The quantum theory of ideal fluids and its lattice investigation

The LSS collaboration

Liquid State ~~Seminar~~ Simulation

T.Burch
Universität Regensburg

G. Torrieri
HIC for FAIR Helmholtz International Center
FIAS Frankfurt Institute for Advanced Studies

Earlier work 1112.4086 (GT) hep-th/0512260 and 1011.6396 (Nicolis,Rattazzi et al) and informal discussions with Christopher Kelly

The conventional widsom: Quantize hydrodynamics? Are you mad?
Hydro is an inherently classical effective "mean field" theory, because it is based on the equation of motion of the average operator $\hat{T}_{\mu\nu}$, whose parameters (sound speed, viscosity,...) are determined by IR limits...

$$\lim_{k\to 0}\frac{1}{k}\int dx dx' e^{ik(x-x')}\left\langle \hat{T}^{\mu\nu}(x)\hat{T}^{\mu\nu}(x')\right\rangle$$

While the underlying degrees of freedom are quantum, everything that appears in hydro are averages. At sizes where quantum uncertainities become important, hydro stops applying.
Why would you quantize an average of an already quantum operator?
Well, there are several reasons to try: (1) It might be possible!

(2) It might teach us something!

In the limit where viscosity is so low that soundwaves

Of amplitude so that momentum $P_{sound} \sim (area)\lambda\,(\delta\rho)\,c_s \gg T$

And wavenumber $k_{sound} \sim P_{sound}$

Survive (ie their amplitude does not decay to $E_{sound} \sim T$) $\tau_{sound} \gg 1/T$

Quantum corrections to sound will be non-negligible, And in "conventional widsom" its not clear how to deal with this!

Is it relevant to physics? good question!

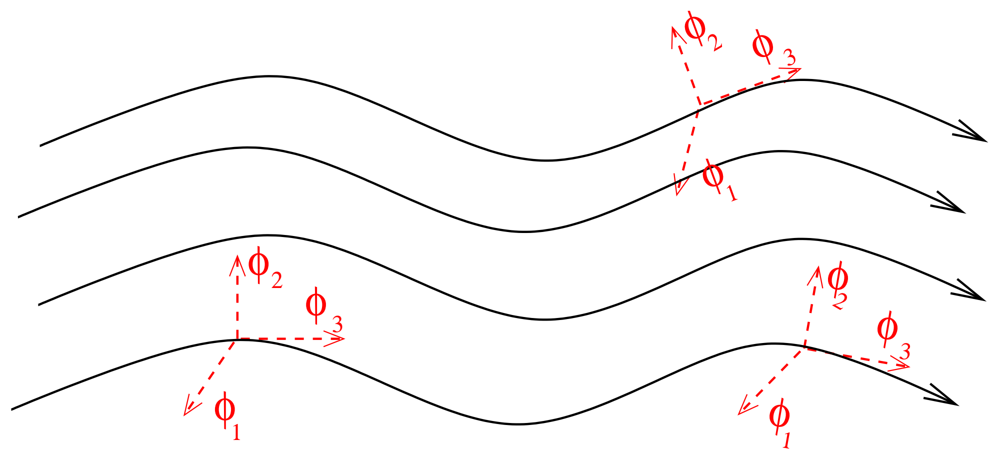
(3) It might teach us about strongly coupled theories in a mysterious limit
Quantum fluctuations $\Leftrightarrow$ Thermal fluctuations. What if viscosity is so low that thermal fluctuations trigger sound waves ?

Landau&Lifshitz (also D.Rishke,B Betz et al): Hydrodynamics has 3 scales

$$\underbrace{l_{micro}}_{\sim s^{-1/3}, n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll \underbrace{L_{macro}}_{size, gradient}$$

Transport Boltzmann eq. good iff $\mathcal{O}\left((1/\rho)^{1/3}\partial_\mu f(...)\right) \ll 1$
Classical AdS/CFT requires $\lambda \gg 1$ but $\lambda N_c^{-1} = g_{YM} \ll 1 \Rightarrow \frac{1}{TN_c^{2/3}} \ll \frac{\eta}{sT}$
Away from this limit microscopic fluctuations drive fluid. This limit is very little known (See also: Kovtun, Moore, Romatschke, 1104.1586) but it might be very relevant for both QGP and cold atoms

Hydro as fields: for simplicity assume no conserved charges except $T_{\mu\nu}$
Continuus mechanics (fluids, solids, jellies,...) is written in terms of 3-coordinates $\phi_I(x^\mu), I = 1...3$ of the position of a fluid cell originally at $\phi_I(t=0, x^i), I = 1...3$. (Lagrangian hydro)!



The system is a Fluid if it's Lagrangian obeys some symmetries (Ideal hydrodynamics $\leftrightarrow$ Isotropy in comoving frame) Excitations (Sound waves, vortices etc) can be thought of as "Goldstone bosons", arising when a theory is expanded around a classical solution.

Translation invariance at Lagrangian level $\leftrightarrow$ Lagrangian can only be a function of $B^{IJ} = \partial_\mu \phi^I \partial^\mu \phi^J$ Now we have a "continuus material"!

Homogeneity/Isotropy means the Lagrangian can only be a function of $B = \mathrm{det} B^{IJ}, \mathrm{diag} B^{IJ}$
The comoving fluid cell must not see a "preferred" direction $\Leftarrow SO(3)$ invariance

Invariance under Volume-preserving diffeomorphisms means the Lagrangian can only be a function of B
In all fluids a cell can be infinitesimally deformed
(with this, we have a fluid. If this last requirement is not met, Nicolis et all call this a "Jelly")

A few exercises for the bored public Check that L = F(B) leads to

$$T_{\mu\nu} = (P+\rho)u_\mu u_\nu - P g_{\mu\nu}$$

provided that

$$\rho = F(B)\ , \qquad p = F(B) - 2F'(B)B\ , \qquad u^\mu = \frac{1}{6\sqrt{B}}\epsilon^{\mu\alpha\beta\gamma}\epsilon_{IJK}\,\partial_\alpha\phi^I\partial_\beta\phi^J\partial_\gamma\phi^K$$

Equation of state chosen by specifying $F(B)$. "Ideal": $\Leftrightarrow F(B) = B^{4/3}$
$\sqrt{B}$ is identified with the entropy and $\sqrt{B}\frac{dF(B)}{dB}$ with the microscopic temperature. You can also show that

$$\partial_\mu \sqrt{B} u^\mu = 0 \qquad , \qquad s = -\frac{dP}{dT} = \frac{p+\rho}{T}$$

Ie, $\sqrt{B}$ is the conserved quantity corresponding to our earlier group.

