MCRG Flow for the Nonlinear Sigma Model

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Asymptotic Safety

RG approach to QFT

- theory at scale k described by effective average action Γ_k
- upper (ultraviolet) cutoff Λ: $\Gamma_k = S_{mic}$
- use RG flow to integrate out fluctuations until lower cutoff $k = \lambda$
- fundamental theory valid on all scales, limit $\Lambda \to \infty$ and $\lambda \to 0$ exists

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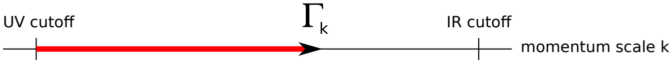
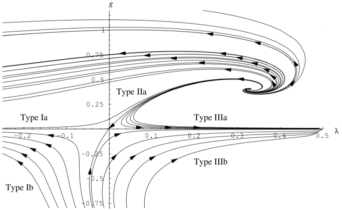


The Gravity situation

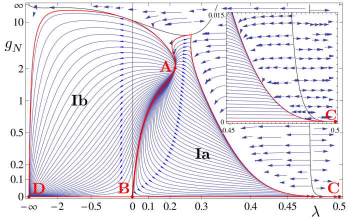
- unified theory requires quantization of spacetime metric
- at IR cutoff: Einstein-Hilbert action
- perturbative approach leads to severe divergences
- theory not renormalizable in perturbative way

Asymptotic Safety

- Asymptotic Safety Scenario [Weinberg '80] : gravity non-perturbatively renormalizable
- needs nongaussian (ultraviolet) fixed point with finite number of relevant directions to protect UV from unphysical divergences
- asymptotic freedom (QCD): theory approaches gaussian fixed point for $k \to \infty$



[cf. Reuter, Saueressig '02]



[cf. Christiansen, Litim, Pawlowski, Rodigast '12]

Setting the stage

- nontrivial UV fixed point suspected in ($D > 2$) nonlinear sigma models by perturbation theory and functional RG calculations [Codello, Percacci '08] [Flore, Wipf, Zanusso '12]

$D = 3$ O(N) NLSM

$$S = \frac{1}{2g^2} \int d^3x\, \partial_\mu \vec{\phi} \partial^\mu \vec{\phi}, \quad \text{where } \vec{\phi}^2 = 1$$

- Flow diagram determinable from the lattice via Monte Carlo Renormalization Group (MCRG) techniques

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Our Setup

- local HMC with O(N)-valued fields to generate Markov Chain
- blockspin transformation to integrate out fluctuations
- demon method to determine effective couplings

RG picture on the lattice

- (discrete) lattice momenta cut off by inverse lattice spacing a^{-1} and inverse linear box size $(aN)^{-1}$
- lattice simulation equivalent to integrating out all fluctuations inbetween
- correlation functions determined by direct measurement

- Blockspin transformation : $a \to 2a$, $N \to N/2$

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Observables

- (discrete) beta function $\tilde{\beta}_i = \tilde{\beta}(g_i) = g_i^{blocked} - g_i$
- (discrete) stability matrix $S_{ij} = \frac{\partial \tilde{\beta}_i}{\partial g_j}$

[Creutz, Gocksch, Ogilvie, Okawa '84] Demon Method

- best match of demon Hamiltonian $H_D = \sum_i g_i O_i(\phi)$ against configurations from source Hamiltonian H_S
- Canonical Demon [Hasenbusch, Pinn, Wieczerkowski '95] avoids systematical small volume errors

- simulate joint partition function including the "Demons" E_D^i

$$Z_C = \sum_{E_D^0 = -E_m}^{E_m} \cdots \sum_{E_D^n = -E_m}^{E_m} \int \mathcal{D}\phi \; e^{-H_S - g_i E_D^i}$$

using microcanonical updates on statistically independent configurations and determine demon couplings

$$\langle E_D^i \rangle = \frac{1}{g_i} - \frac{E_m}{\tanh(g_i E_m)}$$

Systematic Errors

- finite volume effects not visible in $32^3 \to 16^3$ simulations
- discretisation errors small near critical line
- effective action in demon method leads to truncation errors
- half group property of RG transformation R_s (blockspin + demon) violated

$$R_s \circ R_s \neq R_{2s}$$

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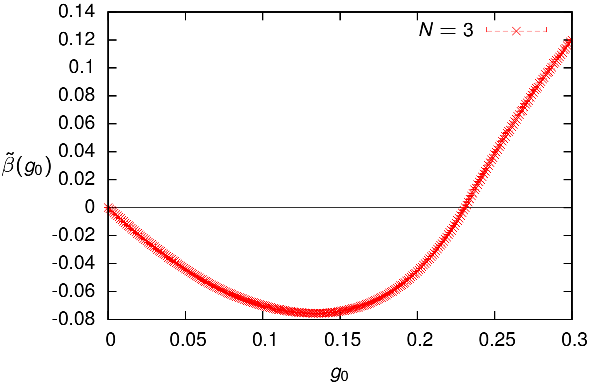
Option 2: Use improved blockspin transformation!

