

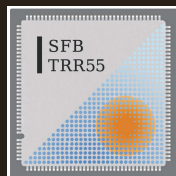
D_s to $\eta(\prime)$ semi-leptonic decay form factors

Issaku Kanamori (University of Regensburg)
for SFBTRR55

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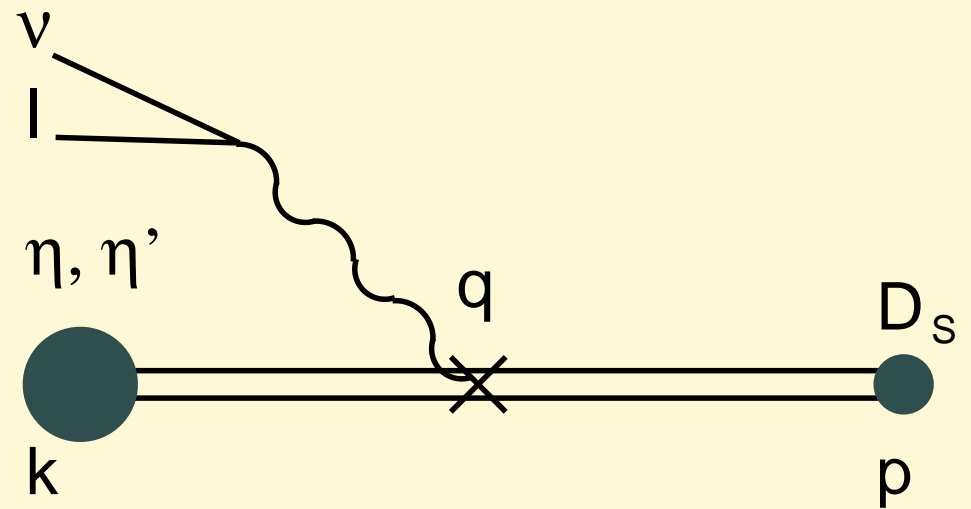
Based on on-going work

cf. PoS LATTICE2011 (2011) 283 [arXiv:1111.4053]
PoS ConfinementX (2012) 143 [arXiv:1302.6087]



Outline

- Introduction
- Extracting η and η' states
- Decay Form Factor
- Conclusion



Introduction

Semileptonic Decay of D -meson (e.g. $D \rightarrow l\nu K$)

- $\frac{d\Gamma}{dq^2} = (\text{kin. factor}) |V_{CKM}|^2 \overbrace{|(\text{form factor})|^2}^{\text{lattice}}$
well-studied in both exp. and lat., high prec. competition

D_S : the major semi-leptonic decay is $D_S \rightarrow l\nu\eta, l\nu\eta'$

- Experiment: only branching fractions in the PDG
- Theory: no lattice calculations so far
(available: light cone QCD sum rule predictions)
- Interesting for η/η' mixing, gluonic contrib.
- Challenging: disconnected fermion loops
- Interesting play ground for QFT:
contributions from anomaly (cf. Witten-Veneziano formula)

Form Factor

$$\langle \eta^{(\prime)}(k) | V^\mu(q^2) | D_s(p) \rangle$$

$$= f_0(q^2) \frac{M_{D_s}^2 - M_{\eta^{(\prime)}}^2}{q^2} q^\mu + f_+(q^2) \left[(p+k)^\mu - \frac{M_{D_s}^2 - M_{\eta^{(\prime)}}^2}{q^2} q^\mu \right]$$

We focus on the scalar form factor:

$$f_0(q^2) = \frac{m_c - m_s}{M_{D_s}^2 - M_{\eta^{(\prime)}}^2} \langle \eta^{(\prime)} | S | D_s \rangle$$

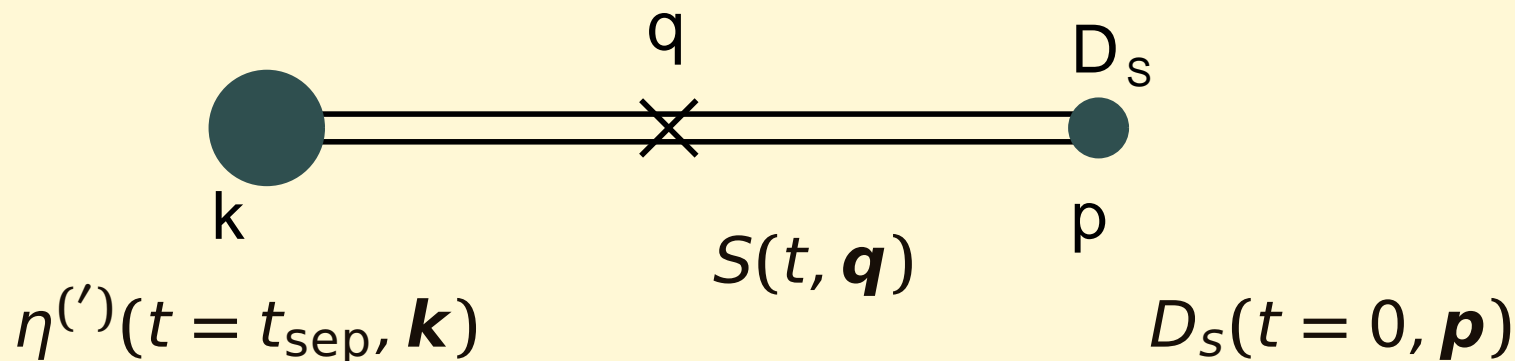
scalar current: $S = \bar{s}c$ $(m_c - m_s)S$: no renormalization

H.Na et al.(HPQCD)

Target observable:

$$\langle 0 | \mathcal{O}_{\eta^{(\prime)}}(t_{\text{sep}}, \mathbf{k}) S(t, \mathbf{q}) \mathcal{O}_{D_s}^\dagger(0, \mathbf{p}) | 0 \rangle$$

$$\sim e^{-E_{\eta^{(\prime)}} t - E_{D_s}(t_{\text{sep}} - t)} \langle \eta^{(\prime)}(k) | S(q^2) | D_s(p) \rangle$$

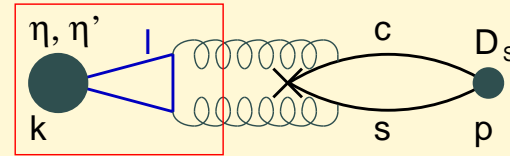


Disconnected fermion loops

$$3 \text{ pt func.} = \text{Diagram} - \sum_{l=u,d,s} \left(\text{Diagram}_1 \quad \text{Diagram}_2 \right)$$

The diagram on the left shows a 3-point function with an incoming fermion line from a dark blue vertex labeled p with momentum p and flavor η, η' . It splits into two paths: an upper path with a fermion line labeled s and a gluon loop labeled q , and a lower path with a fermion line labeled s . Both paths recombine at a red vertex labeled p_b with momentum p_b and flavor D_s . A blue line labeled c connects the two vertices. The two diagrams in the parentheses represent disconnected parts: the first is a fermion loop labeled l with momentum k and flavor η, η' ; the second is a fermion loop with a gluon loop, with a fermion line labeled s and a gluon line labeled q , and a vertex labeled p with momentum p and flavor D_s .