Ideal hydrodynamics and the microscopic scale

The most general Lagrangian is

$$L = T_0^4 F\left(\frac{B}{T_0^6}\right) \quad , \quad B = T_0^6 \det B^{IJ} \quad , \quad B^{IJ} = \left|\partial_\mu \phi^I \partial^\mu \phi^J\right|$$

Where $\phi^{I=1,2,3}$ is the comoving coordinate of a volume element of fluid.

NB: $T_0 \sim \Lambda g$ microscopic scale, includes thermal wavelength and $g \sim N_c^2$ (or μ/Λ for dense systems). $T_0 \to \infty \Rightarrow$ classical limit
It is therefore natural to identify T_0 with the microscopic scale!
At $T_0 < \infty$ quantum and thermal fluctuations can produce sound waves and vortices, "weighted" by the usual path integral prescription! We can now investigate this limit!

Let us linearly expand around the <u>static</u> solution with enthalpy $w_0 \sim T_0^4$

$\phi_I^0 = \vec{X}$ with perturbations: $\phi = \phi_I^0 + \vec{\pi} = \phi_I^0 + \underbrace{\vec{\pi}_L}_{sound} + \underbrace{\vec{\pi}_T}_{vortex}$

$$(\partial\phi)^{-1} = 1 - \partial\pi + \partial\pi^2 + ...$$

$$\det \partial\phi = 1 + [\partial\pi] + \frac{1}{2}\left([\partial\pi]^2 - [\partial\pi^2]\right) + \frac{1}{6}\left([\partial\pi]^3 - 3[\partial\pi][\partial\pi^2] + 2[\partial\pi^3]\right)$$

Two polarizations: $\vec{\pi}$ (sound waves) and π_T (vortices)

And we discover a fundamental problem: Vortices carry arbitray small energies but stay put! No S-matrix in hydrostatic solution!

$$L_{linear} = \underbrace{\dot{\vec{\pi}}_L^{\,2} - c_s^2 (\nabla . \vec{\pi}_L)^2}_{sound \quad wave} + \underbrace{\dot{\pi_T}^2}_{vortex} + Interactions$$

Unlike sound waves , Vortices <u>can not</u> give you a theory of free particles, since they <u>do not propagate</u>: They carry energy and momentum but stay in the same place! Can not expand such a quantum theory in terms of free particles.

Physically: "quantum vortices" can live for an arbitrary long time, and dominate any vacuum solution with their interactions.

A perturbative and suspect solution : Give "quantum vortices" a propagation speed, $E = c_T p$, bring $c_T \to 0$ (see 1112.4086). But some of us are lattice theorists, can do better than that!

$$\int \mathcal{D}\phi_I \exp\left(i\int d^4xL\right) \underbrace{\rightarrow}_{lattice+Wick} \int d\phi_I^i \exp\left[-(T_0\Delta x)^4 \sum_i F(\phi_i)\right]$$

Continuum limit: $\delta \rightarrow \Delta x T_0 \ll 1$. Study the behaviour of $\lim_{\delta\rightarrow 0}\langle X\rangle$.
Some specific considerations:

recall that $B_{IJ} = \partial_\mu\phi^I\partial^\mu\phi^J$ and $u^\mu = \frac{1}{b}\epsilon^{\mu\alpha\beta\gamma}\partial_\alpha\phi^1\partial_\beta\phi^2\partial_\gamma\phi^3$ and: $\mathcal{L} = F(b)$; $b = \sqrt{\det B_{IJ}}$ to avoid problems with periodic boundaries use "shifted" fields ("subtract" the hydrostatic background)...

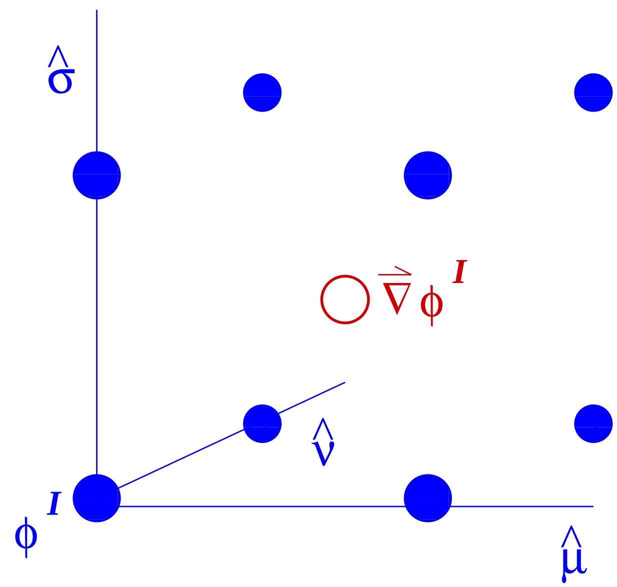
$$\pi^I = \phi^I - x^I \quad \rightarrow \quad \partial_\alpha\phi^I = \partial_\alpha\pi^I + 1\delta^I_\alpha$$

since we expect extended structures (e.g., vortices) we use HMC updates: one therefore needs the variation of the action w.r.t. the local field values...

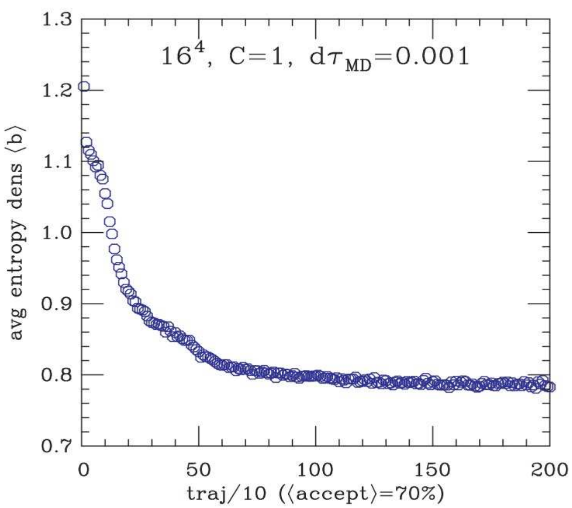
$$\frac{\delta S}{\delta \phi^I(x)} = \frac{\delta S}{\delta b}\frac{\delta b}{\delta(\partial_\alpha \phi^J)}\frac{\delta(\partial_\alpha \phi^J)}{\delta \phi^I(x)} =$$

$$\sum_{y,\mu,\nu,\sigma} \frac{dF}{db}\delta^{IJ}\delta(y - x \pm \hat{\mu}/2 \pm \hat{\nu}/2 \pm \hat{\sigma}/2) \quad \frac{b}{8}B^{-1}_{JK}\left|\epsilon_{\mu\nu\sigma\alpha}\right| \partial_\alpha \phi^K\Big|^{y-\hat{\alpha}/2}_{y+\hat{\alpha}/2}$$

fields (ϕ^I) occupy lattice sites; derivatives (and hence B_{IJ}, b, u_μ, $T_{\mu\nu}$, etc.) defined at body centers of hypercubes:



