Determination of c_A in three-flavour lattice QCD with Wilson fermions and tree-level improved gauge action

John Bulava Michele Della Morte Jochen Heitger
Christian Wittemeier

Lattice 2013, Mainz

31 July 2013

ALPHA Collaboration

Motivation

- Wilson fermions $\rightarrow \mathcal{O}(a)$-discretization errors
- Symanzik improvement:
  - add dimension-five term to action (Sheikoleslami–Wohlert term $\propto c_{\mathrm{SW}}$)
  - add dimension-four terms to quark bilinears

Improvement of Axial Current

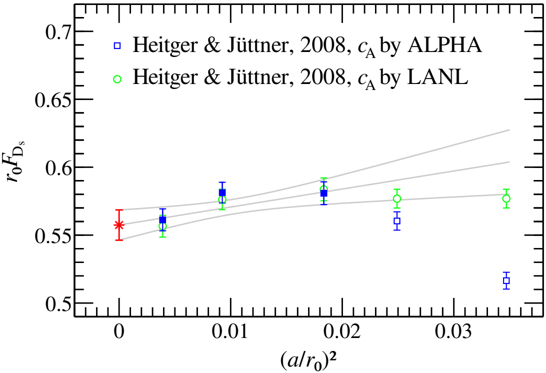
$$(A_{\mathrm{I}})^a_\mu(x) = A^a_\mu(x) + a c_A \frac{1}{2} \left(\partial_\mu + \partial^*_\mu\right) P^a(x)$$

$$A^a_\mu(x) = \bar{\psi}(x) T^a \gamma_\mu \gamma_5 \psi(x) \qquad P^a(x) = \bar{\psi}(x) T^a \gamma_5 \psi(x)$$

applications: e.g. PCAC quark masses, decay constant F_{PS} (in particular for scale setting with f_{K})

improvement coefficient c_A must be determined, ideally in a non-perturbative way

Motivation



Heitger, Jüttner (arxiv:0812.2200)

- two analyses with c_A differing by $\mathcal{O}(a)$ ambiguities
- different behaviour at larger a

$\rightarrow$ we define c_A on a *line of constant physics*
ambiguities are expected to disappear smoothly

Basic Strategy of c_A determination

PCAC quark mass

$$m(x;\alpha,\beta) = r(x;\alpha,\beta) + ac_A \cdot s(x;\alpha,\beta)$$

$$r(x;\alpha,\beta) = \frac{\left\langle \beta \left| \frac{1}{2}(\partial_\mu + \partial^*_\mu)(A(x))^a_0 \right| \alpha \right\rangle}{2\left\langle \beta \left| P(x)^a \right| \alpha \right\rangle}$$

$$s(x;\alpha,\beta) = \frac{\left\langle \beta \left| \partial_\mu \partial^*_\mu (P(x))^a \right| \alpha \right\rangle}{2\left\langle \beta \left| P(x)^a \right| \alpha \right\rangle}$$

ALPHA Collaboration 1996, 2005

- continuum: PCAC mass independent of $|\alpha\rangle$, $|\beta\rangle$, x
- lattice: $\mathcal{O}(a)$ ambiguities without improvement
- c_A defined by improvement condition

$$m(x;\alpha,\beta) \overset{!}{=} m(x;\gamma,\delta) \quad \Longrightarrow \quad c_A = -\frac{1}{a} \cdot \frac{r(x;\alpha,\beta) - r(x;\gamma,\delta)}{s(x;\alpha,\beta) - s(x;\gamma,\delta)}$$

Previous Works in the Alpha Collaboration

$N_f = 0$ (hep-lat/9609035)

- used fields with different periodicity angle θ
  $\rightarrow$ low sensitivity for $L \gtrsim 0.8\,\mathrm{fm}$
  $\rightarrow$ would require individual simulations with $N_f > 0$

$N_f = 2$ (hep-lat/0503003)

$N_f = 3$ and Iwasaki gauge action (hep-lat/0703006)

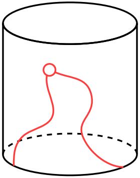
- used states with different wave functions ω
  sensitivity $\propto m^2_{\pi^{(1)}} - m^2_{\pi^{(0)}}$
- physical scales kept fixed (*line of constant physics*)
  $\rightarrow \mathcal{O}(a)$ ambiguities of c_A disappear smoothly

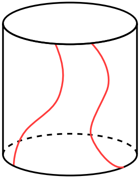
Now

- $N_f = 3$ and tree-level-improved gauge action (*Lüscher–Weisz*)
- NP'ly improved three-flavor Wilson-fermion action: c_{SW} from Bulava and Schaefer (arxiv:1304.7093)