- u, d, s : enhancement by factor 3
- η' : contributions from anomaly

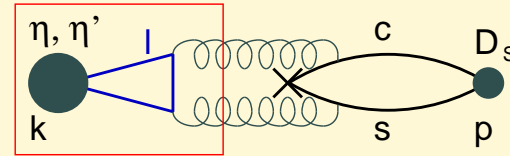


The disconnected part may contribute significantly

Disconnected fermion loops

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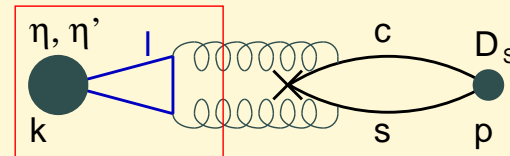
...but noisy and expensive in lattice calculation

(needs **all-to-all** propagators)

Disconnected fermion loops

$$3 \text{ pt func.} = \text{Diagram} - \sum_{l=u,d,s} \left(\text{Diagram}_1 + \text{Diagram}_2 \right)$$

- u, d, s : enhancement by factor 3
- η' : contributions from anomaly



The disconnected part may contribute significantly

...but noisy and expensive in lattice calculation

(needs **all-to-all** propagators)

⇒ **manageable** I.K. Lattice 2011

- stochastic estimation
(also for the connected part Evans-Bali-Collins)
- low mode averaging
- truncated solver method Bali-Collins-Schäfer
- ...

Configurations

QCDSF 2+1 flavor configurations: W.Bietenholz *et al.*[QCDSF collab.]

$$m_u + m_d + m_s = \text{fixed}$$

$$m_u = m_d = m_s \longrightarrow m_u(=m_d) \downarrow, m_s \uparrow$$

SU(3) basis:

$$\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) \quad \eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

- stout link non-perturbative clover
charm quark: relativistic partially quenched approximation
- $\beta = 5.5, 24^3 \times 48, a \sim 0.08$ fm
 - $m_\pi \sim 450$ MeV (SU(3) symmetric) 939 confs
 - $m_\pi \sim 348$ MeV 239 confs
- (planning: $32^3 \times 64$)

Extracting η and η' states

2 point functions

Building blocks: $\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$, $\eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$

$$\begin{pmatrix} (\eta_8 \rightarrow \eta_8) & (\eta_8 \rightarrow \eta_1) \\ (\eta_1 \rightarrow \eta_8) & (\eta_1 \rightarrow \eta_1) \end{pmatrix} \xRightarrow{\text{diagonalize}} \begin{pmatrix} (\eta \rightarrow \eta) & 0 \\ 0 & (\eta' \rightarrow \eta') \end{pmatrix}$$

\Rightarrow masses, interpolators for the physical states \mathcal{O}_η , $\mathcal{O}_{\eta'}$

ex.) $(\eta_8 \rightarrow \eta_8) = \langle \mathcal{O}_8(t) \mathcal{O}_8^\dagger(0) \rangle$

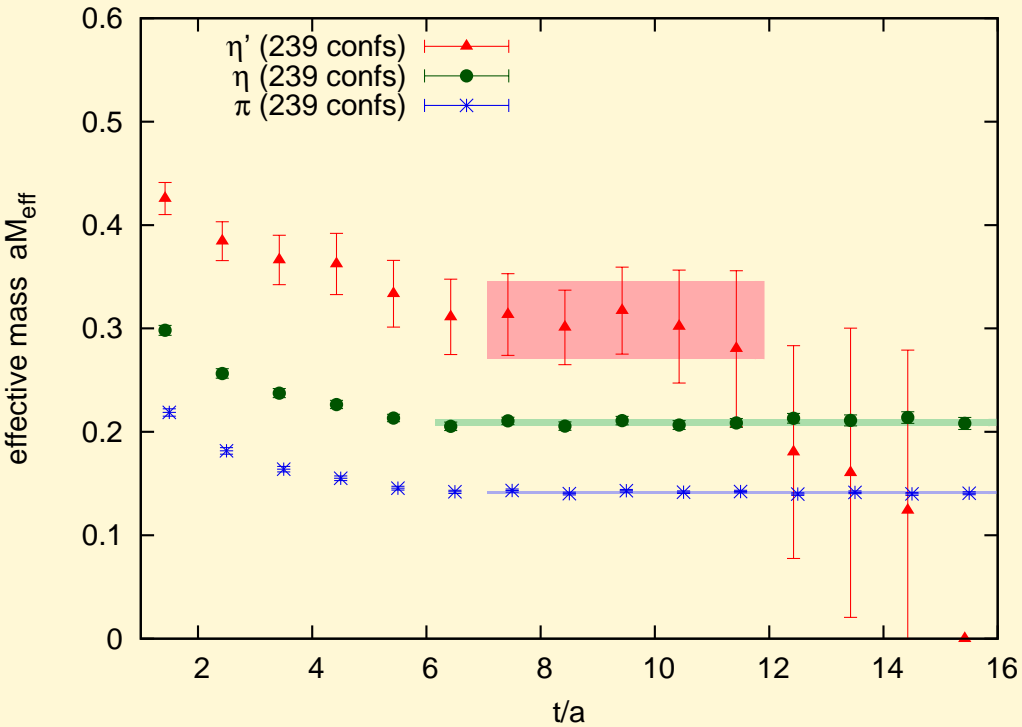
$$= \frac{1}{3} \left[\begin{array}{c} \text{diagram 1} \\ \text{diagram 2} \\ \text{diagram 3} \\ \text{diagram 4} \\ \text{diagram 5} \\ \text{diagram 6} \end{array} \right]$$

The diagrams are:

