D_s to $\eta(')$ semi-leptonic decay form factors

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for SFBTRR55

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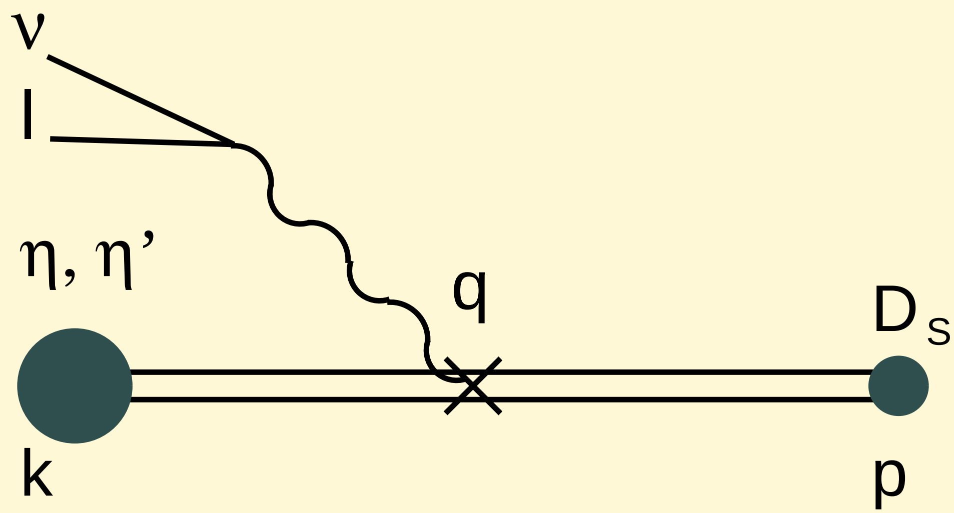
Based on on-going work

cf. PoS LATTICE2011 (2011) 283 [arXiv:1111.4053]
PoS ConfinementX (2012) 143 [arXiv:1302.6087]

Outline

- Introduction
- Extracting η and η' states
- Decay Form Factor
- Conclusion



Introduction

Semileptonic Decay of D-meson (e.g. $D \to l\nu K$)

- $$\frac{d\Gamma}{dq^2} = (\text{kin. factor})|V_{CKM}|^2 \overbrace{|(\text{form factor})|^2}^{\text{lattice}}$$

  well-studied in both exp. and lat., high prec. competition

D_S: the major semi-leptonic decay is $\boxed{D_S \to l\nu\eta, l\nu\eta'}$

- Experiment: only branching fractions in the PDG
- Theory: no lattice calculations so far
  (available: light cone QCD sum rule predictions)
- Interesting for η/η' mixing, gluonic contrib.
- Challenging: disconnected fermion loops
- Interesting play ground for QFT:
  contributions from anomaly (cf. Witten-Veneziano formula)

Form Factor

$$\langle \eta^{(\prime)}(k)|V^{\mu}(q^2)|D_s(p)\rangle$$
$$= f_0(q^2)\frac{M_{D_s}^2 - M_{\eta^{(\prime)}}^2}{q^2}q^{\mu} + f_+(q^2)\left[(p+k)^{\mu} - \frac{M_{D_s}^2 - M_{\eta^{(\prime)}}^2}{q^2}q^{\mu}\right]$$

We focus on the scalar form factor:

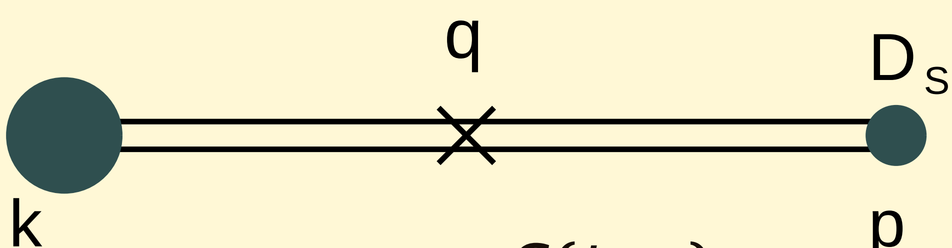
$$\boxed{f_0(q^2) = \frac{m_c - m_s}{M_{D_s}^2 - M_{\eta^{(\prime)}}^2}\langle \eta^{(\prime)}|S|D_s\rangle}$$

scalar current: $S = \bar{s}c$ $\quad (m_c - m_s)S$: no renormalization

H.Na et al.(HPQCD)

Target observable: $\boxed{\langle 0|\mathcal{O}_{\eta^{(\prime)}}(t_{\mathrm{sep}},\boldsymbol{k})S(t,\boldsymbol{q})\mathcal{O}^{\dagger}_{D_s}(0,\boldsymbol{p})|0\rangle}$

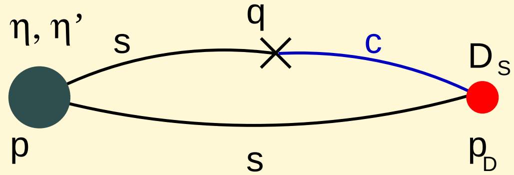
$$\sim e^{-E_{\eta^{(\prime)}}t - E_{D_s}(t_{\mathrm{sep}}-t)}\langle \eta^{(\prime)}(k)|S(q^2)|D_s(p)\rangle$$

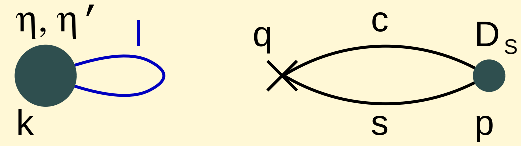


$S(t,\boldsymbol{q})$

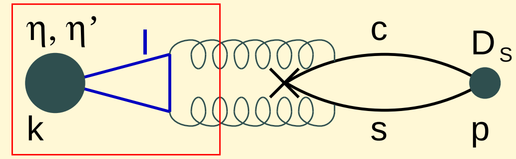
$\eta^{(\prime)}(t = t_{\mathrm{sep}},\boldsymbol{k})$ $\quad D_s(t=0,\boldsymbol{p})$