$$\Phi_{\tilde{x}} \propto e^{C \cdot \Phi_{\tilde{x}} \sum_{x \in \Lambda_{\tilde{x}}} \phi_x}$$

[Hasenfratz, Hasenfratz, Heller, Karsch '84]

- parametrize $C = \sum_i c_i g_i$ such that for $g \to \infty$, $C \to \infty$ holds

1-Parameter effective action



- Gaussian fixed point at zero coupling
- non-Gaussian fixed point with a UV-attractive direction (beta function w.r.t. the inverse lattice spacing)

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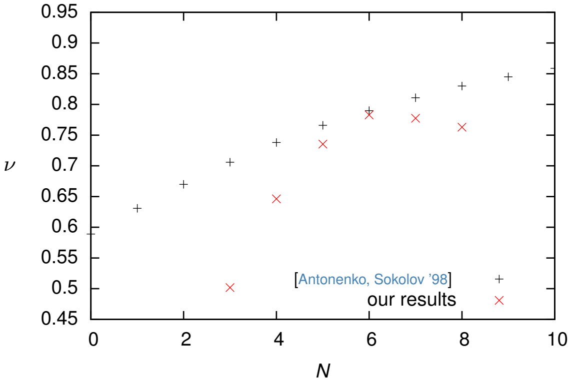
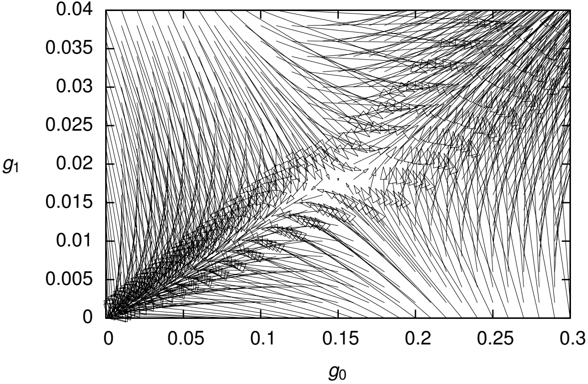


- slope of beta function corresponds to inverse of critical exponent ν of the correlation length
- ν deviates from expected behaviour for $N > 6$

2-Parameter effective action



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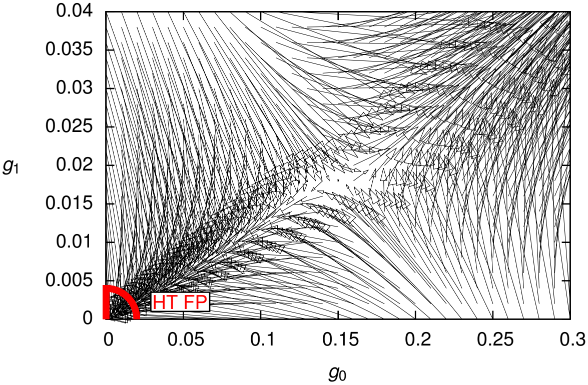

high temperature (Gaussian) FP : spins randomly aligned, absolute disorder

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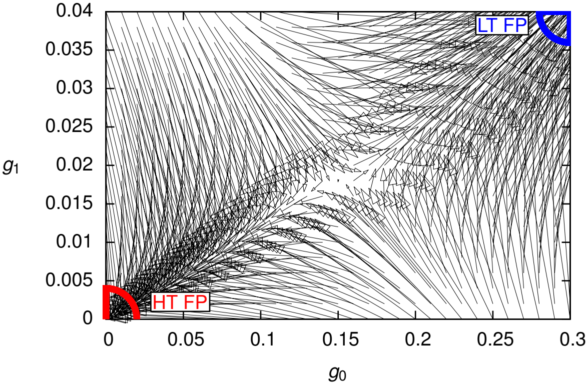

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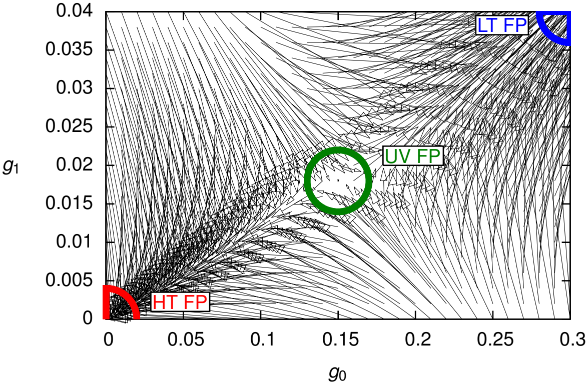