L^4	$C(= aT_0)$	traj	$d\tau_{MD}$	accept
20^4	0.8	4000	0.001 / 0.0005	49% / 85%
16^4	1	10000	0.001	52%
12^4	1.33333	10000	0.0005	61%
10^4	1.6	10000	0.0005	41%
8^4	2	10000	0.00025	72%
6^4	2.66667	10000	0.00025	56%



C/Open MP code

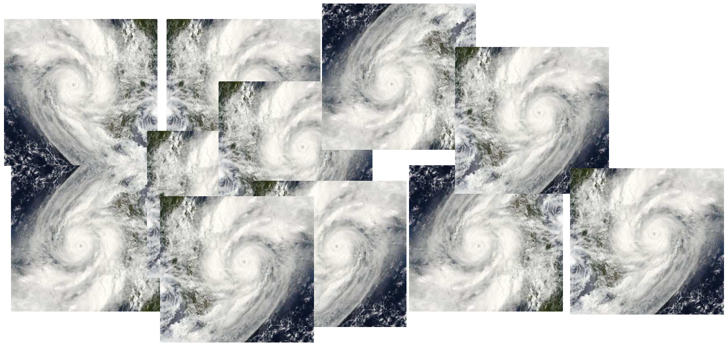
HMC algorithm

Runs with constant physical volume

$L^4=(16/C)^4$

C: assumed fundamental scale $\sim aT_0$

So, what
can we
calculate?



Non−trivial ground state

Is there a phase transition at a critical T_0 between a "classical" hydrostatic vacuum and a vacuum dominated by quantum/thermal turbulence (bound states of quantum vortices and the like)?

Is the theory trivial in the RG group sense?
What $F(B)$ admit to a well-behaved continuum limit?

These questions can be answered by a lattice calculation. No spurious c_T parameter or perturbative expansion needed

Some interesting observables

- "Scalar perturbation" $\frac{\langle B(dF/dB)\rangle}{\left\langle T_\mu^\mu \right\rangle}$

- "Vector perturbation" $\langle u_\mu u_\nu + g_{\mu\nu}\rangle = \left\langle \frac{1}{B_{IJ}}\partial_\mu \phi^I \partial_\nu \phi^J \right\rangle$

- Vorticity $C_P = \oint_P (p+\rho) u_\mu dx^\mu$

Averages and Fluctuation, correlator,spectral function interesting. Modifications of either with T_0 could indicate transition to "quantum turbilence".

Consider a conformal fluid with no degeneracy and one microscopic DoF

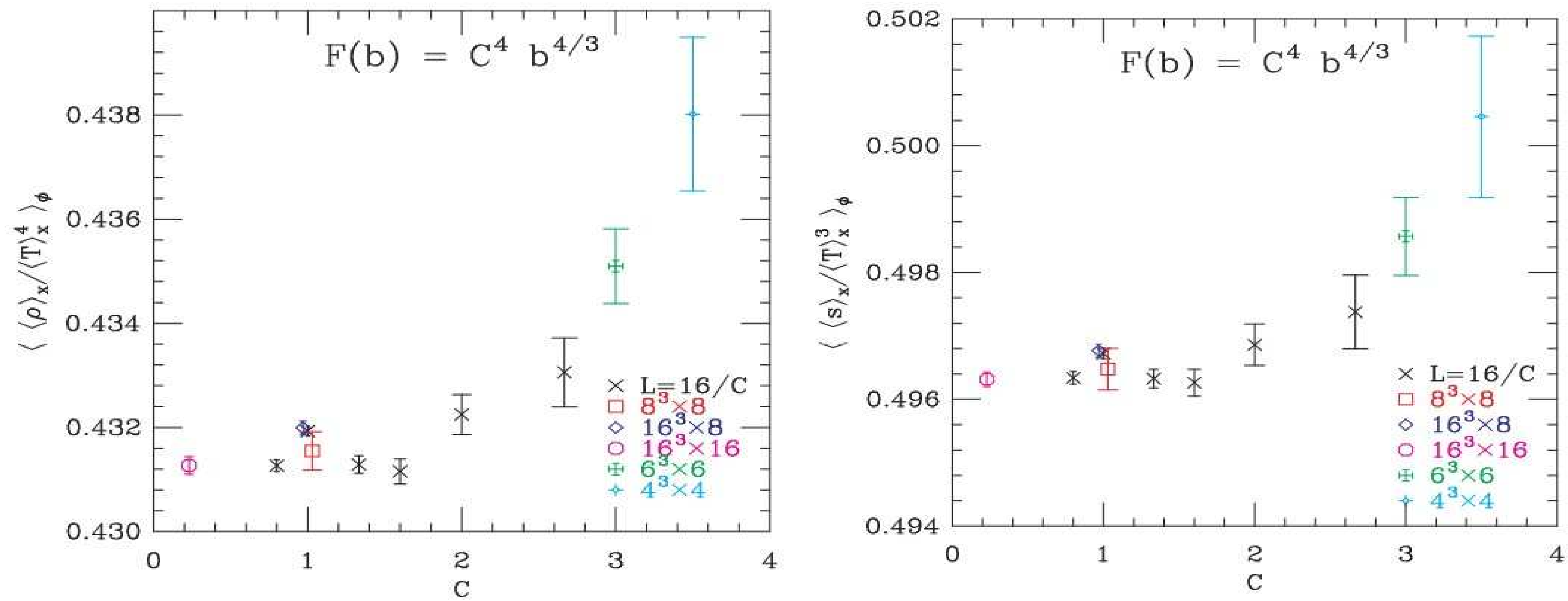
In the classical hydrostatic limit (Where $B=1$)

$$\left.\begin{array}{rcl} e = & T_0^4 B^{2/3} & = \frac{g\pi^2}{60} T^4 \\ s = & T_0^3 \sqrt{B} & = \frac{g\pi^2}{45} T^3 \\ T = & \frac{e+p}{s} & = \frac{4}{3g} T_0 B^{1/6} \end{array}\right\} T = \frac{4}{3g} T_0 = \frac{\chi}{a}?$$