Disconnected fermion loops

3 pt func. = 

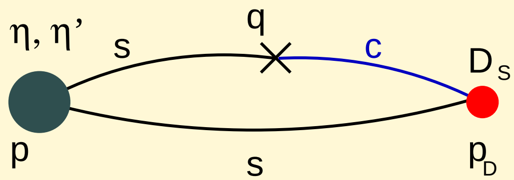
$- \sum_{l=u,d,s} \Bigg($ 

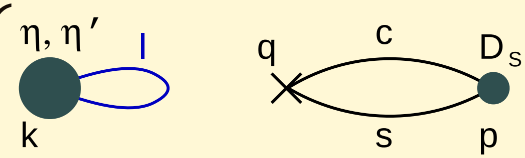
$\Bigg)$

- u, d, s: enhancement by factor 3
- η': contributions from anomaly 

The disconnected part may contribute significantly

Disconnected fermion loops

3 pt func. = 

− $\sum_{l=u,d,s}$ (

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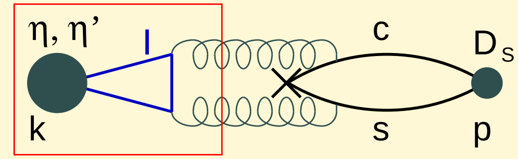
- η': contributions from anomaly 

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...but noisy and expensive in lattice calculation

(needs all-to-all propagators)

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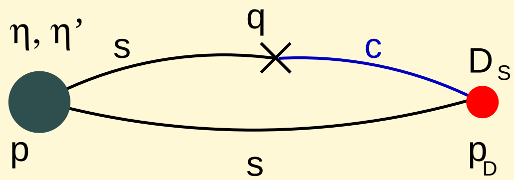
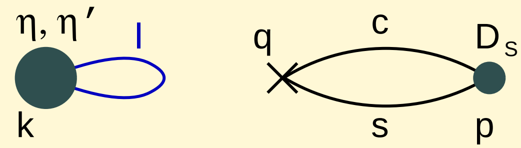
3 pt func. = 

$-\sum_{l=u,d,s}$ (

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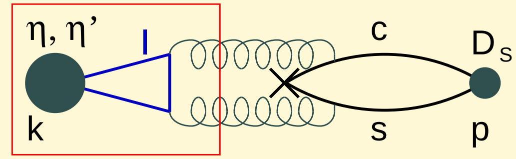
- η': contributions from anomaly 

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$\Rightarrow$ manageable I.K. Lattice 2011

- stochastic estimation
  (also for the connected part Evans-Bali-Collins)
- low mode averaging
- truncated solver method Bali-Collins-Schäfer
- ...

Configurations

QCDSF 2+1 flavor configurations: W.Bietenholz *et al.*[QCDSF collab.]

$$\boxed{m_u + m_d + m_s = \text{fixed}}$$

$$m_u = m_d = m_s \longrightarrow m_u (= m_d) \downarrow, \; m_s \uparrow$$

SU(3) basis:

$$\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) \qquad \eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

- stout link non-perturbative clover

  charm quark: relativistic partially quenched approximation

- $\beta = 5.5$, $24^3 \times 48$, $a \sim 0.08$ fm
  - $m_\pi \sim 450$MeV (SU(3) symmetric) 939 confs
  - $m_\pi \sim 348$MeV 239 confs

- (planning: $32^3 \times 64$)

Extracting η and η' states

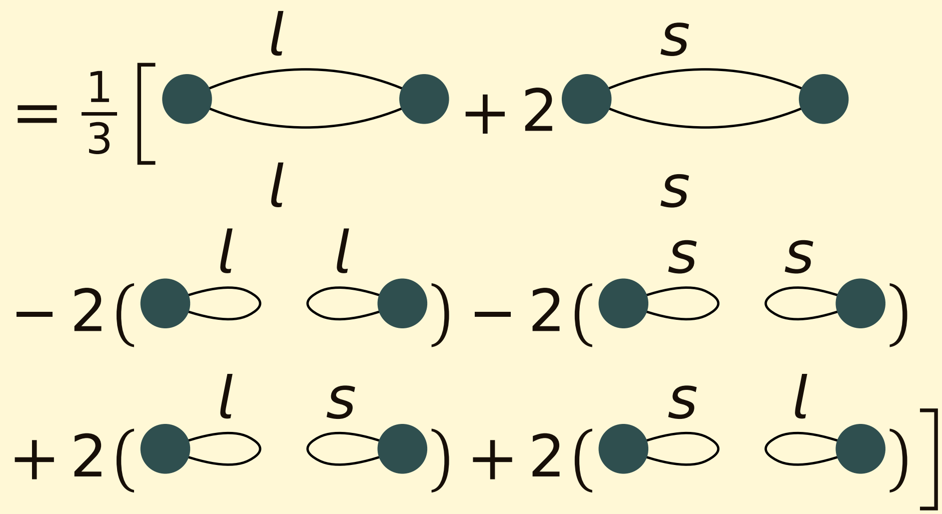
2 point functions

Building blocks: $\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$, $\eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$

$$\begin{pmatrix} (\eta_8 \to \eta_8) & (\eta_8 \to \eta_1) \\ (\eta_1 \to \eta_8) & (\eta_1 \to \eta_1) \end{pmatrix} \underset{\text{diagonalize}}{\Longrightarrow} \begin{pmatrix} (\eta \to \eta) & 0 \\ 0 & (\eta' \to \eta') \end{pmatrix}$$

