(\beta = 5.40)$

Settings, parameters

Following settings are used for the discussed examples

- Operators: $\mathcal{O}^{\mathrm{S}}$, $\mathcal{O}^{\mathrm{V}}_{\mu}$, $\mathcal{O}^{\mathrm{A}}_{\mu}$, $\mathcal{O}^{\mathrm{T}}_{\mu\nu}$, $\mathcal{O}_{v_{2,a}}$, $\mathcal{O}_{v_{2,b}}$
- Clover improved Wilson fermions + plaquette gauge action, $N_f = 2$
- Landau gauge
- $\beta = (5.20, 5.25, 5.29, 5.40)$
- $r_0\,\Lambda_{\overline{\mathrm{MS}}} = (0.700, 0.789) \to g^{\mathcal{S}} \to \Delta\, Z^{\mathcal{S}}(M)$
- $r_0 = 0.501\mathrm{fm}$, $r_0/a = 6.050(\beta = 5.20), 6.603(\beta = 5.25), 7.004(\beta = 5.29), 8.285(\beta = 5.40)$

One-loop subtraction $O(a^2)$

- In *M. Göckeler et al. PR D82 (2010) 114511* we proposed to subtract the complete one-loop lattice artifacts
- Very efficient, but being a purely numerical procedure rather involved
- Look for procedure which can be applied to more general cases - eventually with "less" correction effect
- Cyprus group pioneered diagrammatic $O(g^2a^2)$ approach (see e.g., *M. Constantinou, V. Lubicz, H. Panagopoulos and F. Stylianou, JHEP* **0910** *(2009) 064*)
- Results for local and one-link bilinears and different actions and general mass terms; higher derivative operators are possible (but also not very easy)

One-loop subtraction $O(a^2)$

- In *M. Göckeler et al. PR D82 (2010) 114511* we proposed to subtract the complete one-loop lattice artifacts
- Very efficient, but being a purely numerical procedure rather involved
- Look for procedure which can be applied to more general cases - eventually with "less" correction effect
- Cyprus group pioneered diagrammatic $O(g^2 a^2)$ approach (see e.g., *M. Constantinou, V. Lubicz, H. Panagopoulos and F. Stylianou, JHEP* **0910** *(2009) 064*)
- Results for local and one-link bilinears and different actions and general mass terms; higher derivative operators are possible (but also not very easy)

One-loop subtraction $O(a^2)$

- In *M. Göckeler et al. PR D82 (2010) 114511* we proposed to subtract the complete one-loop lattice artifacts
- Very efficient, but being a purely numerical procedure rather involved
- Look for procedure which can be applied to more general cases - eventually with "less" correction effect
- Cyprus group pioneered diagrammatic $O(g^2 a^2)$ approach (see e.g., *M. Constantinou, V. Lubicz, H. Panagopoulos and F. Stylianou, JHEP* **0910** *(2009) 064)*
- Results for local and one-link bilinears and different actions and general mass terms; higher derivative operators are possible (but also not very easy)

One-loop subtraction $O(a^2)$

- In *M. Göckeler et al. PR D82 (2010) 114511* we proposed to subtract the complete one-loop lattice artifacts
- Very efficient, but being a purely numerical procedure rather involved
- Look for procedure which can be applied to more general cases - eventually with "less" correction effect
- Cyprus group pioneered diagrammatic $O(g^2 a^2)$ approach (see e.g., *M. Constantinou, V. Lubicz, H. Panagopoulos and F. Stylianou, JHEP* **0910** *(2009) 064)*
- Results for local and one-link bilinears and different actions and general mass terms; higher derivative operators are possible (but also not very easy)