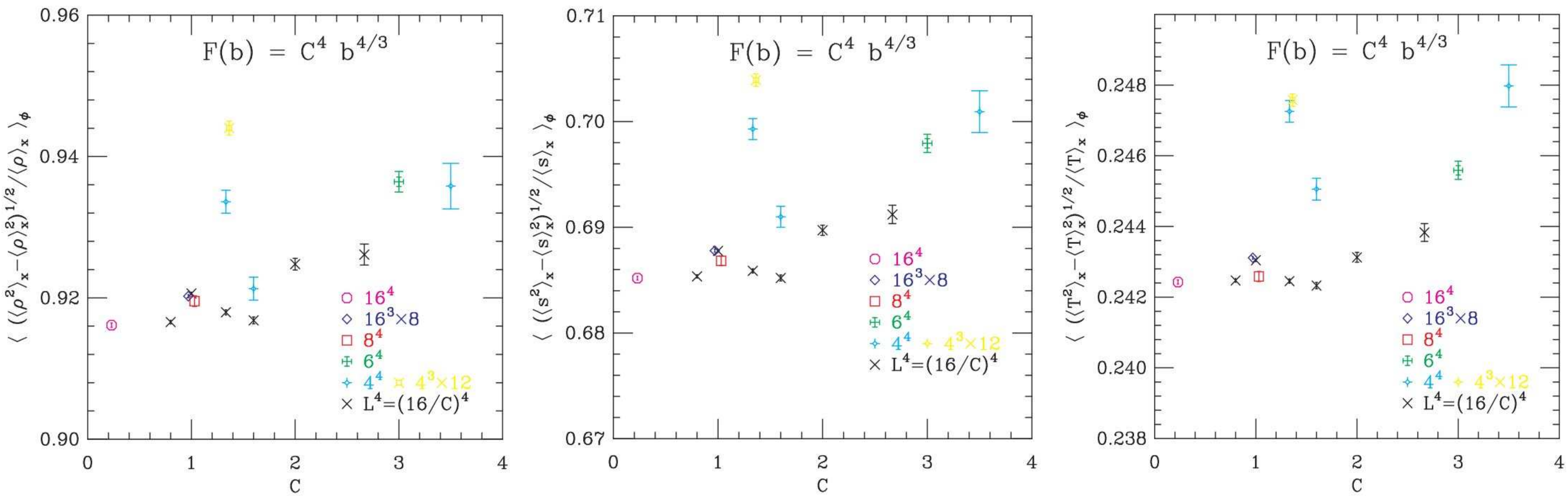
where g is the microscopic degeneracy$\sim N_c^2$. And of course

$$\vec{u^{\mu}} = (1, \vec{0}), \langle u_\mu u_\nu \rangle = \delta_{00} \quad , \quad \langle T_{\mu\nu} \rangle = \frac{\delta \ln Z}{\delta g_{\mu\nu}} \begin{pmatrix} e & 0 & 0 & 0 \\ 0 & e/3 & 0 & 0 \\ 0 & 0 & e/3 & 0 \\ 0 & 0 & 0 & e/3 \end{pmatrix}$$

with higher order correlations vanishing. If quantum vacuum non-trivial $T_{\mu\nu} = (p+\rho) u_\mu u_\nu + p g_{\mu\nu}$ so $e = F(B) \neq T_{00}, p = B \frac{dF(b)}{dB} \neq T_{ii}$ etc.

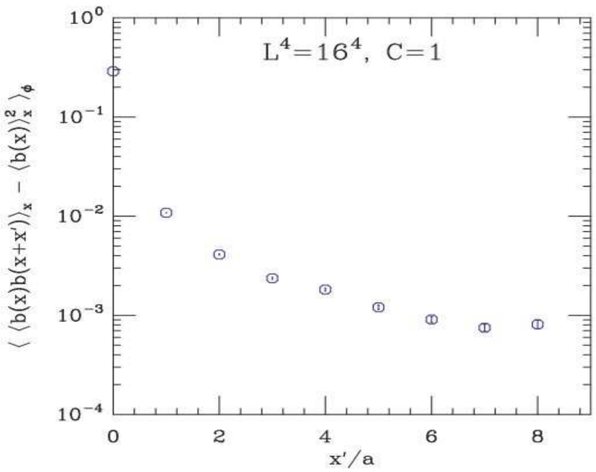


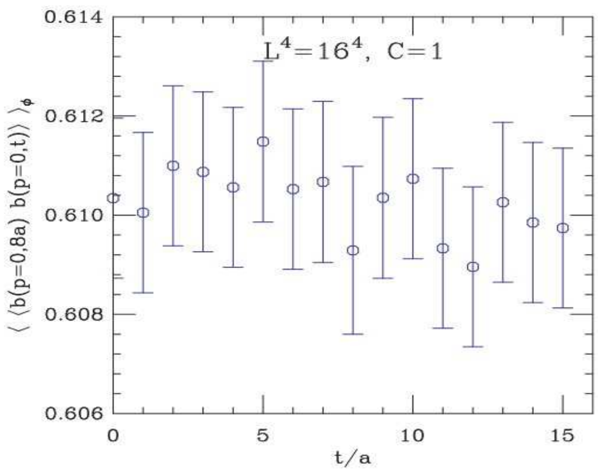
Quantum mechanics means scale a potentially physical "cutoff", dominating dynamics $aT_0 \sim C$. interesting structure at high C... Crossover to collective-dominated regime or lattice artifact?

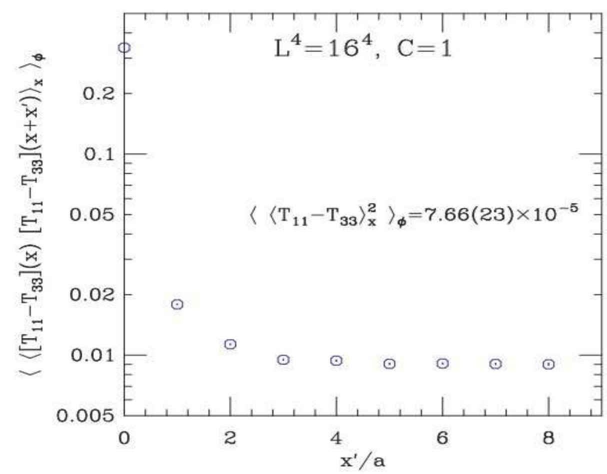


Normalized fluctuations independent of C, but the constants of proportionality non-trivial