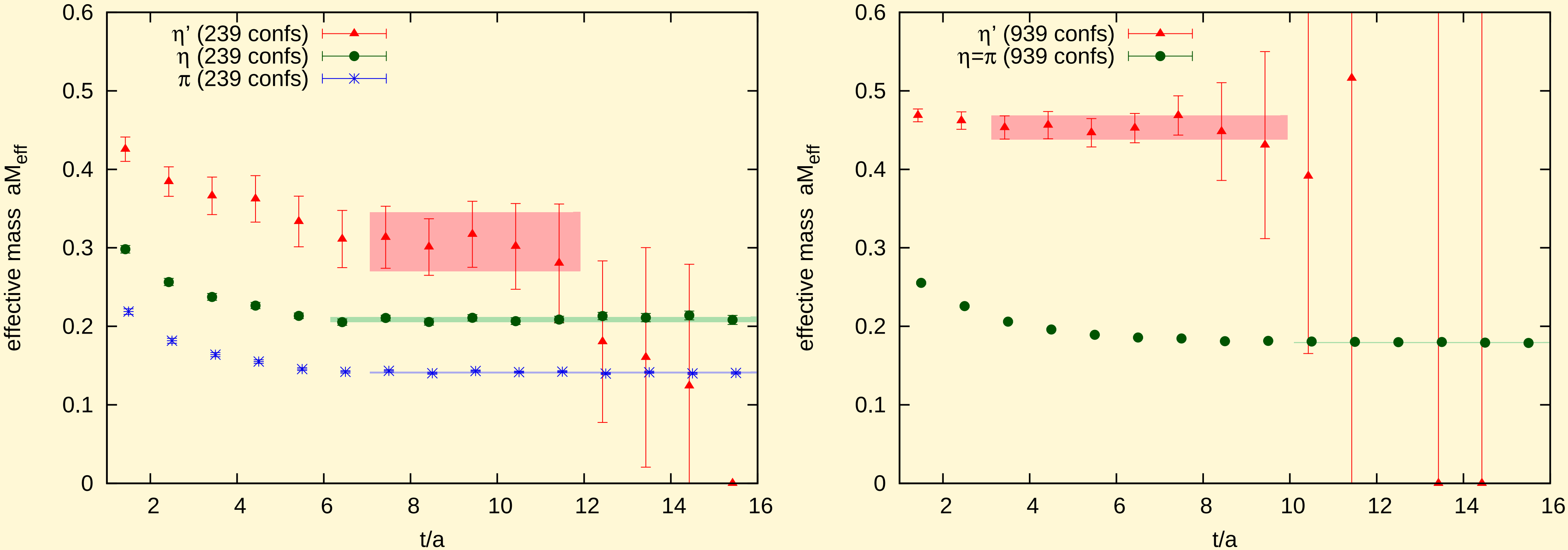
$\Rightarrow$ masses, interpolators for the physical states $\mathcal{O}_\eta$, $\mathcal{O}_{\eta'}$

ex.) $(\eta_8 \to \eta_8) = \langle \mathcal{O}_8(t) \mathcal{O}_8^\dagger(0) \rangle$



effective mass

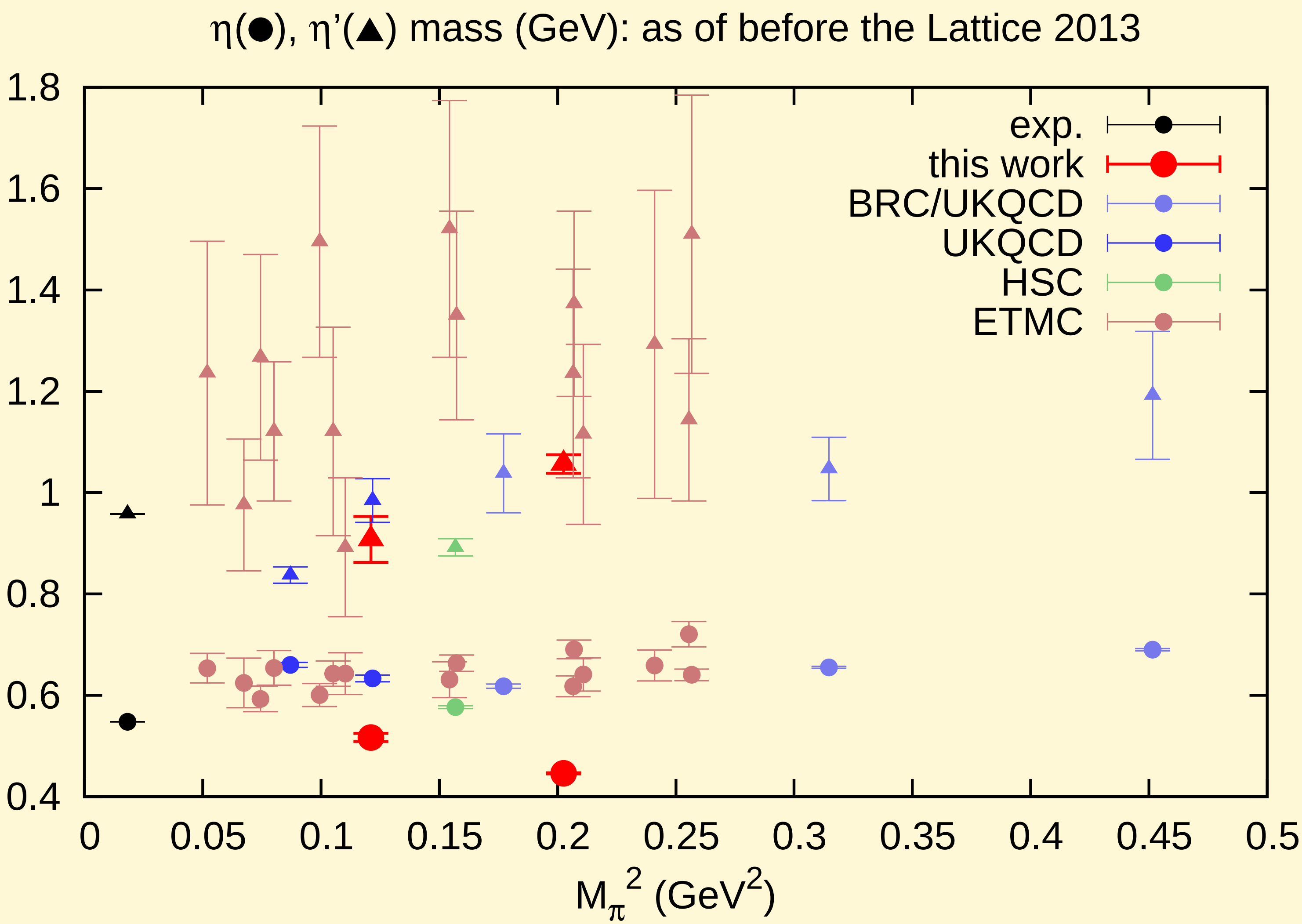
$m_\pi = 348\text{MeV}(m_{u,d} < m_s)$ $m_\pi = 450\text{MeV}(m_{u,d} = m_s)$



(fittings above: with $\boldsymbol{p} = \mathbf{0}$ data only)

We can use $\boldsymbol{p} \neq \mathbf{0}$ data + dispersion relation

mass (preliminary)



η' mass and the topological fluctuation: — finite volume effect —

finite volume effect with fixed topological charge Q:

$$\langle \rho(x)\rho(0)\rangle_Q = -\frac{1}{V_4}\left(\chi_t - \frac{Q^2}{V_4}\right) + \cdots \text{ for } |x| \to \infty$$