One-loop subtraction $O(a^2)$

- In *M. Göckeler et al. PR D82 (2010) 114511* we proposed to subtract the complete one-loop lattice artifacts
- Very efficient, but being a purely numerical procedure rather involved
- Look for procedure which can be applied to more general cases - eventually with "less" correction effect
- Cyprus group pioneered diagrammatic $O(g^2 a^2)$ approach (see e.g., *M. Constantinou, V. Lubicz, H. Panagopoulos and F. Stylianou, JHEP* **0910** *(2009) 064)*
- Results for local and one-link bilinears and different actions and general mass terms; higher derivative operators are possible (but also not very easy)

$\mathcal{O}^{\mathrm{S}}$

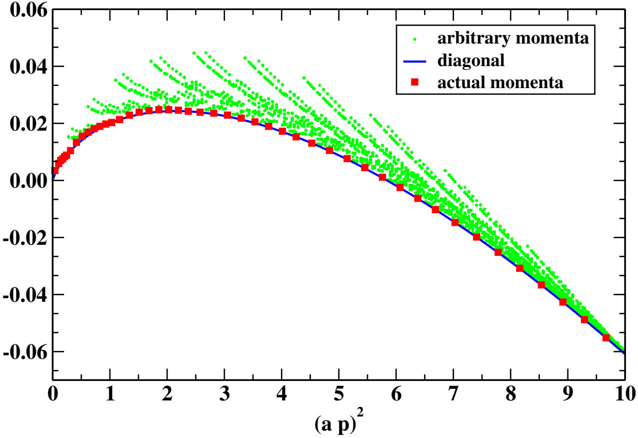
Example: scalar operator $\mathcal{O}^{\mathrm{S}}$

$$\begin{aligned}
Z^{\mathrm{S}}(a,p) &= 1 + \frac{g^2 C_F}{16\,\pi^2}\Bigg(-23.3099453215 + 3\,\log(a^2 S_2) \\
&\quad + (a^2 S_2)\Big(1.6408851782248 - \frac{239}{240}\,\log(a^2 S_2) \\
&\quad + \frac{S_4}{(S_2)^2}\Big(1.9510436778 - \frac{101}{120}\,\log(a^2 S_2)\Big)\Big)\Bigg) \\
&\equiv 1 + \frac{g^2 C_F}{16\,\pi^2}\,Z_{\mathrm{S}}^{(1)} + g^2\,a^2\,Z_{\mathrm{S,1-loop}}^{(a^2)}
\end{aligned}$$

with $S_n = \sum_{\lambda=1}^{4} p_\lambda^n$

$\mathcal{O}^S - O(a^2)$ artifacts

$g^2\, a^2 Z^{(a^2)}_{\mathrm{S,1-loop}}$ for a general momentum set ($24^3 \times 48$)



Subtraction procedures

Following subtraction procedures have been investigated

$$
\begin{aligned}
Z^{\mathrm{RI'-MOM}}_{\mathrm{bare}}(p,a)_{\mathrm{MC,sub,s}} &\overset{(s)}{=} Z^{\mathrm{RI'-MOM}}_{\mathrm{bare}}(p,a)_{\mathrm{MC}} - a^2\, g_\star^2\, Z^{(a^2)}_{1-\mathrm{loop}}(p,a) \\
Z^{\mathrm{RI'-MOM}}_{\mathrm{bare}}(p,a)_{\mathrm{MC,sub,m}} &\overset{(m)}{=} Z^{\mathrm{RI'-MOM}}_{\mathrm{bare}}(p,a)_{\mathrm{MC}} \times \left(1 - a^2\, g_\star^2\, Z^{(a^2)}_{1-\mathrm{loop}}(p,a)\right)
\end{aligned}
$$

$g_\star$ can be chosen to be either the bare lattice coupling g or the boosted coupling g_{B}.