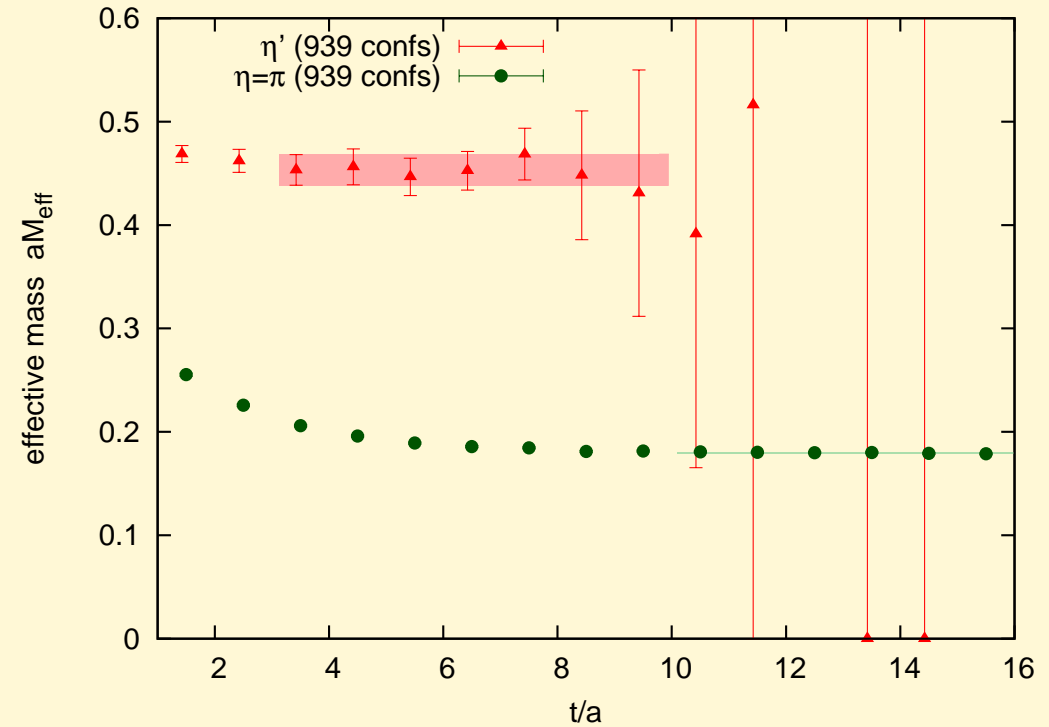
- Diagram 1: Two vertices connected by two arcs, both labeled l .
- Diagram 2: Two vertices connected by two arcs, both labeled s .
- Diagram 3: Two vertices, each with a self-loop labeled l .
- Diagram 4: Two vertices, each with a self-loop labeled s .
- Diagram 5: Two vertices, the left one has a self-loop labeled l and the right one has a self-loop labeled s .
- Diagram 6: Two vertices, the left one has a self-loop labeled s and the right one has a self-loop labeled l .

effective mass

$m_\pi = 348\text{MeV} (m_{u,d} < m_s)$



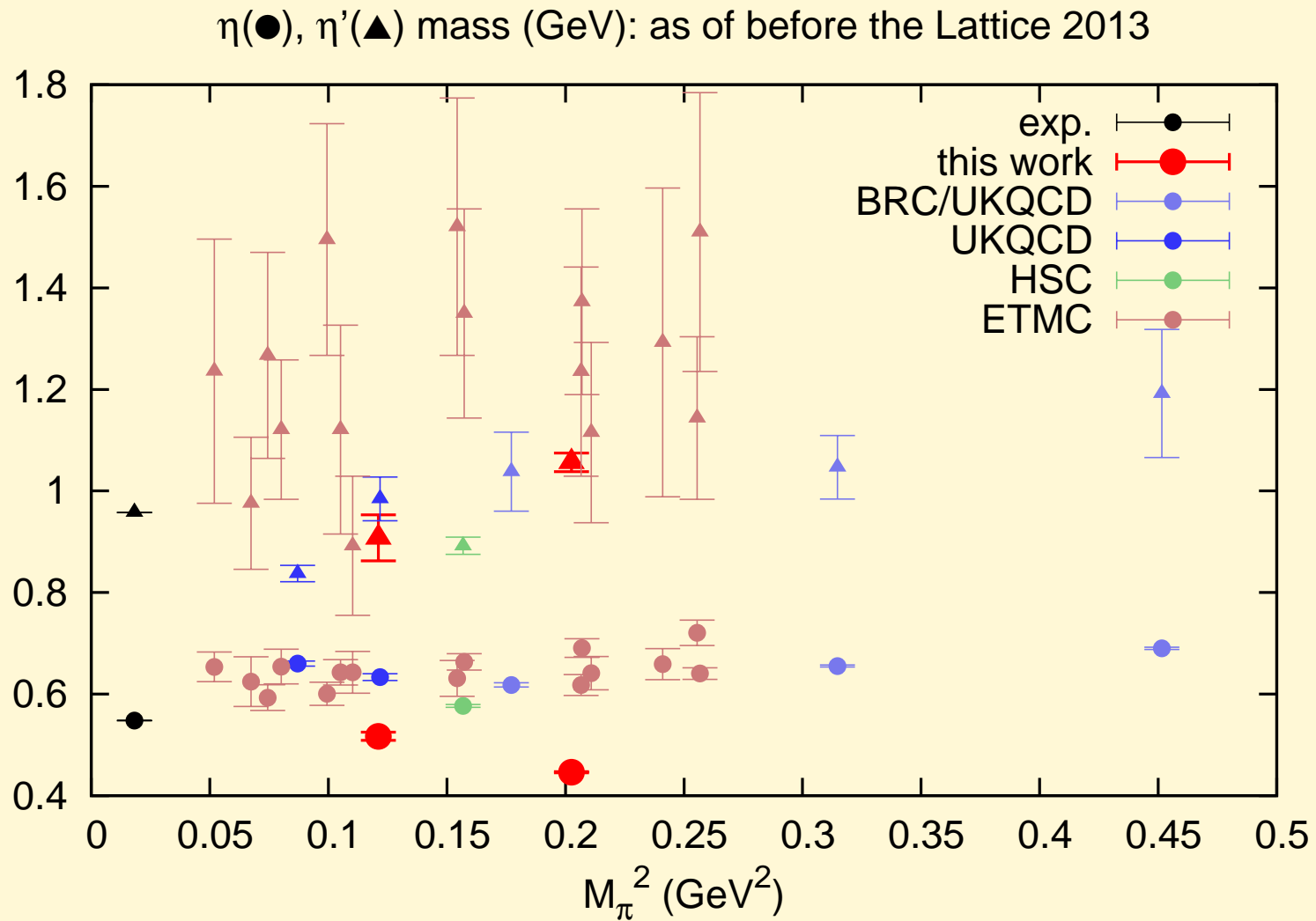
$m_\pi = 450\text{MeV} (m_{u,d} = m_s)$



(fittings above: with $\mathbf{p} = \mathbf{0}$ data only)

We can use $\mathbf{p} \neq \mathbf{0}$ data + dispersion relation

mass (preliminary)



η' mass and the topological fluctuation: — finite volume effect —

finite volume effect with fixed topological charge Q :

$$\langle \rho(x)\rho(0) \rangle_Q = -\frac{1}{V_4}(\chi_t - \frac{Q^2}{V_4}) + \dots \text{ for } |x| \rightarrow \infty$$