ρ: topological charge density

Aoki-Fukaya-Hashimoto-Onogi

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Since $\rho \sim \eta_1$, if the statistics is not large enough, one may observe the constant part:

$$\langle \mathcal{O}_{\eta_1}(\boldsymbol{p}=0,t)\mathcal{O}^\dagger_{\eta_1}(\boldsymbol{p}=0,0)\rangle \xrightarrow{t\to\infty} A\exp(-m_{\eta_1}t) + c$$

(at $m_u = m_d = m_s$ point)

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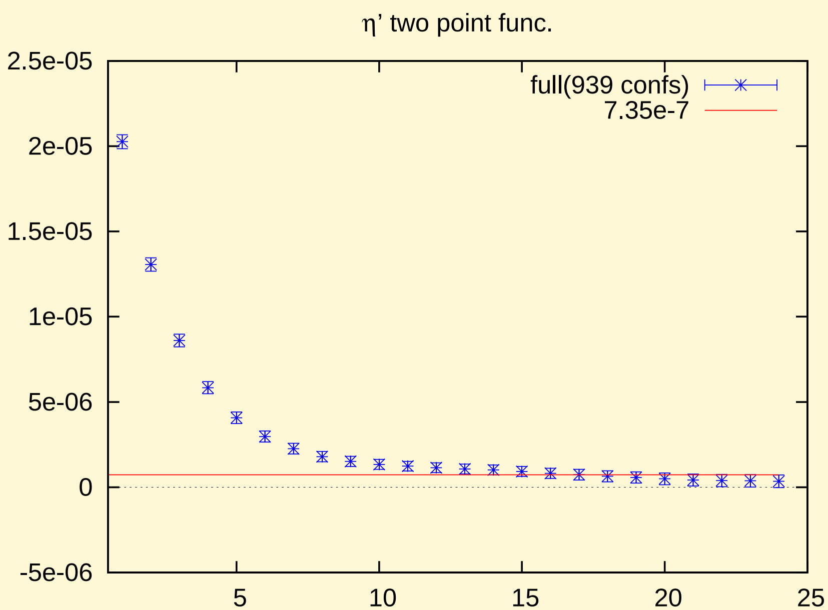
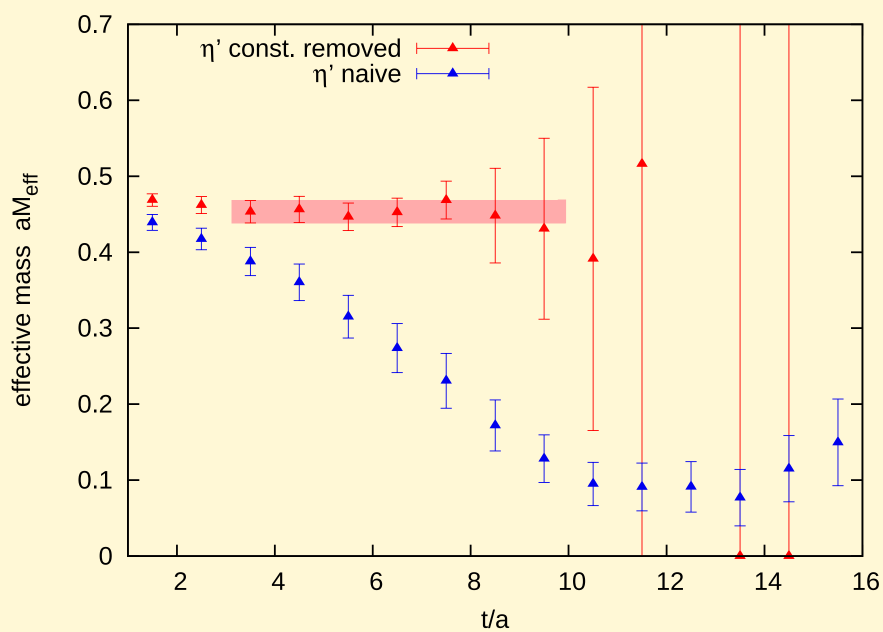




needed to remove the constant part

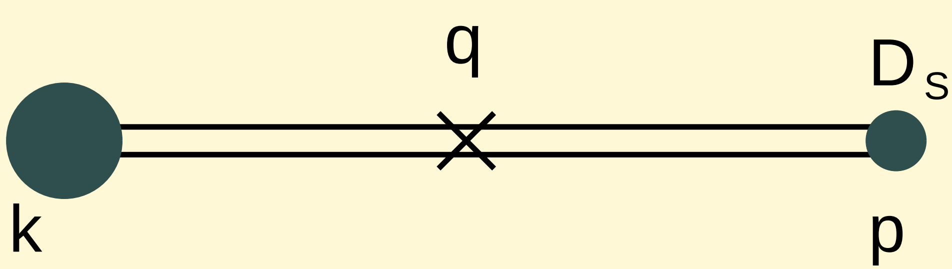
Decay Form Factor

Correlation functions to matrix element

Now we have $\mathcal{O}_\eta$ and can calculate: (the same for η')

- $C_{3\text{pt}}(t) = \langle 0|\mathcal{O}_\eta(\boldsymbol{k}, t_{\text{sep}})S(\boldsymbol{q}, t)\mathcal{O}^\dagger_{D_s}(\boldsymbol{p}, 0)|0\rangle$
  $= \frac{Z_\eta}{2E_\eta}\frac{Z_{D_s}}{2E_{D_s}}\exp(-E_{D_s}t - E_\eta(t_{\text{sep}} - t)) \times \left[\langle\eta|S|D_s\rangle + \cdots\right]$
- $C^\eta_{2\text{pt}}(t, \boldsymbol{k}) = \frac{|Z_\eta(\boldsymbol{k})|^2}{2E_\eta(\boldsymbol{k})}\exp(-E_\eta(\boldsymbol{k})t) + \cdots$
- $C^{D_s}_{2\text{pt}}(t, \boldsymbol{p}) = \frac{|Z_{D_s}(\boldsymbol{p})|^2}{2E_{D_s}(\boldsymbol{p})}\exp(-E_{D_s}(\boldsymbol{p})t) + \cdots$

$\Rightarrow \langle\eta(k)|S(q^2)|D_s(p)\rangle \Rightarrow$ form factor $f_0(q^2)$