Subtraction procedures

Following subtraction procedures have been investigated

$$Z_{\rm bare}^{\rm RI'-MOM}(p,a)_{\rm MC,sub,s} \stackrel{(s)}{=} Z_{\rm bare}^{\rm RI'-MOM}(p,a)_{\rm MC} - a^2\, g_\star^2\, Z_{\rm 1-loop}^{(a^2)}(p,a)$$

$$Z_{\rm bare}^{\rm RI'-MOM}(p,a)_{\rm MC,sub,m} \stackrel{(m)}{=} Z_{\rm bare}^{\rm RI'-MOM}(p,a)_{\rm MC} \times \left(1 - a^2\, g_\star^2\, Z_{\rm 1-loop}^{(a^2)}(p,a)\right)$$

$g_\star$ can be chosen to be either the bare lattice coupling g or the boosted coupling $g_{\rm B}$.

Effect of subtraction

For the scalar operator we get

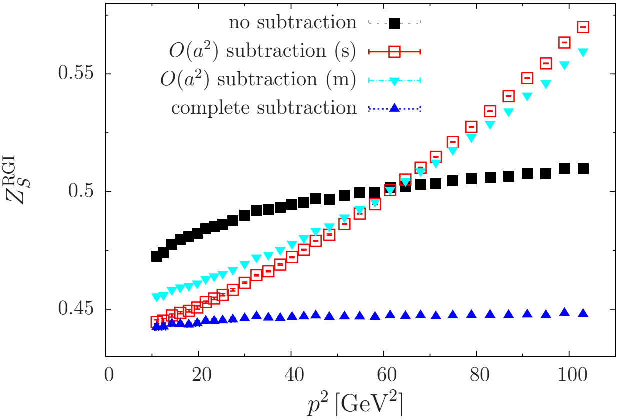


Figure: Z_{S} and $Z_{\mathrm{S,sub,i}}$ as function of $a^2 p^2$.

Fit of lattice artifacts

Parametrizing the remaining lattice artifacts

$$\left(Z^{\mathrm{RGI}}(a) = \Delta\, Z^{\widetilde{MOMgg}}(p)\, Z^{\widetilde{MOMgg}}_{\mathrm{RI'-MOM}}(p)\, Z^{\mathrm{RI'-MOM}}_{\mathrm{bare}}(p,a)_{\mathrm{MC,sub}} \right)$$

$$Z^{\mathcal{S}}_{\mathrm{RI'-MOM}}(p) Z^{\mathrm{RI'-MOM}}_{\mathrm{bare}}(p,a)_{\mathrm{MC,sub}} = \frac{Z^{\mathrm{RGI}}(a)}{\Delta Z^{\mathcal{S}}(p)\,\left[1 + b_1\,(g^{\mathcal{S}})^8\right]} + a^2\left(c_1\,S_2 + c_2\,\frac{S_4}{S_2} + c_3\,\frac{S_6}{(S_2)^2}\right) + a^4\left(c_4\,(S_2)^2 + c_5\,S_4\right) + a^6\left(c_6\,(S_2)^3 + c_7\,S_4\,S_2 + c_8\,S_6\right).$$

c_i fit the lattice artifacts ($S_n = \sum_i p_i^n$)

Fit procedure

1. In order to avoid the region of breakdown of perturbation theory we fit for $p^2 \geq p^2_{min} = 10\,\mathrm{GeV}^2$.
2. We vary the type of subtraction ((s),(m)) and type of coupling (g, g_B)
3. We vary the number/kind of parameters (c_i) $\rightarrow$ strength of hypercubic lattice artifacts
4. Assuming that one-loop complete subtraction is “benchmark” $\rightarrow$ compare the $O(a^2)$ subtraction results with these numbers
5. **Fitting routines**: Nelder-Mead/differential evolution algorithms cross checked with MINUIT

Fit procedure

1. In order to avoid the region of breakdown of perturbation theory we fit for $p^2 \geq p^2_{min} = 10\,\mathrm{GeV}^2$.
2. We vary the type of subtraction ((s),(m)) and type of coupling (g, g_B)
3. We vary the number/kind of parameters (c_i) $\rightarrow$ strength of hypercubic lattice artifacts
4. Assuming that one-loop complete subtraction is "benchmark" $\rightarrow$ compare the $O(a^2)$ subtraction results with these numbers
5. **Fitting routines**: Nelder-Mead/differential evolution algorithms cross checked with MINUIT