ρ : topological charge density

Aoki-Fukaya-Hashimoto-Onogi

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Since $\rho \sim \eta_1$, if the statistics is not large enough, one may observe the constant part:

$$\langle \mathcal{O}_{\eta_1}(\mathbf{p} = 0, t) \mathcal{O}_{\eta_1}^\dagger(\mathbf{p} = 0, 0) \rangle \xrightarrow{t \rightarrow \infty} A \exp(-m_{\eta_1} t) + c$$

(at $m_u = m_d = m_s$ point)

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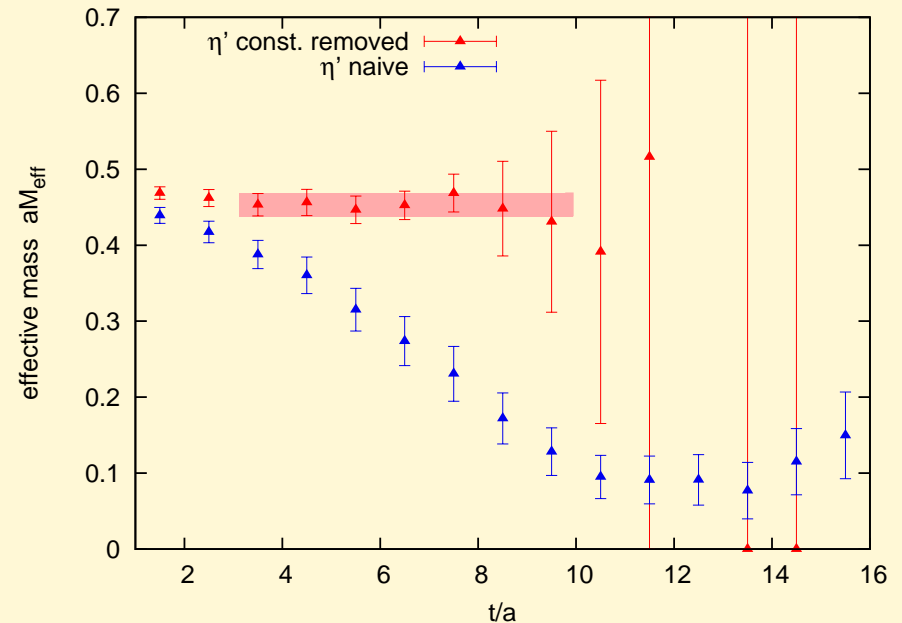
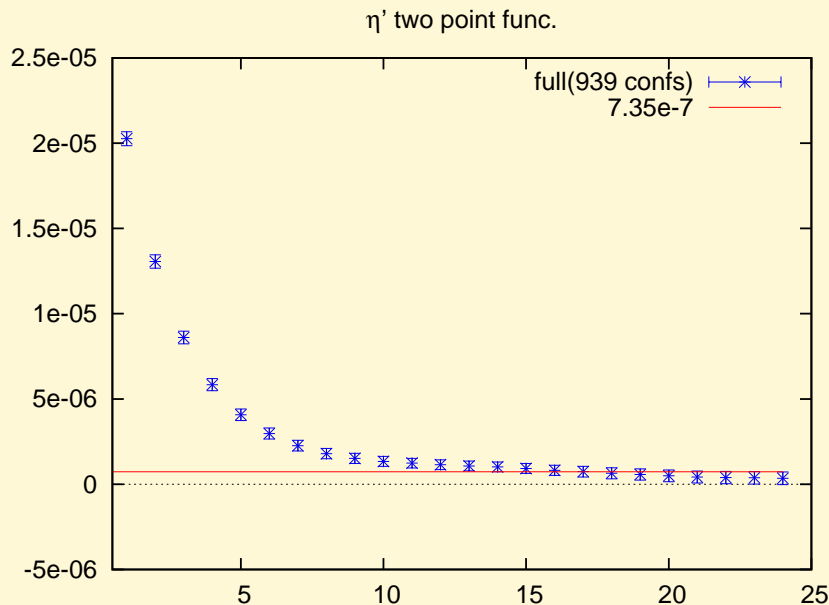
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needed to remove the constant part

Decay Form Factor

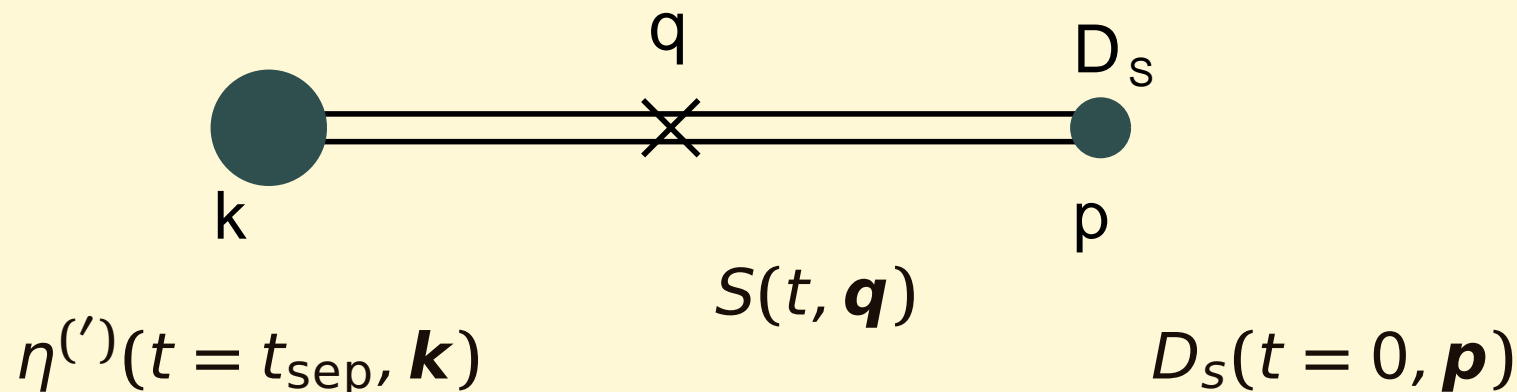
Correlation functions to matrix element

Now we have \mathcal{O}_η and can calculate: (the same for η')