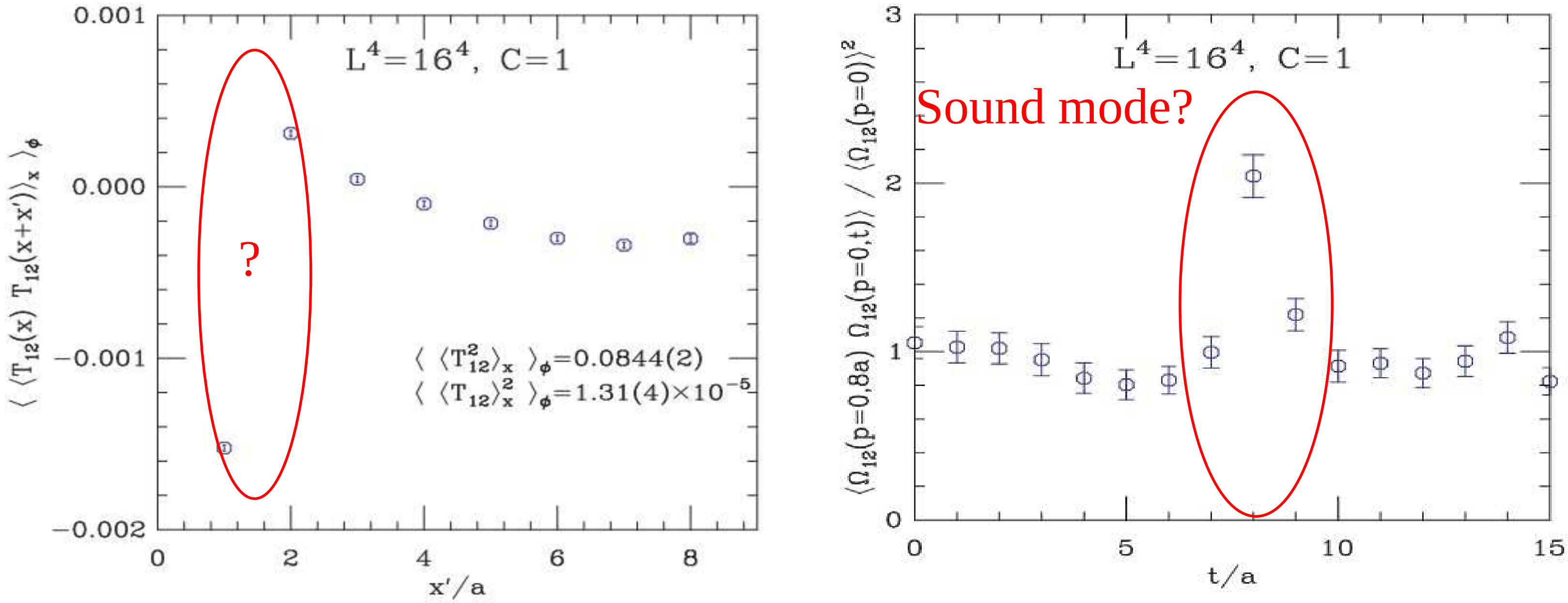
Fluctuations high Are we "missing" phase transition by measuring average observables? Are fluctuations part of "new phase" ?







Entropy and energy density correlators, "quantum corrections" to equations of state?



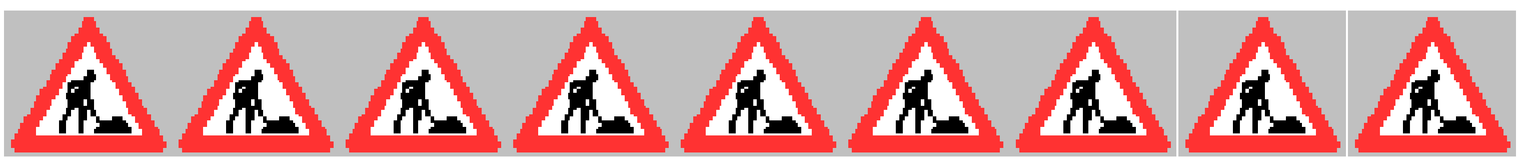
Off diagonal and diagonal elements have long-time correlation. To what extent is this "similar to a quantum viscosity" ?

Instead of a conclusion: further steps

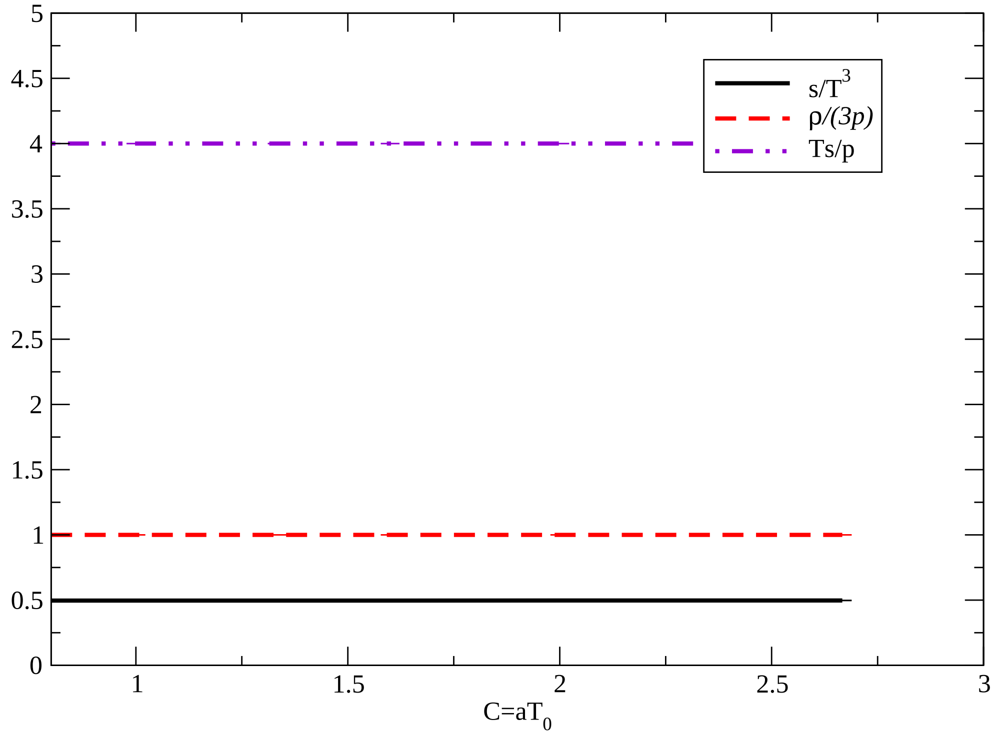
Understand the continuum limit How do observables diverge when it is approached? Langevan semi-classical limit? (Relativistic generalization of T.Koide, T.Kodama, 1105.6256)

Understanding the vacuum Under what circumstances is the hydrostatic limit stable? If its not, what are the effective degrees of freedom? What happens at finite temperature? Are there phase transitions In temperature or T_0 ?

Connecting to the "usual" transport theory BBGKY hyerarchy, gradient expansion



Spare slides



Rescaling C by appropriate factor gives us "right" thermodynamics

If $\lim_{\delta\to 0} \langle X \rangle \sim \langle X_0 \rangle$, the thermostatic expectation value, ($\langle X \rangle$ is stable).

If $\lim_{\delta\to 0} \langle X \rangle / \langle X_0 \rangle \sim f(B)$, vacuum non-trivial but "well-behaved".

If $\lim_{\delta\to 0} \langle X \rangle / \langle X_0 \rangle \sim \delta^{-\alpha}$ or $\sim \exp(\alpha\delta^{-1})$ for universal degrees of diverge α, the theory is renormalizeable: δ is needed to set an absolute scale, but dimensionless ratios are independent of it.

If $\lim_{\delta\to 0} \langle X \rangle / \langle X_0 \rangle \sim \delta^{-\alpha}$ or $\sim \exp(\alpha\delta^{-1})$ for αs that are $\langle X \rangle$-specific (One α for the scalar and another for the tensor,defined below) the theory is "trivial" , in that taking $\delta \to 0$ makes the vacuum diverge. In this case, step **(ii)** of the previous section is strictly impossible.