Fit procedure

1. In order to avoid the region of breakdown of perturbation theory we fit for $p^2 \geq p^2_{min} = 10\,\mathrm{GeV}^2$.
2. We vary the type of subtraction ((s),(m)) and type of coupling (g, g_B)
3. We vary the number/kind of parameters (c_i) $\rightarrow$ strength of hypercubic lattice artifacts
4. Assuming that one-loop complete subtraction is "benchmark" $\rightarrow$ compare the $O(a^2)$ subtraction results with these numbers
5. **Fitting routines**: Nelder-Mead/differential evolution algorithms cross checked with MINUIT

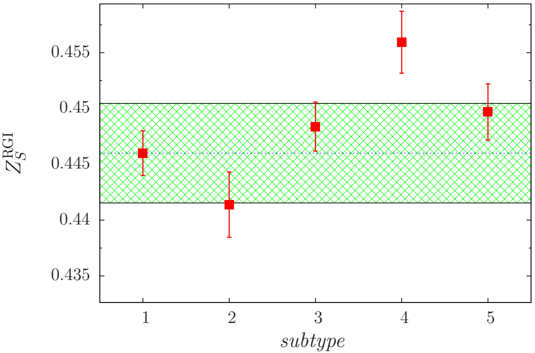
Fit procedure

1. In order to avoid the region of breakdown of perturbation theory we fit for $p^2 \geq p^2_{min} = 10\,\mathrm{GeV}^2$.
2. We vary the type of subtraction ((s),(m)) and type of coupling (g, g_B)
3. We vary the number/kind of parameters (c_i) $\rightarrow$ strength of hypercubic lattice artifacts
4. Assuming that one-loop complete subtraction is “benchmark” $\rightarrow$ compare the $O(a^2)$ subtraction results with these numbers
5. **Fitting routines**: Nelder-Mead/differential evolution algorithms cross checked with MINUIT

Fit procedure

1. In order to avoid the region of breakdown of perturbation theory we fit for $p^2 \geq p^2_{min} = 10\,\mathrm{GeV}^2$.
2. We vary the type of subtraction ((s),(m)) and type of coupling (g, g_B)
3. We vary the number/kind of parameters (c_i) $\rightarrow$ strength of hypercubic lattice artifacts
4. Assuming that one-loop complete subtraction is “benchmark” $\rightarrow$ compare the $O(a^2)$ subtraction results with these numbers
5. **Fitting routines**: Nelder-Mead/differential evolution algorithms cross checked with MINUIT

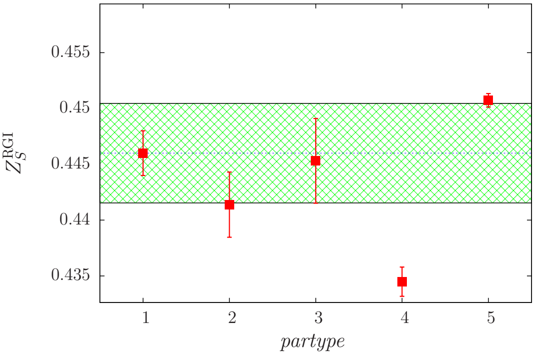
Results, $\mathcal{O}^S$ as example: subtraction type



1: complete subtraction with g_B, **2**: (s) with g_B, **3**: (m) with g_B, **4**: (s) with g, **5**: (m) with g

Fit using all c_i, Shaded area: 1% deviation from **1**.