Setup

- Schrödinger functional (periodic BC in space, Dirichlet in time)
- correlators with wave functions at the boundaries:





$$f_A(x_0;\omega) = -\frac{a^3}{3L^6}\sum_{\mathbf{x}}\langle A_0^a(x)O^a(\omega)\rangle$$

$$f_P(x_0;\omega) = -\frac{a^3}{3L^6}\sum_{\mathbf{x}}\langle P^a(x)O^a(\omega)\rangle$$

$$f_1(\omega',\omega) = -\frac{1}{3L^6}\left\langle O'^a(\omega')O^a(\omega)\right\rangle$$

$$O^a(\omega) = a^6\sum_{\mathbf{xy}}\bar{\zeta}(\mathbf{x})\cdot T^a\gamma_5\cdot\omega(\mathbf{x}-\mathbf{y})\cdot\zeta(\mathbf{y})$$

$$r(x_0;\omega) = \frac{\frac{1}{2}(\partial_\mu+\partial_\mu^*)f_A(x_0;\omega)}{2f_P(x_0;\omega)} \qquad s(x_0;\omega) = \frac{\partial_\mu\partial_\mu^* f_P(x_0;\omega)}{2f_P(x_0;\omega)}$$

Wave Functions

choose WF $\omega_{\pi^{(0)}}$ and $\omega_{\pi^{(1)}}$ that couple only to the ground and first excited state

- define (periodic) basis functions

$$\bar{\omega}_1(r) = \mathrm{e}^{-r/r_0} \qquad \bar{\omega}_2(r) = r \cdot \mathrm{e}^{-r/r_0} \qquad \bar{\omega}_3 = \mathrm{e}^{-r/(2r_0)}$$

$$\omega_i(x) = N_i \sum_{\mathbf{n}\in\mathbb{Z}^3} \bar{\omega}_i(|x - \mathbf{n}L|)$$

(with some physical length scale r_0 and normalization N_i)

- determine eigenvalues $\lambda^{(0)} > \lambda^{(1)} > \lambda^{(2)}$ and eigenvectors $\eta^{(0)}$, $\eta^{(1)}$, $\eta^{(2)}$ of 3×3 matrix $f_1(\omega_i, \omega_j)$
- approximate $\omega_{\pi^{(0)}}$ and $\omega_{\pi^{(1)}}$ by eigenvectors

$$\omega_{\pi^{(0)}} \approx \sum_i \eta_i^{(0)} \omega_i \qquad \omega_{\pi^{(1)}} \approx \sum_i \eta_i^{(1)} \omega_i$$

Simulations

- generation of gauge configurations by openQCD code with Schrödinger-functional BC

  Lüscher, Schaefer (`arxiv:1206.2809`)
  (poster by Stefan Schaefer)

  - frequency splitting of the quark determinant
  - twisted-mass regularization was not necessary in most cases
  - Zolotarev rational approximation for third quark $\rightarrow$ reweighting
  - two- and three-level integration schemes

- Schrödinger-functional setup
  - $\theta = 0$, vanishing background field
  - $T = 3/2 \cdot L$
    $\rightarrow$ potential re-use in determination of Z_A

Line of Constant Physics

- β tuned to keep L constant for various lattice sizes via perturbative formula ($g_0 < g_0'$)

$$\frac{a(g_0^2)}{a(g_0'^2)} = \mathrm{e}^{-\left(g_0^{-2}-g_0'^{-2}\right)/(2b_0)} \left(g_0^2/g_0'^2\right)^{-b_1/(2b_0^2)} \times \left[1 + q\left(g_0^2 - g_0'^2\right) + \mathcal{O}(g_0'^4)\right]$$

  3-loop contrib. from β-function in q not known for LW action
  $\rightarrow$ only universal parts used

L/a	12	16	20	24
β	3.3	3.512	3.676	3.810

  $\rightarrow L \approx 1.2\,\mathrm{fm}$

- κ tuned to get almost vanishing (PCAC) quark mass ($|aM| < 0.015$), first estimate of κ_c can be guessed from Bulava, Schaefer (`arxiv:1304.7093`)