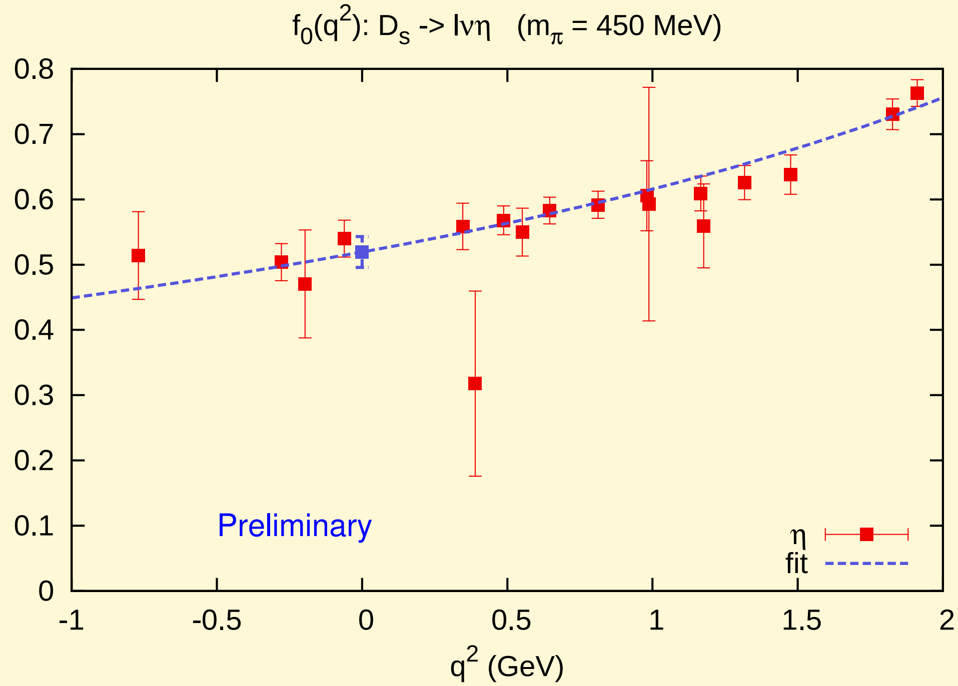


$S(t, \boldsymbol{q})$

$\eta^{(\prime)}(t = t_{\text{sep}}, \boldsymbol{k})$ $D_s(t = 0, \boldsymbol{p})$

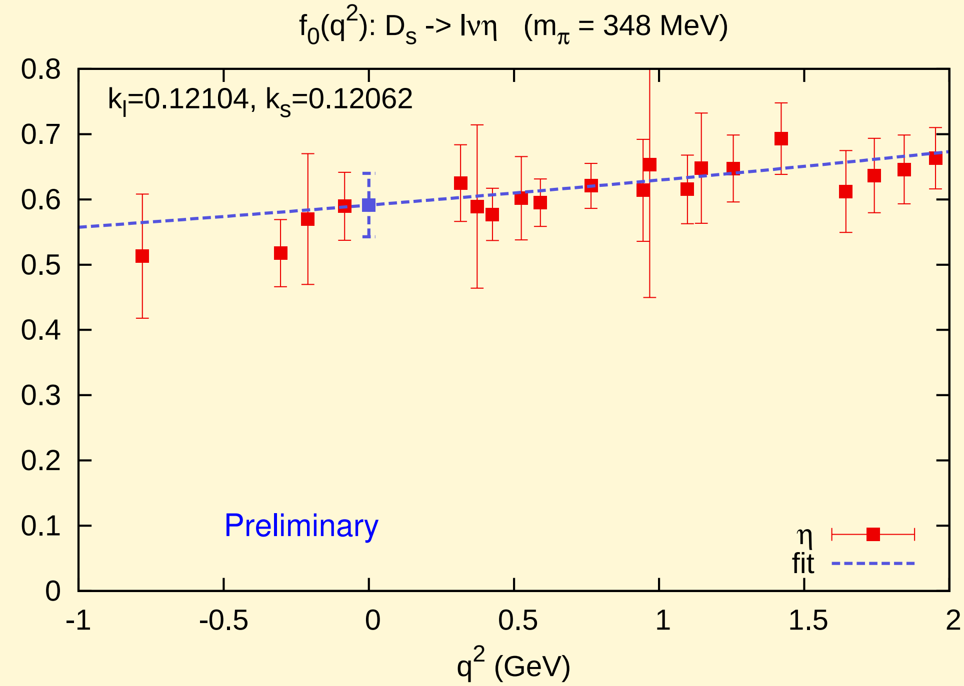
results: $D_s \to l\nu\eta$

$m_\pi = 450\text{MeV (SU(3))}$



$$f_0^\eta(0) = 0.52(2)$$

$m_\pi = 348\text{MeV}$



$$f_0^\eta(0) \sim 0.58(5)$$

(fit with $f_0(q^2) = \frac{a}{1-bq^2}$)

results: $D_s \to l\nu\eta'$

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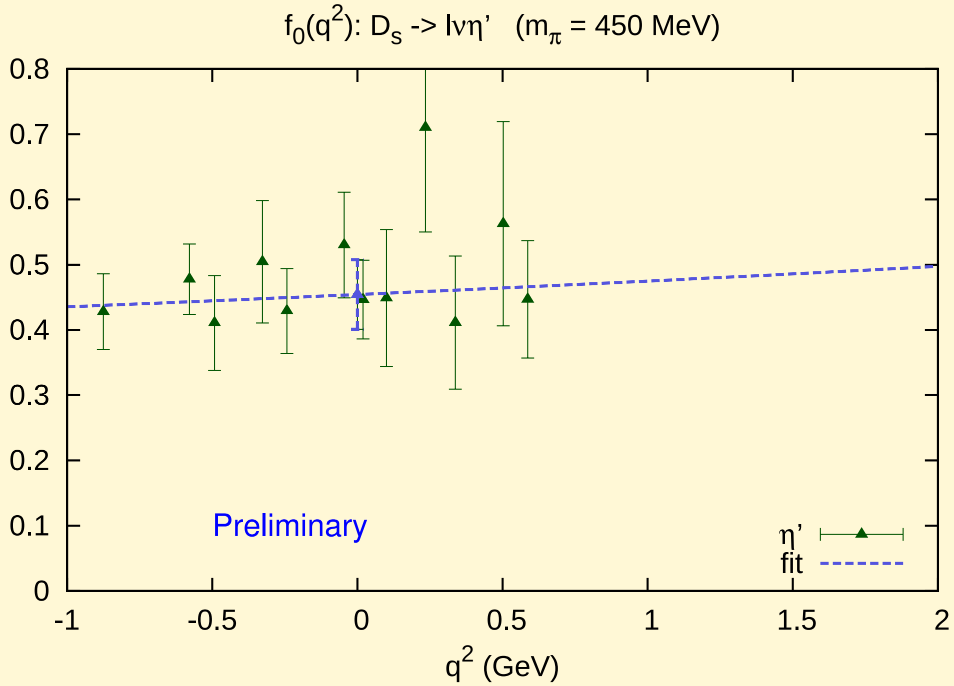