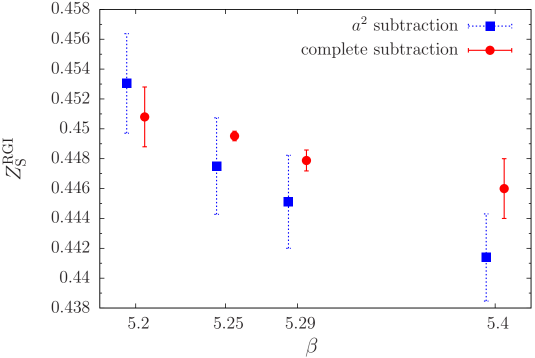
Results, $\mathcal{O}^S$ as example: parameter type



1: complete subtraction, **2**: all c_i, **3**: $O(4)$-invariants, **4**: a^2, a^4 - invariants, **5**: a^4, a^6 - invariants

Fit for (s) and g_B, Shaded area: 1% deviation from **1**.

Results, $\mathcal{O}^S$ as example: final comparison



Z_S^{RGI} at $r_0\,\Lambda_{\overline{\mathrm{MS}}} = 0.700$

Results for a^2 subtraction

Op.	$r_0\,\Lambda_{\overline{\rm MS}}$	$Z^{\rm RGI}\vert_{\beta=5.20}$	$Z^{\rm RGI}\vert_{\beta=5.25}$	$Z^{\rm RGI}\vert_{\beta=5.29}$	$Z^{\rm RGI}\vert_{\beta=5.40}$
$\mathcal{O}^S$	0.700	0.4530(34)	0.4475(33)	0.4451(32)	0.4414(30)
	0.789	0.4717(44)	0.4661(65)	0.4632(54)	0.4585(27)
$\mathcal{O}^V$	0.700	0.7163(26)	0.7253(26)	0.7308(25)	0.7451(24)
	0.789	0.7238(72)	0.7319(94)	0.7365(99)	0.7519(50)
$\mathcal{O}^A$	0.700	0.7460(41)	0.7543(40)	0.7590(39)	0.7731(37)
	0.789	0.7585(46)	0.7634(77)	0.7666(81)	0.7805(30)
$\mathcal{O}^T$	0.700	0.8906(43)	0.9036(42)	0.9108(41)	0.9319(39)
	0.789	0.8946(85)	0.9041(111)	0.9075(120)	0.9316(49)
$\mathcal{O}^{v_2,a}$	0.700	1.4914(55)	1.5131(55)	1.5266(54)	1.5660(53)
	0.789	1.4635(108)	1.4776(112)	1.4926(90)	1.5397(58)
$\mathcal{O}^{v_2,b}$	0.700	1.5061(37)	1.5218(37)	1.5329(36)	1.5534(35)
	0.789	1.4601(151)	1.4727(206)	1.4863(165)	1.5115(140)

Summary

- Described subtraction procedures for getting rid of lattice artifacts
- Numerical procedure for subtraction of all lattice artifacts in one-loop very efficient
- Not suited for more complicated operators and/or actions
- Fit procedure must be performed and adjusted for each operator individually
- Used data do not favour strongly an unique choice of g, subtraction type or parameter set c_i - compared to the complete one-loop subtraction results
- "External" arguments from boosted perturbation theory ($\rightarrow$ g_B), simplicity considerations ($\rightarrow$ (s)) and data momenta ($\rightarrow$ all c_i) served as guide lines
- Z^{RGI} from a^2 subtraction differ at most by 2% from the corresponding results obtained from the complete one-loop subtraction

Summary

- Described subtraction procedures for getting rid of lattice artifacts
- Numerical procedure for subtraction of all lattice artifacts in one-loop very efficient
- Not suited for more complicated operators and/or actions
- Fit procedure must be performed and adjusted for each operator individually
- Used data do not favour strongly an unique choice of g, subtraction type or parameter set c_i - compared to the complete one-loop subtraction results
- "External" arguments from boosted perturbation theory ($\rightarrow$ g_B), simplicity considerations ($\rightarrow$ (s)) and data momenta ($\rightarrow$ all c_i) served as guide lines
- $Z^{\rm RGI}$ from a^2 subtraction differ at most by 2% from the corresponding results obtained from the complete one-loop subtraction