- $$C_{3\text{pt}}(t) = \langle 0 | \mathcal{O}_\eta(\mathbf{k}, t_{\text{sep}}) S(\mathbf{q}, t) \mathcal{O}_{D_s}^\dagger(\mathbf{p}, 0) | 0 \rangle$$

$$= \frac{Z_\eta}{2E_\eta} \frac{Z_{D_s}}{2E_{D_s}} \exp(-E_{D_s}t - E_\eta(t_{\text{sep}} - t)) \times \left[\langle \eta | S | D_s \rangle + \dots \right]$$
- $$C_{2\text{pt}}^\eta(t, \mathbf{k}) = \frac{|Z_\eta(\mathbf{k})|^2}{2E_\eta(\mathbf{k})} \exp(-E_\eta(\mathbf{k})t) + \dots$$
- $$C_{2\text{pt}}^{D_s}(t, \mathbf{p}) = \frac{|Z_{D_s}(\mathbf{p})|^2}{2E_{D_s}(\mathbf{p})} \exp(-E_{D_s}(\mathbf{p})t) + \dots$$

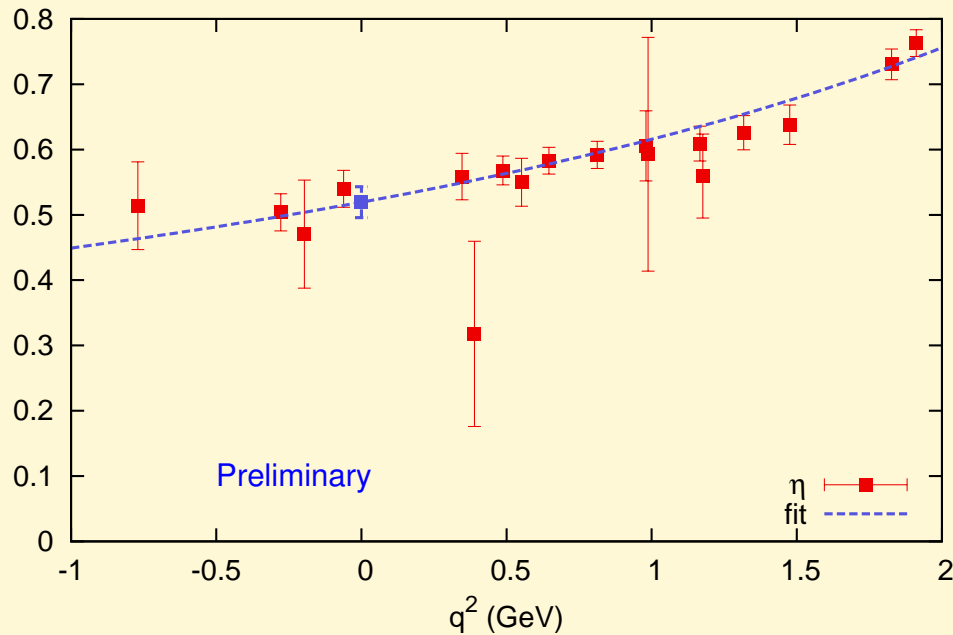
$\Rightarrow \langle \eta(k) | S(q^2) | D_s(p) \rangle \Rightarrow$ form factor $f_0(q^2)$



results: $D_s \rightarrow l\nu\eta$

$m_\pi = 450\text{MeV}$ (SU(3))

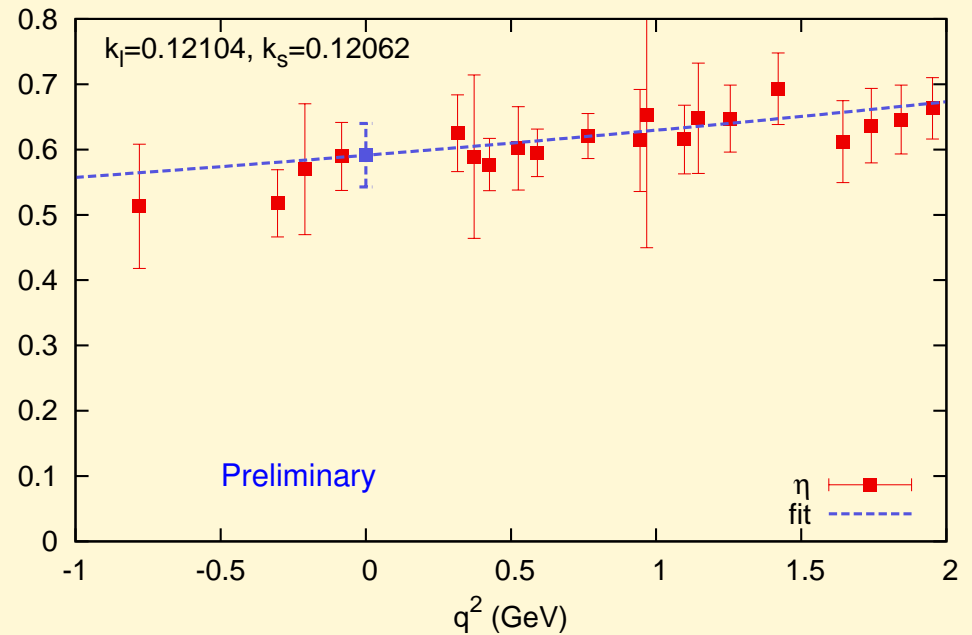
$f_0(q^2): D_s \rightarrow l\nu\eta$ ($m_\pi = 450\text{ MeV}$)



$$f_0^\eta(0) = 0.52(2)$$

$m_\pi = 348\text{MeV}$

$f_0(q^2): D_s \rightarrow l\nu\eta$ ($m_\pi = 348\text{ MeV}$)



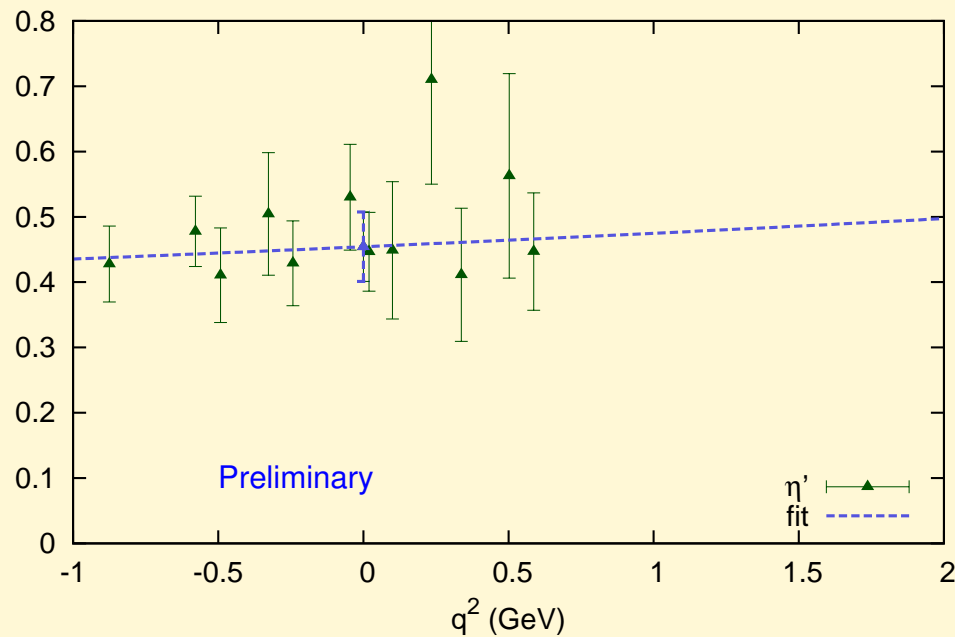
$$f_0^\eta(0) \sim 0.58(5)$$

$$(\text{fit with } f_0(q^2) = \frac{a}{1-bq^2})$$

results: $D_s \rightarrow l\nu\eta'$

$m_\pi = 450\text{MeV}$ (SU(3))