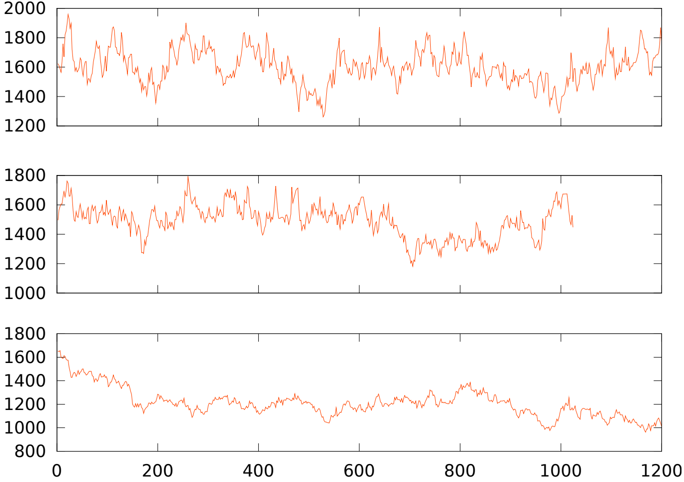
Histories

- plaquette actions at Wilson flow time $t = (cL)^2/8$ with $c = 0.35$

$L/a = 12$
$T/a = 19$
$\kappa \approx 0.13673$

$L/a = 16$
$T/a = 23$
$\kappa = 0.1369$

$L/a = 24$
$T/a = 35$
$\kappa \approx 0.13712$



Histories

- topological charges at Wilson flow time $t = (cL)^2/8$ with $c = 0.35$

$L/a = 12$
$T/a = 19$
$\kappa \approx 0.13673$

$L/a = 16$
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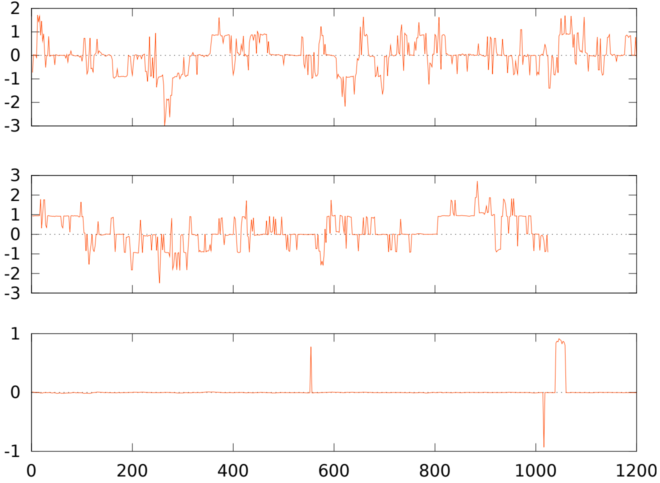

Histories

- reweighting factors for rational approximation

$L/a = 12$
$T/a = 19$
$\kappa \approx 0.13673$

$L/a = 16$
$T/a = 23$
$\kappa = 0.1369$

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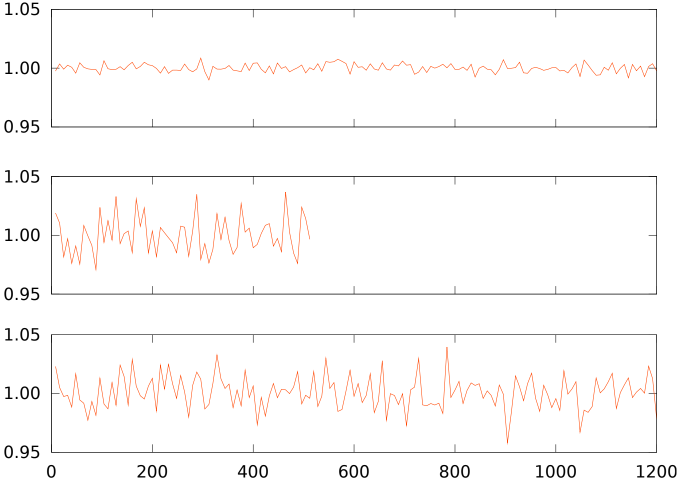
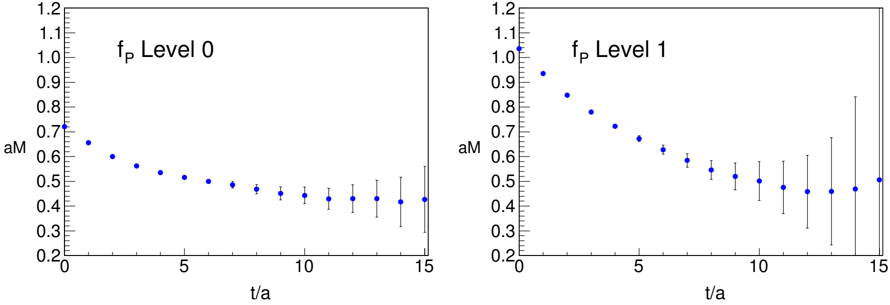


Effective Masses



effective masses in lattice units determined from f_P on $L = 24$ lattice for approximated wave functions of ground and first excited state

Summary

- improved axial current needed for reliable decay constants, in mass tuning etc.

Method

- via PCAC mass
- improvement condition uses wave functions that resemble ground and first excited state $\rightarrow$ sensitivity $\propto m^2_{\pi^{(1)}} - m^2_{\pi^{(0)}}$
- Schrödinger-functional setup
- simulations along a line of constant physics

Status

- tuning the mass, producing configurations, measuring correlators
- preliminary results probably in the proceedings