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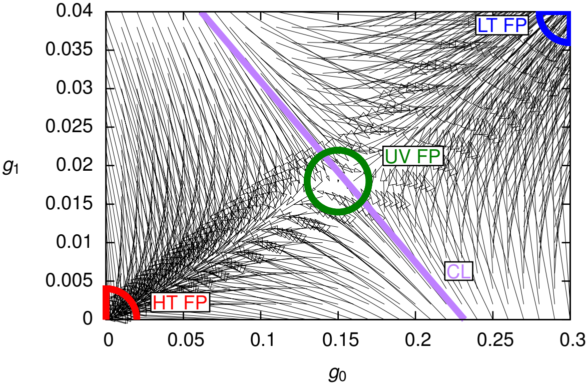

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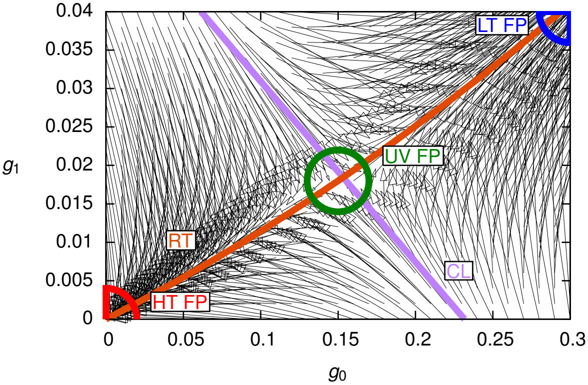

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renormalized trajectory : attractor for the RG trajectories

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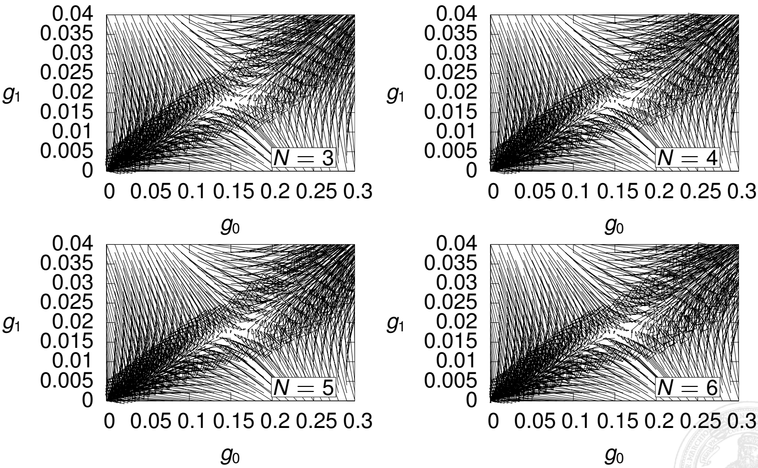

- leading N-dependence absorbed into couplings $g_i \equiv \frac{g_i}{N}$

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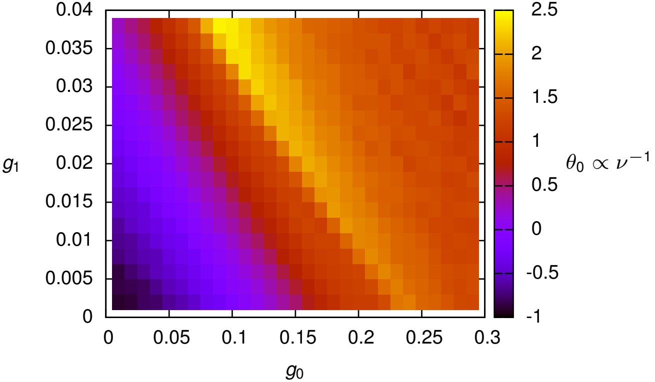

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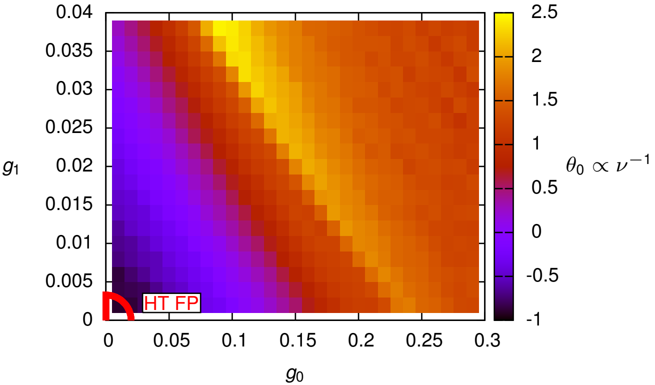