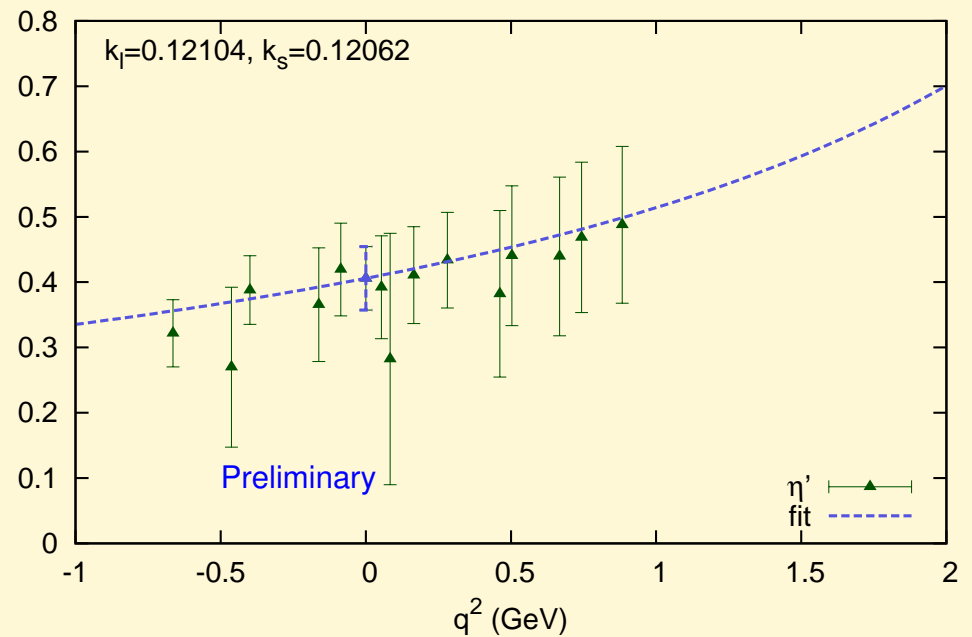
$f_0(q^2): D_s \rightarrow l\nu\eta'$ ($m_\pi = 450\text{ MeV}$)



$$f_0^{\eta'}(0) = 0.45(5)$$

$m_\pi = 348\text{MeV}$

$f_0(q^2): D_s \rightarrow l\nu\eta'$ ($m_\pi = 348\text{ MeV}$)



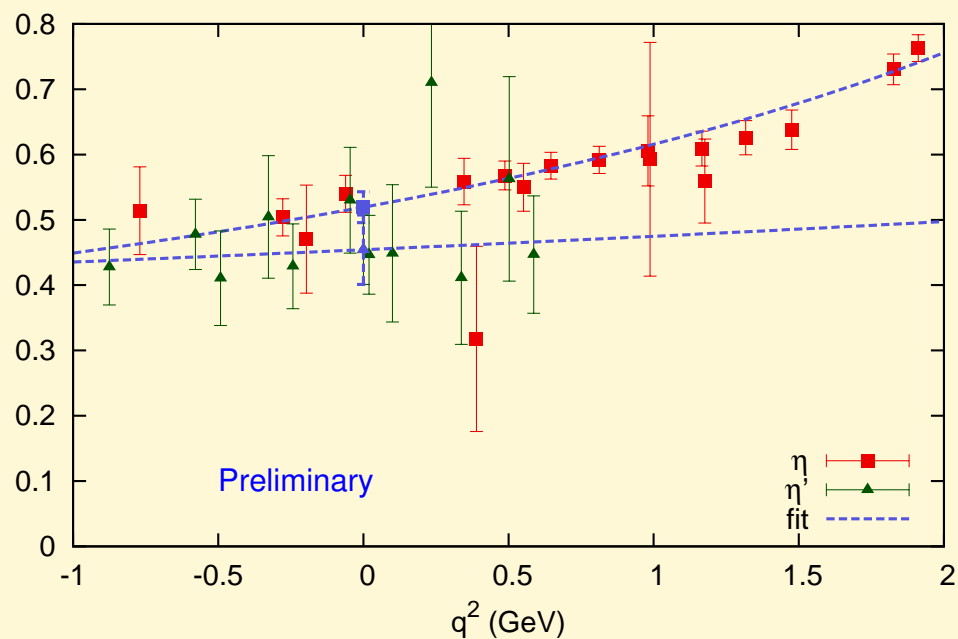
$$f_0^{\eta'}(0) \sim 0.42(5)$$

$$(\text{fit with } f_0(q^2) = \frac{a}{1-bq^2})$$

results

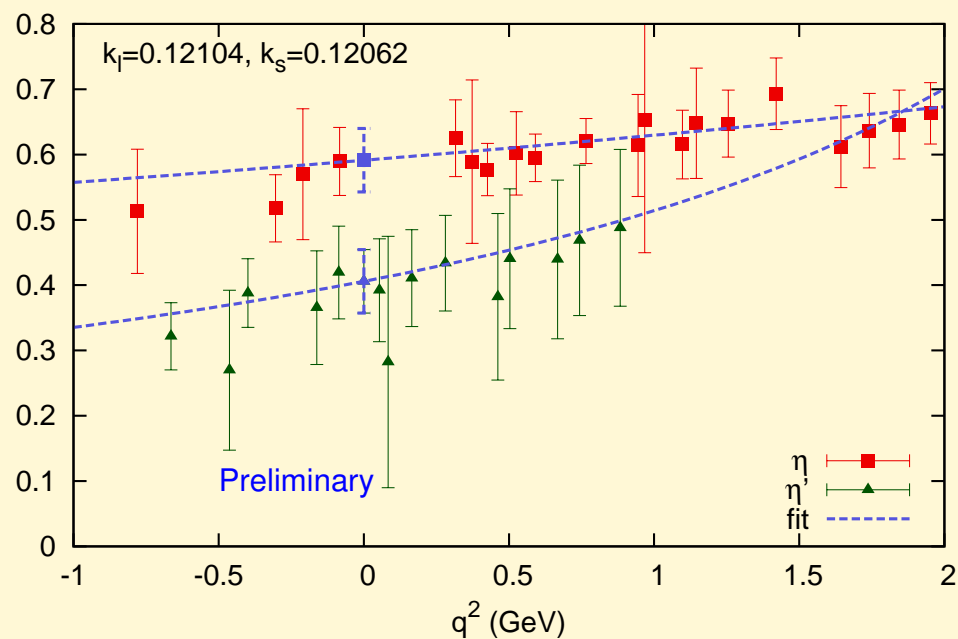
$m_\pi = 450\text{MeV}$ (SU(3))