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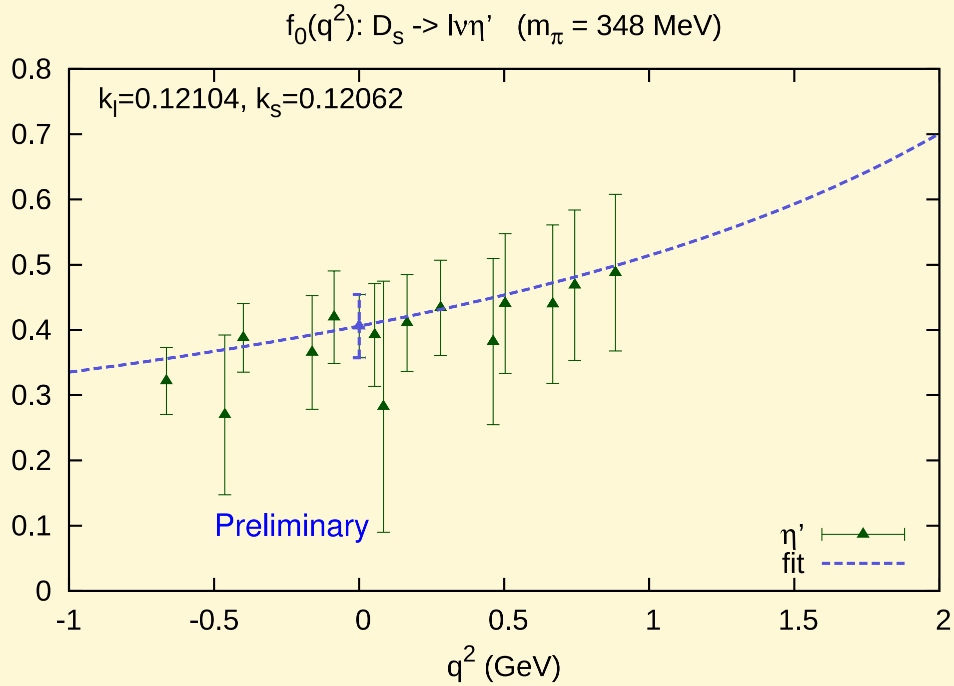


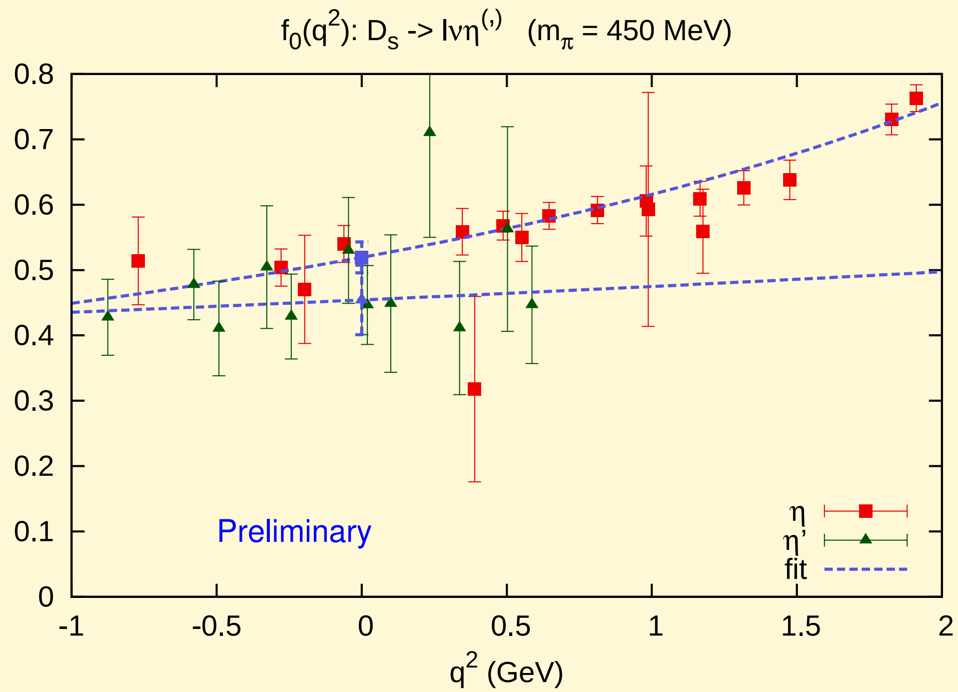
$$f_0^{\eta'}(0) = 0.45(5)$$

$$f_0^{\eta'}(0) \sim 0.42(5)$$

(fit with $f_0(q^2) = \frac{a}{1-bq^2}$)

results

$m_\pi = 450\text{MeV (SU(3))}$



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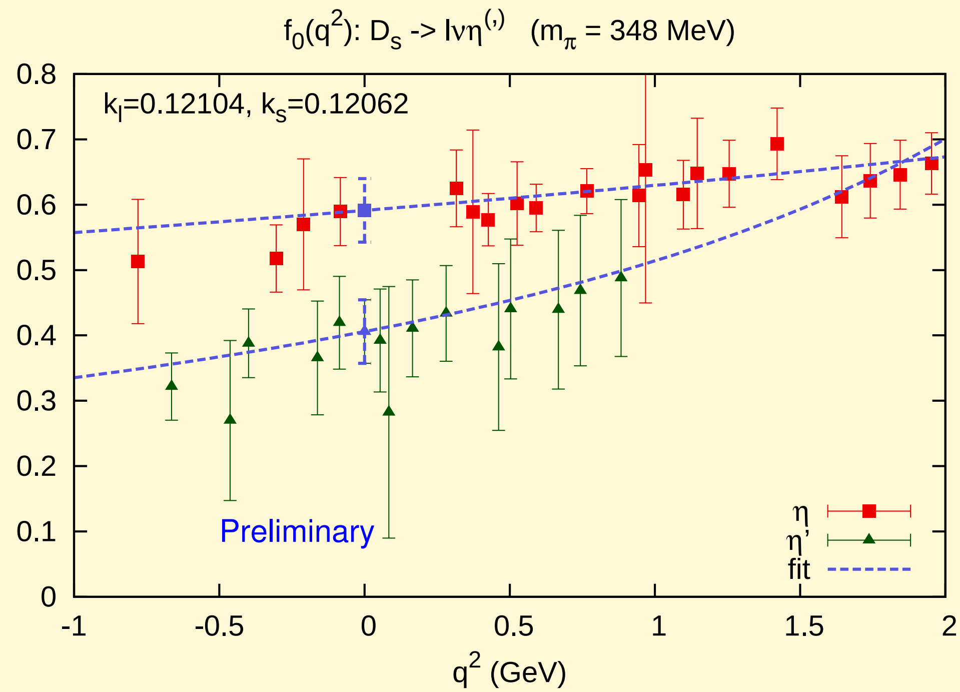