Summary

- Described subtraction procedures for getting rid of lattice artifacts
- Numerical procedure for subtraction of all lattice artifacts in one-loop very efficient
- Not suited for more complicated operators and/or actions
- Fit procedure must be performed and adjusted for each operator individually
- Used data do not favour strongly an unique choice of g, subtraction type or parameter set c_i - compared to the complete one-loop subtraction results
- "External" arguments from boosted perturbation theory ($\rightarrow$ g_B), simplicity considerations ($\rightarrow$ (s)) and data momenta ($\rightarrow$ all c_i) served as guide lines
- Z^{RGI} from a^2 subtraction differ at most by 2% from the corresponding results obtained from the complete one-loop subtraction

Summary

- Described subtraction procedures for getting rid of lattice artifacts
- Numerical procedure for subtraction of all lattice artifacts in one-loop very efficient
- Not suited for more complicated operators and/or actions
- Fit procedure must be performed and adjusted for each operator individually
- Used data do not favour strongly an unique choice of g, subtraction type or parameter set c_i - compared to the complete one-loop subtraction results
- "External" arguments from boosted perturbation theory ($\rightarrow$ g_B), simplicity considerations ($\rightarrow$ (s)) and data momenta ($\rightarrow$ all c_i) served as guide lines
- Z^{RGI} from a^2 subtraction differ at most by 2% from the corresponding results obtained from the complete one-loop subtraction

Summary

- Described subtraction procedures for getting rid of lattice artifacts
- Numerical procedure for subtraction of all lattice artifacts in one-loop very efficient
- Not suited for more complicated operators and/or actions
- Fit procedure must be performed and adjusted for each operator individually
- Used data do not favour strongly an unique choice of g, subtraction type or parameter set c_i - compared to the complete one-loop subtraction results
- "External" arguments from boosted perturbation theory ($\rightarrow$ g_B), simplicity considerations ($\rightarrow$ (s)) and data momenta ($\rightarrow$ all c_i) served as guide lines
- Z^{RGI} from a^2 subtraction differ at most by 2% from the corresponding results obtained from the complete one-loop subtraction

Summary

- Described subtraction procedures for getting rid of lattice artifacts
- Numerical procedure for subtraction of all lattice artifacts in one-loop very efficient
- Not suited for more complicated operators and/or actions
- Fit procedure must be performed and adjusted for each operator individually
- Used data do not favour strongly an unique choice of g, subtraction type or parameter set c_i - compared to the complete one-loop subtraction results
- “External” arguments from boosted perturbation theory ($\rightarrow$ g_B), simplicity considerations ($\rightarrow$ (s)) and data momenta ($\rightarrow$ all c_i) served as guide lines
- Z^{RGI} from a^2 subtraction differ at most by 2% from the corresponding results obtained from the complete one-loop subtraction

Summary

- Described subtraction procedures for getting rid of lattice artifacts
- Numerical procedure for subtraction of all lattice artifacts in one-loop very efficient
- Not suited for more complicated operators and/or actions
- Fit procedure must be performed and adjusted for each operator individually
- Used data do not favour strongly an unique choice of g, subtraction type or parameter set c_i - compared to the complete one-loop subtraction results
- “External” arguments from boosted perturbation theory ($\rightarrow$ g_B), simplicity considerations ($\rightarrow$ (s)) and data momenta ($\rightarrow$ $\text{all} \quad c_i$) served as guide lines
- Z^{RGI} from a^2 subtraction differ at most by 2% from the corresponding results obtained from the complete one-loop subtraction

Acknowledgements

We thank:

SFB/TR55 (Hadron Physics from Lattice QCD)

EU grant 283286 (HadronPhysics3)

Cyprus Research Promotion Foundation under Contract No. TECHNOLOGY/ΘΕΠΙΣ/ 0311(BE)/16