$f_0(q^2): D_s \rightarrow l\nu\eta^{(\prime)}$ ($m_\pi = 450\text{ MeV}$)



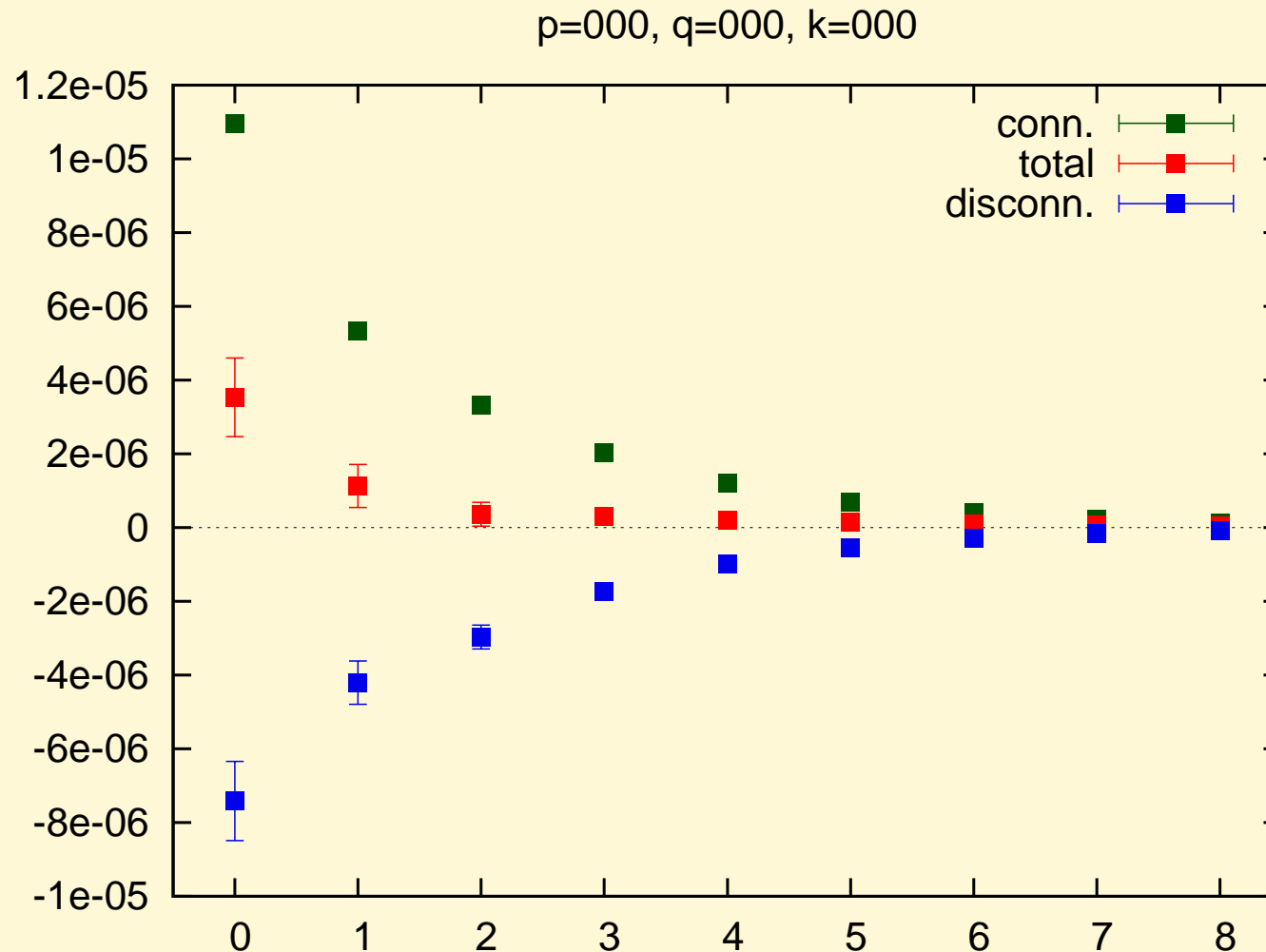
$m_\pi = 348\text{MeV}$

$f_0(q^2): D_s \rightarrow l\nu\eta^{(\prime)}$ ($m_\pi = 348\text{ MeV}$)



$$\left(\text{fit with } f_0(q^2) = \frac{a}{1-bq^2}\right)$$

Connected vs. Disconnected



3 point function for $\langle \eta' | S(t) | D_S \rangle$, $m_u = m_d = m_s$:
the disconnected part is significantly non-zero

Removing the Excited contributions

$$C_{3\text{pt}}(t) = \frac{Z_\eta}{2E_\eta} \frac{Z_{D_s}}{2E_{D_s}} \exp(-E_{D_s}t - E_\eta(t_{\text{sep}} - t))$$
$$\times \left[\langle \eta | S | D_s \rangle + \underbrace{A_1 \exp(-\Delta E_{D_s}t) + B_1 \exp(-\Delta E_\eta(t_{\text{sep}} - t)) + \dots}_{\text{excited contributions}} \right]$$

