The Gradient Flow Coupling in Minimal Walking Technicolor

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- Technicolor requires (nearly) conformal field theories
- Direct lattice calculations of the β-function expensive
    - High accuracy required to distinguish slow running
    - Large coupling, massless fermions
- Running coupling from gradient flow a promising method
- Minimal Walking Technicolor:
    - SU(2) gauge field theory
    - 2 fermions in the adjoint representation
    - Schrödinger functional coupling shows conformal behaviour
    - Relatively easy to simulate on the lattice

Wilson Flow

- Flow along the gradient of the Wilson gauge action [1]

$$\begin{aligned} \partial_t B_{t,\mu} &= D_{t,\mu} B_{t,\mu\nu}, \\ B_{0,\mu} &= A_\mu \\ B_{t,\mu\nu} &= \partial_\mu B_{t,\nu} - \partial_\nu B_{t,\mu} + [B_{t,\mu}, B_{t,\nu}] \end{aligned}$$

- Diffusive flow with scale $\sqrt{8t}$
- Produces a finite, smoothed field
- Correlators of the flow field renormalized, encode physical properties

[1] M. Luscher and P. Weisz, JHEP **1102** (2011) 051 [arXiv:1101.0963 [hep-th]].

Wilson Flow

- For example, the energy density:

$$\begin{aligned}\langle E(t)\rangle &= \frac{1}{4}\langle G_{\mu\nu}(t)G_{\mu\nu}(t)\rangle \\ &= Ng_0^2/t^2 + \mathcal{O}(g_0^4)\end{aligned}$$

- Define a renormalized coupling

$$g_{GF}^2 = \frac{t^2\langle E(t)\rangle}{N}$$

Wilson Flow

$$g_{GF}^2 = \frac{t^2 \langle E(t) \rangle}{N}$$

- Integrate the flow equation to a time t and calculate the $t^2 \langle E(t) \rangle$
- Compared to the SF coupling expecting:
  - Small statistical errors
  - Smaller mass dependence
  - Noticeable discretization effects

Step Scaling Function

- For step scaling, define coupling with a single scale
- Fix the flow time t to the lattice size L

$$\sqrt{8t} = cL$$

- Here we use $c = 0.5$
- Measure the running by the step scaling function

$$\Sigma(u, \mathrm{a}/L) = g^2_{GF}(g_0, 2L/\mathrm{a})\Big|_{g^2_{GF}(g_0, L/\mathrm{a})=u}$$

$$\sigma(g^2) = \lim_{\mathrm{a}\to 0} \Sigma(g^2_{GF}, \mathrm{a}/L)$$

The model

$$S = (1 - c_g) S_G(U) + c_g S_G(V) + S_F(V) + c_{SW} \delta S_{SW}(U)$$

- Standard gauge action $S_G(U)$
- Smeared Wilson fermion action $S_F(V)$
- and gauge action $S_G(V)$
- HEX Smearing:
  - three sequential stout smearing steps
  - taking only orthogonal directions
- Bulk correction term $\delta S_{SW}(U)$
- Here, $c_g = 0.5$ and $c_{SW} = 1$

Boundary Conditions

- Schrödinger Functional boundary conditions [2]
  - Unique global minimum: simpler perturbation theory
- Periodic spatial boundary conditions, trivial time boundaries

$$U_k(x) = 1, \text{ when } x_0 = 0, L$$
$$U_\mu(x + L\hat{k}) = U_\mu(x)$$

- Fermion fields disappear at time boundaries, twisted in spatial directions

$$\psi(x + L\hat{k}) = e^{i\pi/5}\psi(x).$$

[2] P. Fritzsch and A. Ramos, arXiv:1301.4388 [hep-lat].

Boundary Conditions

- Time translation symmetry broken
- Measure the coupling using the middle time slice

$$M_{GF} = N g_{GF}^2 = t^2 \langle E(t, x_0) \rangle , \quad x_0 = L/2, t = c^2 L^2 / 8$$

- Because of the smearing $N(\mathrm{a}/L)$ is unknown
- Continuum value for N, with $c = 0.5$ and $N_c = 2$

$$N_{\mathrm{cont}} = 288.527$$

The SF coupling

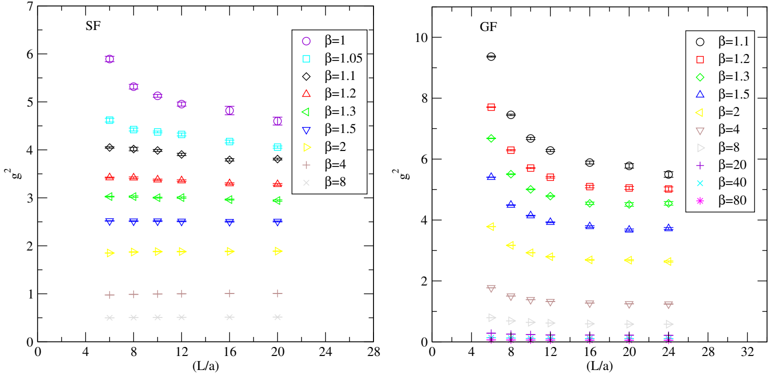
- Comparison to the Schrödinger Functional coupling
- Non-trivial boundaries in time direction

$$U(x_0 = 0) = U_0(\eta),$$
$$U(x_0 = T) = U_T(\eta)$$

- Coupling response to the boundaries

$$\left\langle \frac{\partial S}{\partial \eta} \right\rangle = \frac{k}{g^2_{SF}(g_0, L/\mathrm{a})}$$