Expectation: $\delta \sim (aT_0)$, α is what you get from dimensional analysis in the <u>classical limit</u> , but something else <u>beyond</u> the transition

A prelude: Kovtun, Moore, Romatschke, 1104.1586

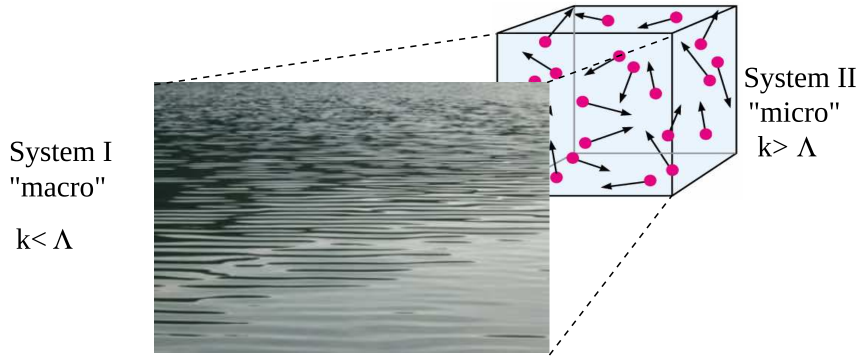
Basic idea: As viscosity decreases, energy-momentum tensor becomes correlated by soundwaves ("infinite propagation of soundwaves" inpacts "IR limit of Kubo formula").

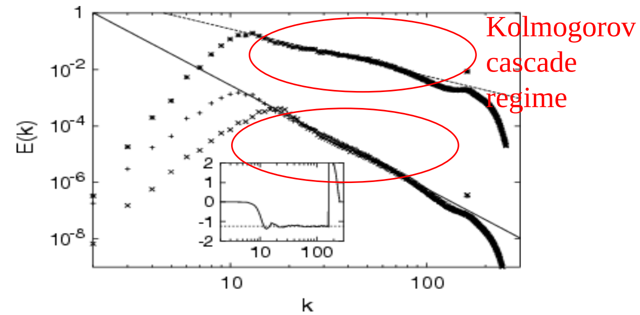
$$G^{xyxy}_{\mathrm{R,shear-shear}}(\omega) \simeq -i\omega \frac{7Tp_{max}}{60\pi^2\gamma_\eta} + (i+1)\omega^{\frac{3}{2}} \frac{7T}{240\pi\gamma_\eta^{\frac{3}{2}}}$$

where p_{max} is the maximum momentum scale and $\gamma_\eta = \eta/(e+p)$
The authors analyze p_{max} in terms of $\tau_\pi \sim \eta/(Ts)$, but we want to see what happens in <u>ideal</u> fluids!

$$\eta_{reno} = \eta_{bare} + \frac{17p_{max}\frac{\eta_{bare}}{e+p}T(\epsilon+P)^2}{120\pi^2\eta^2_{bare}}$$

Naively, $\eta_{reno} \sim p_{max}/\eta \sim \eta_{bare}^{-2}$, quadratic divergence . But...





A classical low-viscosity fluid is <u>turbulent</u>. Typically, low-k modes cascade into higher and higher k modes via sound and vortex emission (phase space looks more "fractal"). Classically this process goes on until dissipation, $k \sim \eta/(Ts)$. By essneitally dimensional analysis, Kolmogorov has showsn that provided $\eta/(sT) \ll L_{eddy} \ll L_{boundary}$, $E(k) \sim \left(\frac{dE}{dt}\right)^{2/3} k^{-5/3}$ For a classical ideal fluid, it can go on forever, since $\delta E(k) \sim \delta \rho k c_s$ can be arbitrarily small for arbitrarily high k by making $\delta\rho$ even smaller. but for <u>quantum</u> perturbations, $E \geq k$ so conservation of energy <u>has</u> to cap cascade

Now we are all set...

Cross-sections have been computed using these Feynman rules.

$$\sigma_{TT\leftrightarrow TT} = \frac{1}{256\pi}\left(\frac{13}{15}\right)\frac{1}{p^2}\left(\frac{p^4}{w_0 c_T}\right)^2 \quad , \quad \sigma_{LL\leftrightarrow LT} \sim \frac{c_T}{c_s}\frac{1}{p^2}\left(\frac{p^4}{w_0 c_s}\right)^2$$

$$\sigma_{LT\leftrightarrow LT} = \frac{1}{105\pi}\frac{1}{p^2}\left(\frac{p^4}{w_0 c_s}\right)^2 [1 + 7c_s^4] + \mathcal{O}(c_T)$$

$$\sigma_{LL\leftrightarrow LL} = \frac{1}{256\pi}\frac{1}{p^2}\left(\frac{p^4}{w_0 c_s}\right)^2 \left[2\alpha^2 + \frac{4\alpha\beta}{3} + \frac{2\beta^2}{5}\right] \sim \frac{1}{p^2}\left(\frac{p^4}{w_0 c_s}\right)^2$$

where p is the exchanged momentum, w_0 is the *microscopic* enthalpy density ($= Ts$) of the background fluid and $\alpha \equiv (f_4/c_s^2 - 2f_3^2/c_s^4 + 3c_s^2 + 2f_3 + c_s^4) = \mathcal{O}(1) + \mathcal{O}(c_s^2) + \mathcal{O}(c_s^4)$.

NB the singularities at $c_T \to 0$

Now we are all set...

As S.Jeon, hep-ph/9409250 at tree level, η from Kubo's formula

$$\eta = \frac{\beta}{20} \lim_{\omega \to 0} \lim_{\mathbf{q} \to 0} \int d^3\mathbf{x}\, dt\, e^{-i\mathbf{q}\cdot\mathbf{x}+i\omega t} \left\langle \pi_{lm}(t,\mathbf{x})\pi^{lm}(0)\right\rangle_{\rm eq}$$

equivalent to "Lifshitz-Landau formula"

$$\eta = \# \langle p \rangle \langle n \rangle l_{mfp} \quad , \quad l_{mfp} = \frac{1}{\langle\langle n\sigma \rangle\rangle}$$

where

$$\langle X \rangle = \frac{1}{N} \int d^3p X(\vec{p}) f(p,T) \quad , \quad \langle\langle X \rangle\rangle = \frac{1}{N} \int d^3p_1 d^3p_2 X(\vec{p_1}-\vec{p_2}) f(\vec{p_1},T) f(\vec{p}$$

Note: What about bulk viscosity

In this approximation

$$\zeta = \# \left(c_{ss}^2 - \frac{1}{3} \right)^2 \eta$$

c_{ss}^2 is the "speed of sound in a gas of sound-waves". In an ideal gas, $F(B) = T_0^4 B^{4/3}$ the speed of sound of a sound wave is $c_s = 1/\sqrt{s}$, but

$$c_{ss}^2 \sim \frac{1}{\ln T} \left(\ln \frac{d}{dT} \int d^3 p \exp \left[- \frac{p}{\sqrt{3} T} \right] \right) \neq \frac{1}{3}$$

So "quantum sound" generally introduces a bulk viscosity <u>also</u> in an ideal gas .