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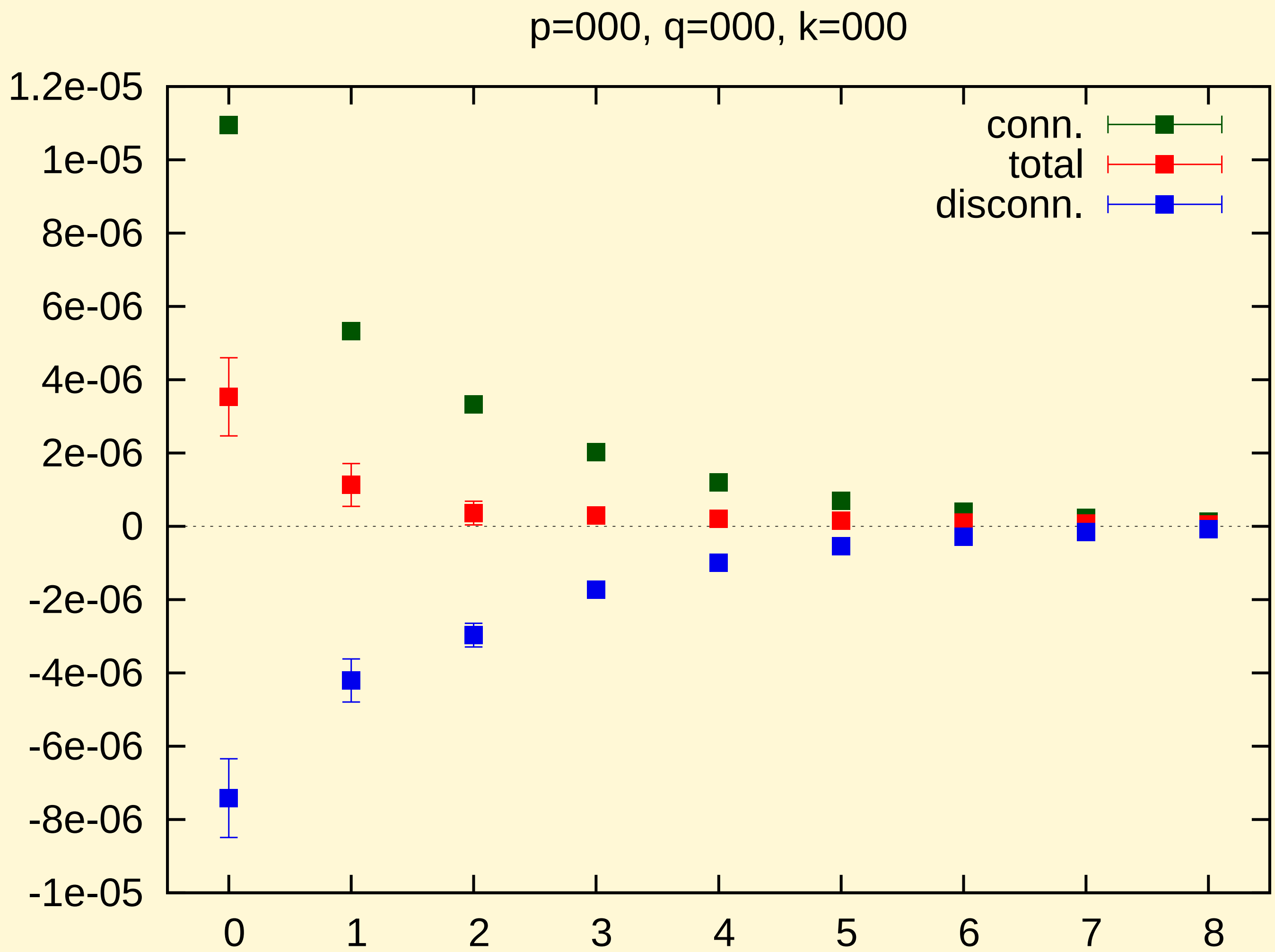
Connected vs. Disconnected



3 point function for $\langle \eta' | S(t) | D_s \rangle$, $m_u = m_d = m_s$:
the disconnected part is significantly non-zero

Removing the Excited contributions

$$C_{3\text{pt}}(t) = \frac{Z_\eta}{2E_\eta}\frac{Z_{D_s}}{2E_{D_s}}\exp(-E_{D_s}t - E_\eta(t_{\text{sep}} - t))$$

$$\times\left[\langle\eta|S|D_s\rangle + \underbrace{A_1\exp(-\Delta E_{D_s}t) + B_1\exp(-\Delta E_\eta(t_{\text{sep}} - t)) + \cdots}_{\text{excited contributions}}\right]$$