The coupling



Statistical errors

	SF		GF		
L/a	20	20	20	20	24
$\beta = 4/g_0^2$	2	1.2	2	1.2	1.2
N_{traj}	151203	158828	14065	13693	9648
$(\delta g^2/g^2)_{\text{statistical}}$	0.0059	0.0109	0.0112	0.0121	0.0150
$\left(\delta g^2/g^2\right)^2 N_{\text{traj}}$	5.225	18.75	1.765	2.048	2.250
t_{traj}	1221.11	1277.04	585.58	769.64	1787.13

Volume Scaling

SF, $\beta = 2$

L/a	$\left(\delta g^2/g^2\right)^2 N_{\mathrm{traj}}$
6	0.093
8	0.259
10	0.265
12	0.880
16	3.791
20	5.225

GF, $\beta = 1.2$

L/a	$\left(\delta g^2/g^2\right)^2 N_{\mathrm{traj}}$
6	0.577
8	0.373
10	0.988
12	0.885
16	1.191
20	2.048
24	2.250

Continuum Limit

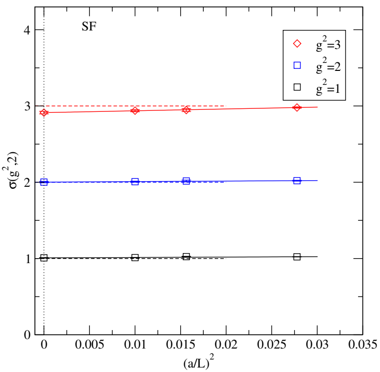
- Continuum limit of σ at constant g^2
- Since we measure at constant g_0^2, need to extrapolate

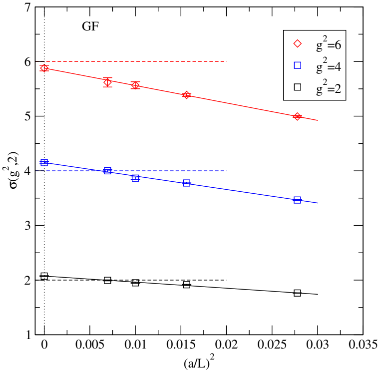
$$g_{GF}^2(g_0, L/\mathrm{a}) = a_0 g_0^2 \frac{\prod_{k=1}^{2}(1 + a_k g_0^2)}{\prod_{k=1}^{3}(1 + b_k g_0^2)},$$
$$g_{SF}^2(g_0, L/\mathrm{a}) = g_0^2 \frac{\prod_{k=1}^{3}(1 + a_k g_0^2)}{\prod_{k=1}^{3}(1 + b_k g_0^2)}$$

- a_0 parametrizes a discretization error
  $\rightarrow$ Scaled coupling $g_{GF}^2/a_0(L/\mathrm{a})$
- Expecting leading discretization effect to be order a^2

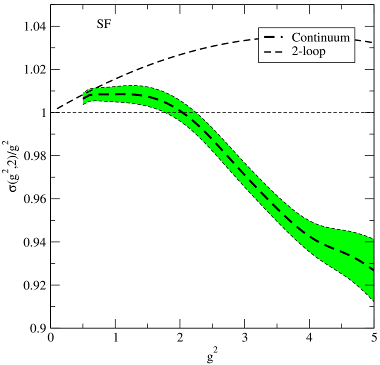
$$\Sigma(g^2, \mathrm{a}/L) = \sigma(g^2) + c\mathrm{a}^2/L^2$$

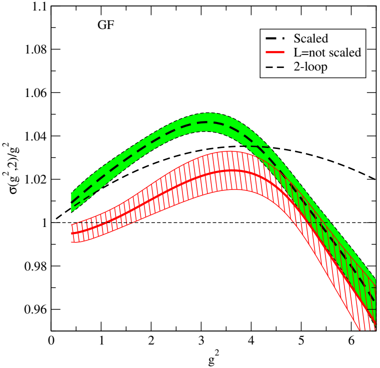
Continuum Limit





Continuum Limit





To conclude

- Step scaling calculated from the gradient flow coupling
- Higher statistical accuracy
- Better scaling with volume
- Worse discretization effects
- Fixed point appears at a higher bare coupling

Measuring the anomalous dimension

- The mass anomalous dimension γ_m can be measured from the pseudoscalar density renormalization constant

$$Z_P(g_0, L/\mathrm{a}) = \frac{\sqrt{3f_1}}{f_P(L/2\mathrm{a})}$$

- Scales like $\frac{1}{\overline{m}(\mathrm{a}/L)}$.

Measuring the anomalous dimension

$$\Sigma_P(u, a/L) = \left.\frac{Z_P(g_0, 2L/a)}{Z_P(g_0, L/a)}\right|_{g^2(g_0, L/a)=u}$$

$$\sigma_P(g^2) = \lim_{a \to 0} \Sigma_P(g^2, a/L)$$

Mass anomalous dimension