Is a <u>thermalized</u> $\mathcal{N} = 4$ SYM Stable against $1/N_c$ corrections?

My calculation (Boltzmann approximation):

$$\eta = \frac{1}{3}\frac{\left(\langle p_L\rangle + \langle p_T\rangle\right)\left(\langle n_L\rangle + \langle n_T\rangle\right)^2}{\langle\langle\sigma\rangle\rangle_{LL\leftrightarrow LL} + \langle\langle\sigma\rangle\rangle_{LL\leftrightarrow LT} + 2\,\langle\langle\sigma\rangle\rangle_{LT\leftrightarrow LT} + \langle\langle\sigma\rangle\rangle_{LT\leftrightarrow LL} + \langle\langle\sigma\rangle\rangle_{TT\leftrightarrow TT}}$$

$$-\int_0^\Lambda f(\beta, c_T, p)\ln f(\beta, c_T, p)dp = -\lim_{c_T\to 0}\lim_{\Lambda\to\infty}\int_0^\Lambda c_T\beta p^3 e^{-c_T\beta p}dp$$

where $\langle\langle\sigma_{XY\leftrightarrow AB}\rangle\rangle = \int d^3p_x \int d^3p_y n(p_x)n(p_y)\sigma_{XY\to AB}(\vec{p_x} - \vec{p_y})$ and

$$s = -\int_0^\Lambda f(\beta, c_T, p)\ln f(\beta, c_T, p)dp = -\lim_{c_T\to 0}\lim_{\Lambda\to\infty}\int_0^\Lambda c_T\beta p^3 e^{-c_T\beta p}dp$$

in the limit

$$c_T \to 0, \Lambda \to \infty$$

Three possible divergences

$p_{max} = \Lambda \to \infty$

$c_T \to 0$

$g \to \infty$ (Microscopic degeneracy)
NOT T_0: That goes away in rations like η/s

Adjusting thes three paramters, it should always possible to get a finite η/s even if η, s both diverge differently.

At $\Lambda \to \infty$ integral analytically solvable! Key term

$$\int d^3p_1 \int d^3p_2 e^{-c_1 p_1/T - c_2 p_2/T} \frac{1}{(p_1-p_2)^2} \left(\frac{(p_1-p_2)^4}{w_0} \right)^2 =$$

$$\left(\frac{4\pi}{(2\pi)^3} \right)^2 80640 H(c_1,c_2) \frac{T^{12}}{w_0^2}$$

$$H(c_1,c_2) = \frac{\left(2\zeta(3)\zeta(9)\left(c_1^6+c_2^6\right)+3c_1^2c_2^2\zeta(5)\zeta(7)\left(c_1^2+c_2^2\right)\right)}{c_1^9c_2^9}$$

$$\frac{\eta}{s} = \frac{\left(\frac{\mathcal{O}(1)}{c_s^4} + \frac{\mathcal{O}(1)}{c_T^4}\right)\left(\frac{\mathcal{O}(1)}{c_s^3} + \frac{\mathcal{O}(1)}{c_T^3}\right)}{\frac{\mathcal{O}(1)}{c_s^3} + \frac{\mathcal{O}(1)}{c_T^3}} \frac{w_0^2}{T^8} \times$$

$$\times \left(\alpha_{LL\to LL} H(c_s, c_s) + \alpha_{TT\to TT} H(c_T, c_T) + \right.$$
$$\left. + \alpha_{LT\to LT} H(c_T, c_s) + \alpha_{LL\to LT} H(c_s, c_s)\right)^{-1}$$

Where α_{ii} functions of $c_{T,L}$ and $d^n F/dB$.

Objective: counterbalance $c_T \to 0$ with $g \to \infty$

Similar to "counterterm" in renormalization theory, except c_T an infrared deformation, not a UV one

$$\frac{\eta}{s} \sim \frac{\mathcal{O}\left(w_0^2\right)}{c_T^4 T^8} \left(\underbrace{\mathcal{O}\left(1\right)}_{LL\to LL} + \underbrace{\frac{\mathcal{O}\left(1\right)}{c_T^{14}}}_{TT\to TT} + \underbrace{\frac{\mathcal{O}\left(1\right)}{c_T^{9}}}_{LT\to LT} + \underbrace{\mathcal{O}\left(c_T\right)}_{LL\to LT} \right)^{-1}$$

Remembering that

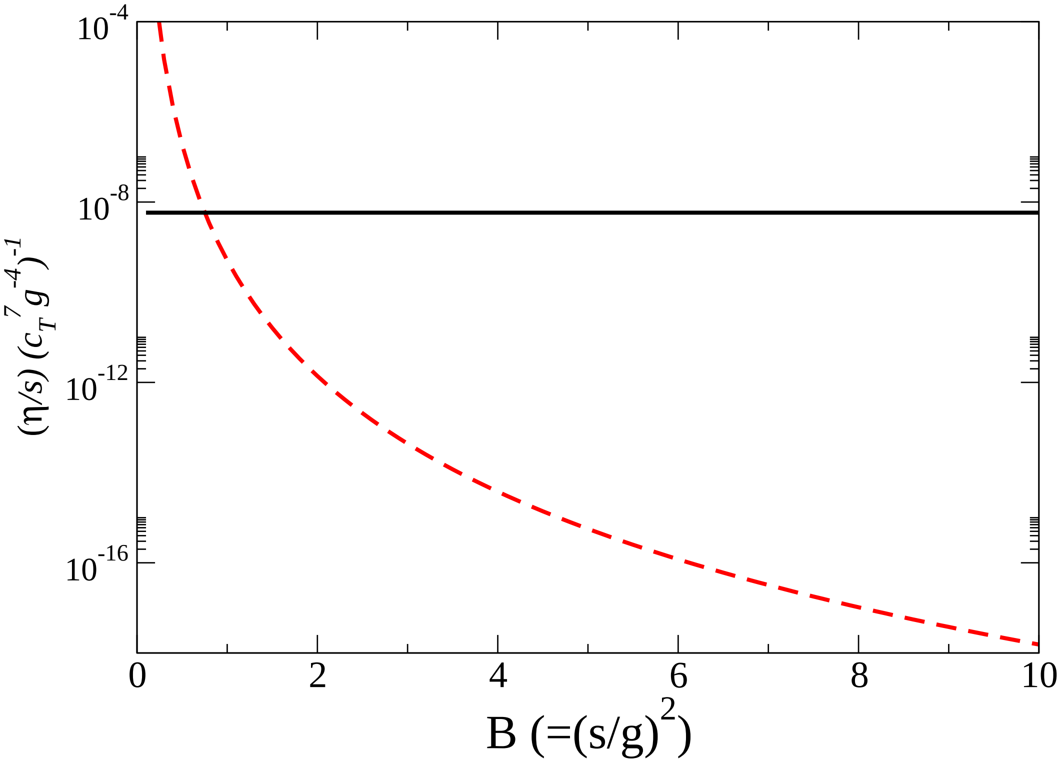
$$T = \frac{w_0}{s} \sim \frac{\sqrt{B}(dF/dB)}{g}$$