$$A_1, B_1 \sim Z^*_\bullet / Z_\bullet$$

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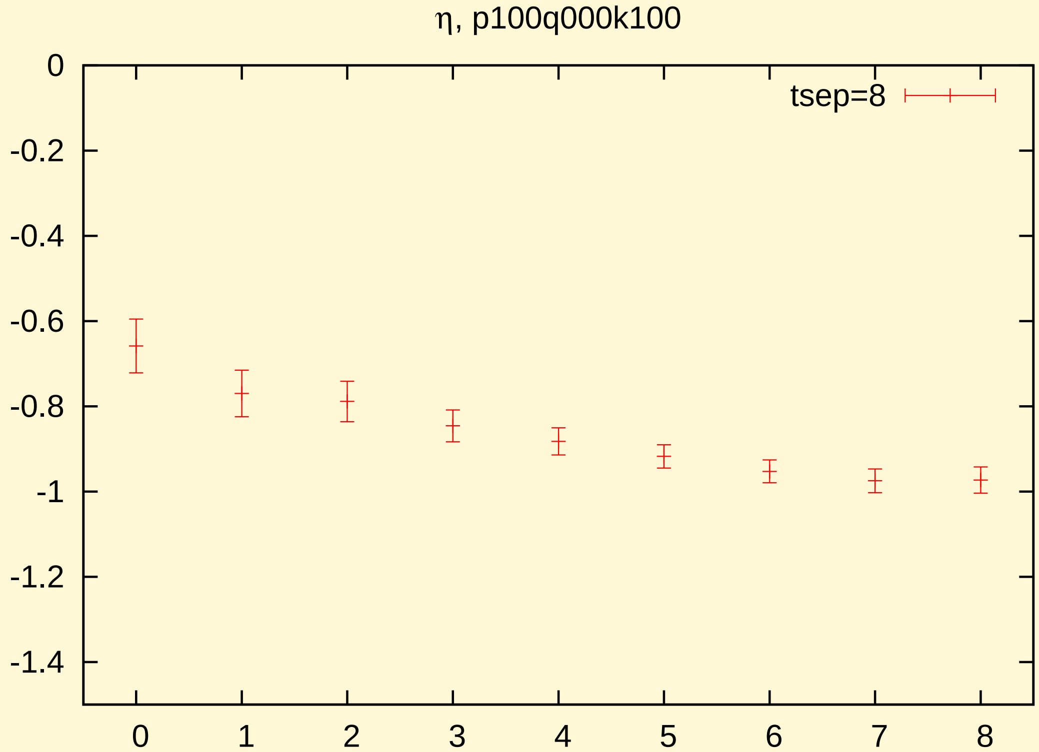

$$\frac{C_{3\mathrm{pt}}}{\frac{Z_\eta}{2E_\eta}\frac{Z_{D_s}}{2E_{D_s}}\exp(-E_{D_s}t - E_\eta(t_{\mathrm{sep}}-t))}$$
$$= \left[\langle\eta|S|D_s\rangle + \cdots\right]$$
$$= \text{const. ?}$$

Fit using the Excited States

$$\text{input: } \overbrace{\Delta E_{D_s},\ \Delta E_{\eta}}^{\text{from 2 pt func.}},\ t_{\text{sep}} = 8, 10, 16$$

$$\Rightarrow \text{fit: } \underbrace{c}_{= \langle \eta | S | D_s \rangle} + A_1 \exp(-\Delta E_{D_s} t) + B_1 \exp(-\Delta E_{\eta}(t_{\text{sep}} - t))$$

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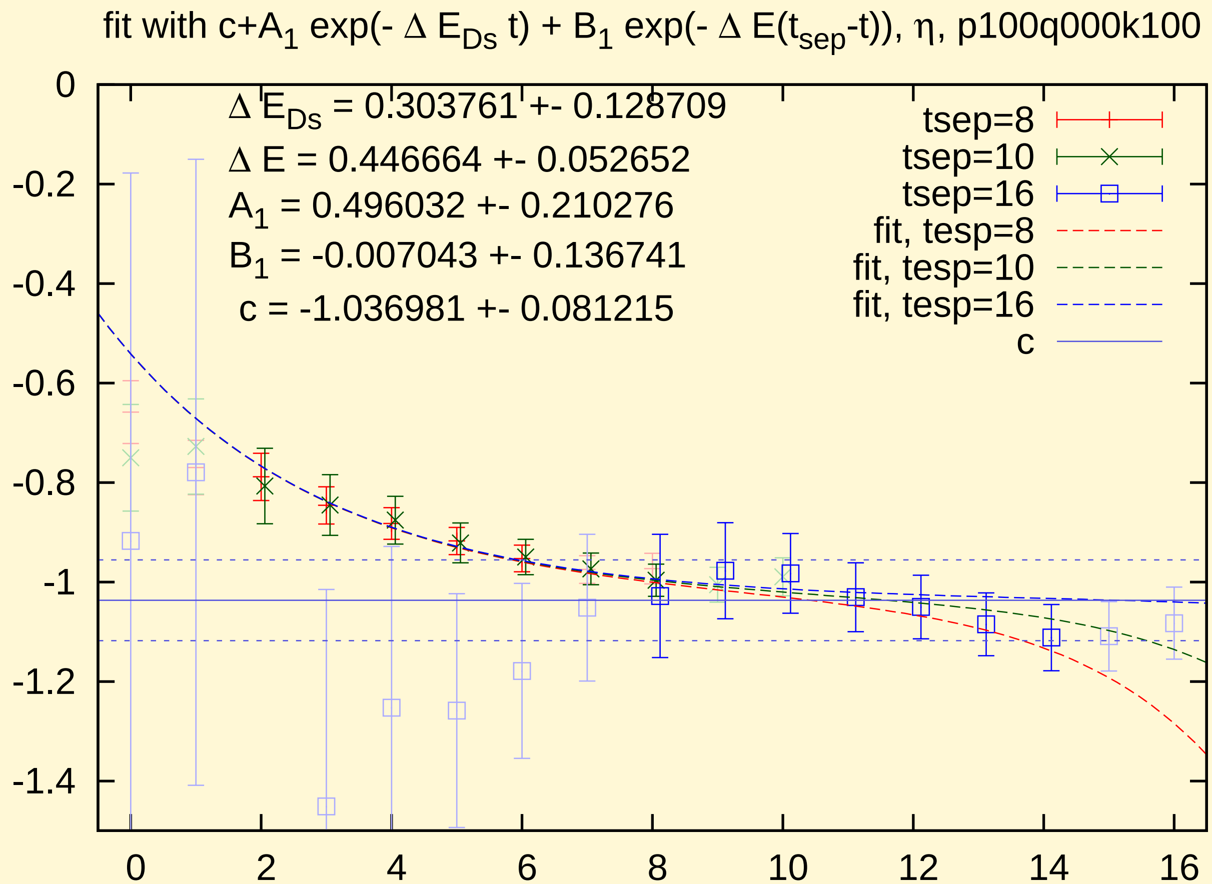

Conclusions

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Semi-leptonic Decay Form Factor for $D_s \to \eta l \nu$, $D_s \to \eta' l \nu$

- Calculable using lattice, including fermion disconnected loops: First Result
- $f_0(q^2 = 0)$
  (stats. error) $\lesssim$ 10-15% at $m_\pi = 348$MeV, 450MeV

outlook

- larger (and finer) lattice
- mixing angle of η-η'
- $f_+(q^2)$
- Decay to ϕ

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