$A_1, B_1 \sim Z_\bullet^* / Z_\bullet$

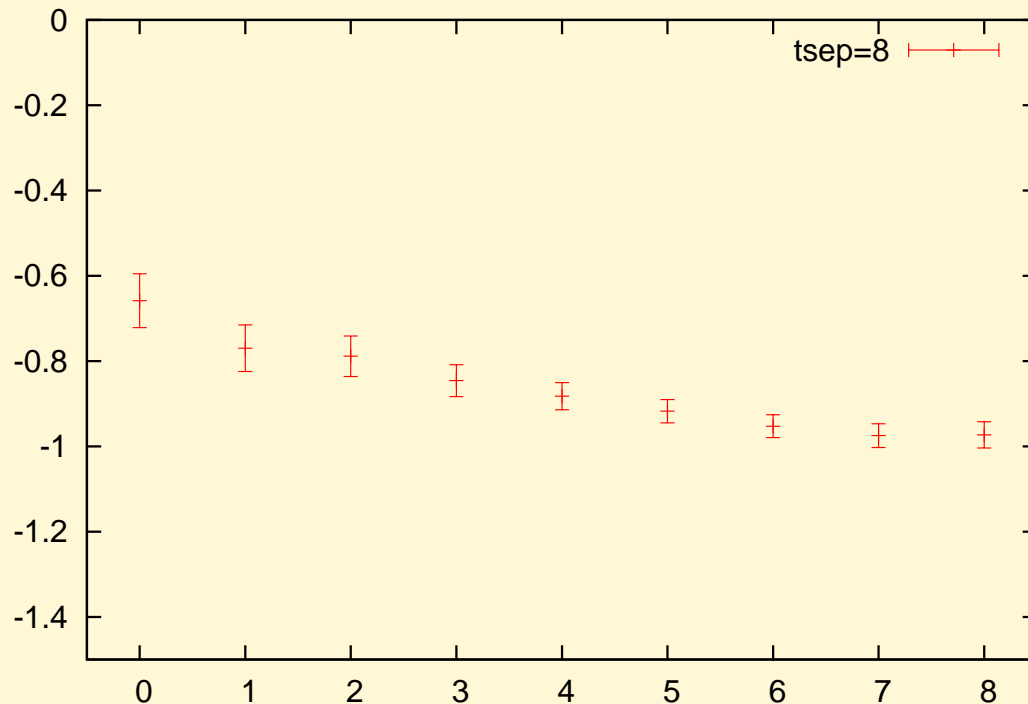
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$\eta, p100q000k100$



$$\frac{C_{3\text{pt}}}{\frac{Z_\eta}{2E_\eta} \frac{Z_{D_S}}{2E_{D_S}} \exp(-E_{D_S}t - E_\eta(t_{\text{sep}} - t))}$$

$$= [\langle \eta | S | D_S \rangle + \dots]$$

$$= \text{const. ?}$$

Fit using the Excited States

from 2 pt func.

$$\begin{aligned} \text{input: } & \overbrace{\Delta E_{D_s}, \Delta E_{\eta}} \\ \Rightarrow \text{fit: } & \underbrace{c}_{=} + A_1 \exp(-\Delta E_{D_s} t) + B_1 \exp(-\Delta E_{\eta}(t_{\text{sep}} - t)) \\ & = \langle \eta | S | D_s \rangle \end{aligned}$$

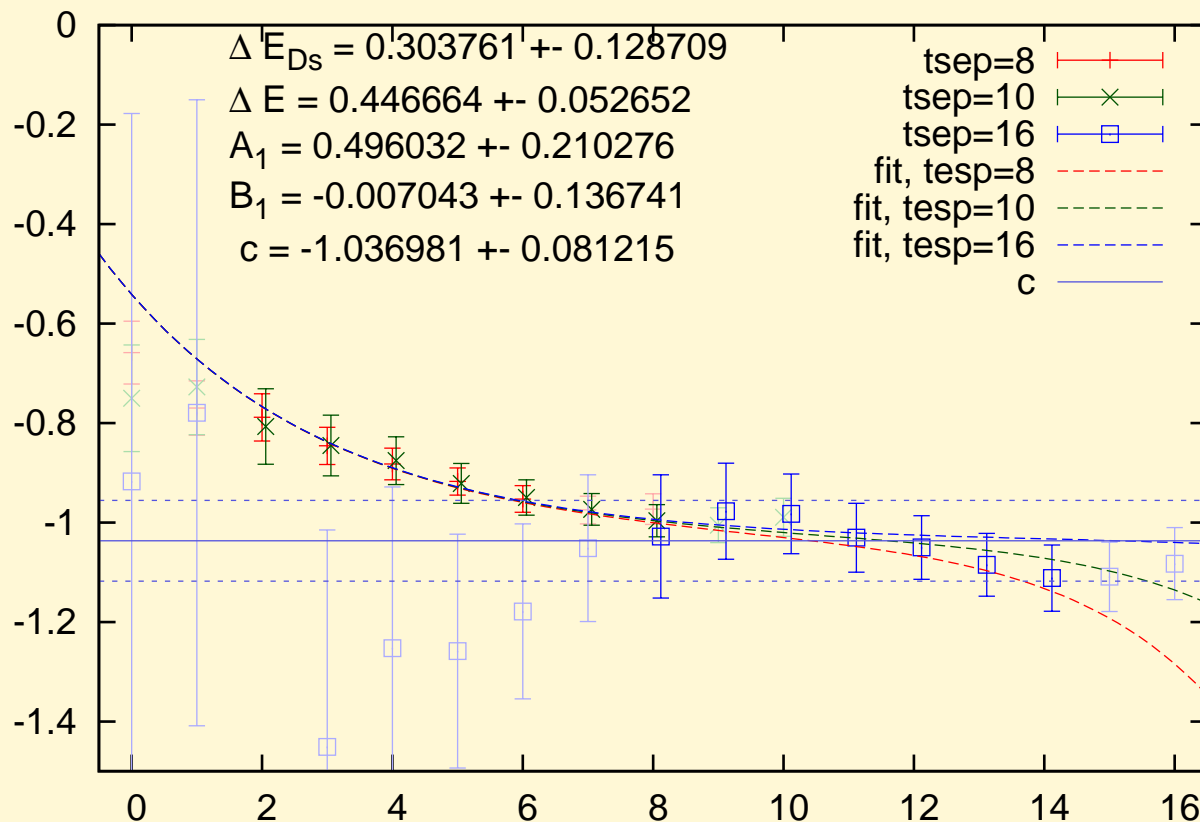
Fit using the Excited States

from 2 pt func.

input: $\overbrace{\Delta E_{D_s}, \Delta E_{\eta}}^{\text{from 2 pt func.}}, t_{\text{sep}} = 8, 10, 16$

$$\Rightarrow \text{fit: } \underbrace{c}_{= \langle \eta | S | D_s \rangle} + A_1 \exp(-\Delta E_{D_s} t) + B_1 \exp(-\Delta E_{\eta} (t_{\text{sep}} - t))$$

fit with $c + A_1 \exp(-\Delta E_{D_s} t) + B_1 \exp(-\Delta E_{\eta} (t_{\text{sep}} - t))$, η , p100q000k100



Conclusions

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Semi-leptonic Decay Form Factor for $D_s \rightarrow \eta l\nu$, $D_s \rightarrow \eta' l\nu$

- Calculable using lattice, including fermion disconnected loops: **First Result**
- $f_0(q^2 = 0)$
(stats. error) $\lesssim 10\text{-}15\%$ at $m_\pi = 348\text{MeV}$, 450MeV

outlook

- larger (and finer) lattice
- mixing angle of η - η'
- $f_+(q^2)$
- Decay to ϕ

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