The grand result

$$\frac{\eta}{s} = K_0 \frac{c_T^{14} g^8}{B^2 (dF/dB)^6}$$

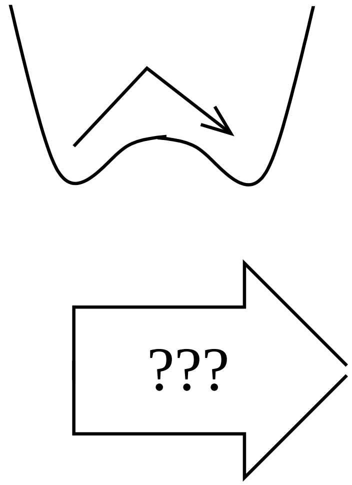
$$K_0 = \frac{\zeta(3)^2 \zeta(9)}{80640} \frac{4}{256\pi} \frac{13}{45} \frac{\pi^2}{15} \left(\frac{4\pi^4}{45}\right)^{-1} \simeq 1.96(10^{-9})$$

The meat: If $c_T^{14} g^8 \sim \mathcal{O}\left(1\right)$, η/s finite and evolves with EoS.



Result for $F(B) = \begin{cases} B^{2/3} & \text{Ideal gas} \\ f_1 B^{\zeta}/B_0^{f_2/2-2/3} & \text{lQCD fit} \end{cases}$

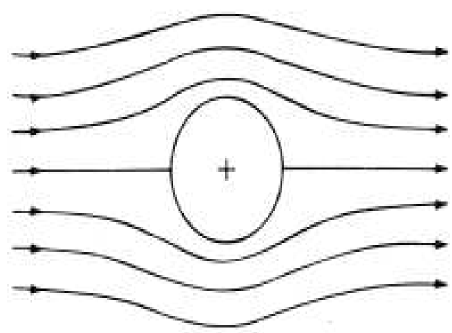
Non-perturbative dissipation loss (“quantum” turbulence)



Classical hydrodynamics has infinitely many solutions arbitrarily close together.
Could WKB-type jumps among solutions with different entropy content be allowed? work in progress!

Example: The Dalambert problem

Analytical solution Euler equation

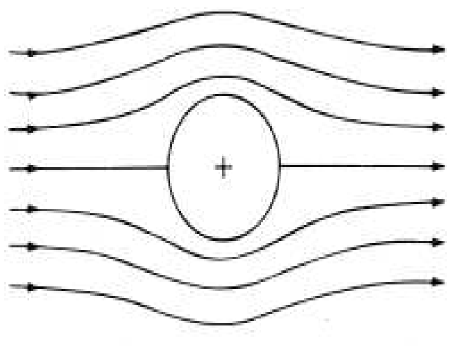


Cylinder and asymptotic flow

Analytically solvable:

$$v_r = U\left(1 - \frac{R^2}{r^2}\right)\cos\theta \quad , \quad v_\theta = -U\left(1 + \frac{R^2}{r^2}\right)\sin\theta$$

Analytical solution Euler equation

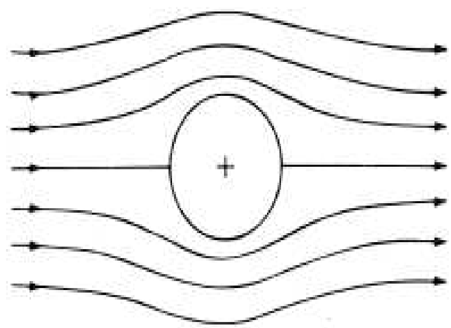


Cylinder and asymptotic flow

For Energy to be the same $\frac{\rho(U_1)}{\rho(U_2)} = \left(\frac{U_2}{U_1}\right)^2$

NB: Entropy density <u>different</u> for each U

Analytical
solution
Euler equation



Cylinder
and
asymptotic flow

Rewrite in ϕ_I and find minima in $\left\langle \phi^I_{\vec{x_0},U,\mathcal{E}} \middle| \middle| \phi^I_{\vec{x_0}',U',\mathcal{E}'} \right\rangle \sim \exp\left[-\Delta S_{U,U'}\right]$

$$\Delta S_{U,U'} = \int d^4x \sum_{IJ} \left.\frac{\delta^2 S}{\delta\phi^I \delta\phi^J}\right|_{\phi^{I,J}=\phi^I_{\vec{x_0},U,\mathcal{E}}} \sum_{IJ} \left(\phi^I_{\vec{x_0},U,\mathcal{E}} - \phi^I_{\vec{x}'_0,U',\mathcal{E}}\right)\left(\phi^J_{\vec{x_0},U,\mathcal{E}} - \phi^J_{\vec{x}'_0,U}\right.$$

What does this mean?

Why does a quintessentially unitary theory (quantum mechanics!) set a lower limit to dissipative processes?
How does one reconcile quantum viscosity with Von Neumann's theorem?

$$\frac{d}{dt}\mathrm{Tr}\hat{\rho}\ln\hat{\rho} = 0$$

My tentative answer: Quantum field theory also sets limit to scale beyond which we measure! Quantum correlations in a many particle system inevitably go over that scale.

What the hell does this all mean? II

Loss of unitarity at the renormalization scale. A quantum field with many particles obeys the fully quantum equation of motion

$$\frac{d\hat{\rho}}{dt} = i\left[\mathcal{H}, \hat{\rho}\right]$$

where $\hat{\rho}$ is the density operator for the field

$$\hat{\rho}(x) = \sum_{k,k'} A_{k,k'} a^{+}_{k,k'} |0 >< 0| a_{k,k'}$$

and $\mathcal{H}$ is the Hamiltonian density.

Like all QFT equations, this has to be regulated by a momentum scale Λ (plus, fluid theory non-renormalizable). Generally, information should flow across the cut-off (ie, get lost among the "fast" degrees of freedom), so effective theory dissipative

Conclusions

Ie, what needs to be done before I have a result

Understand divergences Under what circumstances, if any, can g, c_T, p_{max} diverge while η/s is constant

Understanding how does this constrain the "running" of η/s with $\sqrt{B}, c_T$

Understanding whether this makes any sense...

Work in progress... if you think you can help, Id like to hear from you!