high temperature (Gaussian) FP: $\nu \approx -1$

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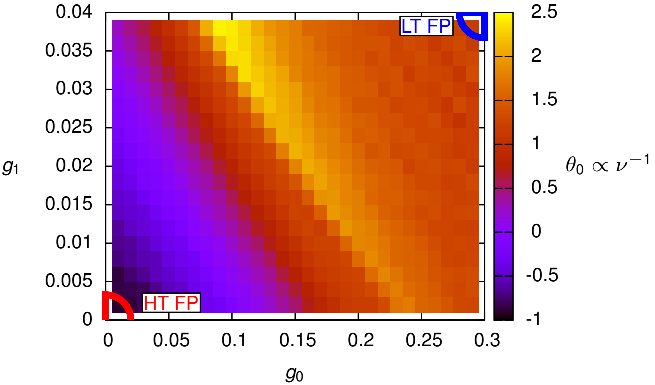

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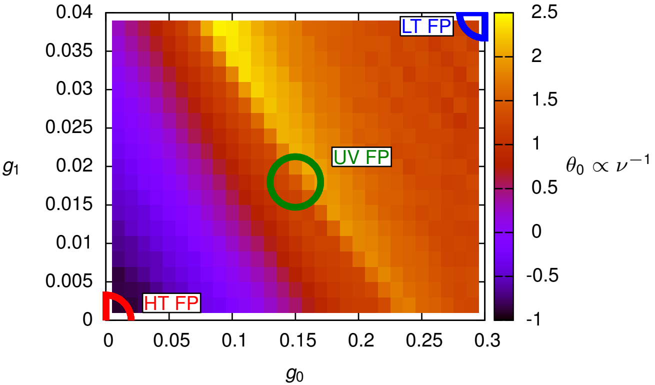


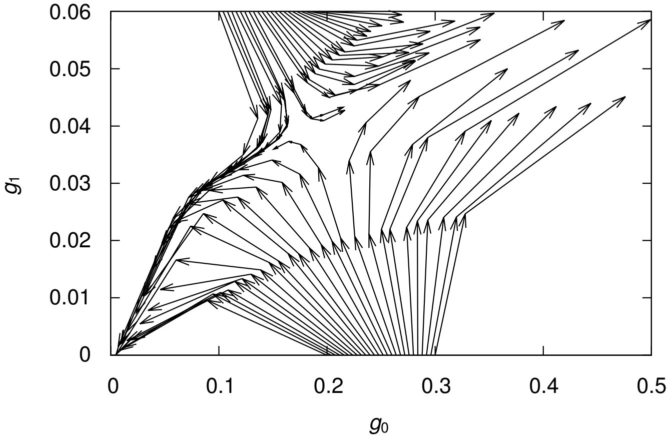
high temperature (Gaussian) FP: $\nu \approx -1$

low temperature FP: $\nu \approx 1$

non-Gaussian FP: $\nu = 0.64(3)$ (1P $\nu = 0.50$, FSS $\nu = 0.7112(5)$

[Campostrini, Hasenbusch, Pelissetto, Rossi, Vicari '02]) *work in progress!*

3-Parameter effective action



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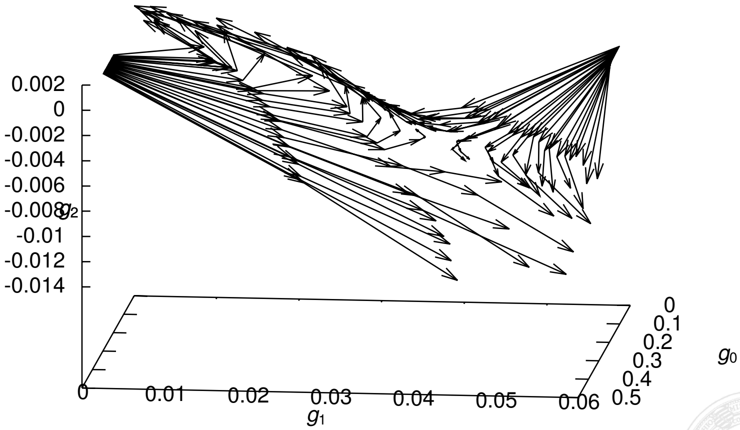

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Summary

- MCRG+Demon Method suitable to obtain flow diagram
- trajectories can be computed in *parallel*
- in principle arbitrarily long trajectories without large lattices
- systematic errors of demon method reduced by improved blockspin transformation

- flow diagram reveals two trivial IR fixed points (absolute order and absolute disorder) and one nontrivial UV fixed point
- fixed point structure stable against change of truncation
- only one relevant direction $\rightarrow$ asymptotic safety scenario realized

Outlook

- finish detail simulations for critical exponents
- compare to large N and FRG results
- extend to fermionic / CDT models

Optimization of the RG transformation