Study of the $a_0(980)$ on the lattice

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Introduction, motivation (1)

- The nonet of light scalar mesons ($J^P = 0^+$)
  - $\sigma \equiv f_0(500)$, $I = 0$, $400 \ldots 550\,$MeV,
  - $\kappa \equiv K_0^*(800)$, $I = 1/2$, $682 \pm 29\,$MeV,
  - $a_0(980)$, $f_0(980)$, $I = 1$, $980 \pm 20\,$MeV, $990 \pm 20\,$MeV

  is poorly understood:
  - All nine states are unexpectedly light (should rather be close to the corresponding $J^P = 1^+, 2^+$ states around $1200 \ldots 1500\,$MeV).
  - The ordering of states is inverted compared to expectation:
    - E.g. in a $q\bar{q}$ picture the $I = 1$ states $a_0(980)$, $f_0(980)$ must necessarily be formed by two u/d quarks, while the $I = 1/2$ κ states are made from an s and a u/d quark; since $m_s > m_{u/d}$ one would expect $m(\kappa) > m(a_0(980)), m(f_0(980))$.

Introduction, motivation (2)

* In a tetraquark picture the quark content could be the following: $\kappa \equiv \bar{s}l\bar{l}l$, while $a_0(980)$, $f_0(980) \equiv \bar{s}l\bar{l}s$; this would naturally explain the observed ordering.

– Certain decays also support a tetraquark interpretation: e.g. $a_0(980)$ readily decays to $K + \bar{K}$, which indicates that besides the two light quarks required by $I = 1$ also an $s\bar{s}$ pair is present.

→ Study these states by means of lattice QCD to confirm or to rule out their interpretation in terms of tetraquarks.

- Examples of heavy mesons, which are tetraquark candidates:
  - $D_{s0}^*(2317)^\pm$ ($I(J^P) = 0(0^+)$), $D_{s1}(2460)^\pm$ ($I(J^P) = 0(1^+)$),
  - charmonium states $X(3872)$, $Z(4430)^\pm$, $Z(4050)^\pm$, $Z(4250)^\pm$, ...

Outline

(1) Wilson twisted mass study of $a_0(980)$.

[C. Alexandrou *et al.* [ETM Collaboration], JHEP **1304**, 137 (2013) [arXiv:1212.1418 [hep-lat]]]

(2) Recent advances:

- Lattice discretization changed, now Wilson + clover fermions (generated by the PACS-CS Collaboration).

  [S. Aoki *et al.* [PACS-CS Collaboration], Phys. Rev. D **79**, 034503 (2009) [arXiv:0807.1661 [hep-lat]]]

- Inclusion of disconnected diagrams.

Part 1:
Wilson twisted mass study of $a_0(980)$

Tetraquark creation operators

- $a_0(980)$:
  - Quantum numbers $I(J^{PC}) = 1(0^{++})$.
  - Mass $980 \pm 20\,\text{MeV}$.
- Tetraquark creation operators:
  - Need two light quarks due to $I = 1$, e.g. $u\bar{d}$.
  - $a_0(980)$ decays to $K\bar{K}$ ... suggests an $s\bar{s}$ component.
  - Molecule type (models a bound $K\bar{K}$ state):

$$\mathcal{O}_{a_0(980)}^{K\bar{K}\text{ molecule}} = \sum_{\mathbf{x}} \Big(\bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x})\Big)\Big(\bar{d}(\mathbf{x})\gamma_5 s(\mathbf{x})\Big).$$

  - Diquark type (models a bound diquark-antidiquark):

$$\mathcal{O}_{a_0(980)}^{\text{diquark}} = \sum_{\mathbf{x}} \Big(\epsilon^{abc}\bar{s}^b(\mathbf{x})C\gamma_5\bar{d}^{c,T}(\mathbf{x})\Big)\Big(\epsilon^{ade}u^{d,T}(\mathbf{x})C\gamma_5 s^e(\mathbf{x})\Big).$$

Two-particle creation operators (1)

- There are two-particle states, which have the same quantum numbers as $a_0(980)$, $I(J^{PC}) = 1(0^{++})$,
  - $K + \bar{K}$ ($m(K) \approx 500\,\textrm{MeV}$),
  - $\eta_s + \pi$ ($m(\eta_s \equiv \bar{s}\gamma_5 s) \approx 700\,\textrm{MeV}$, $m(\pi) \approx 300\,\textrm{MeV}$ in our lattice setup),

  which are both around the expected $a_0(980)$ mass $980 \pm 20\,\textrm{MeV}$.

- To determine, whether there is a bound $a_0(980)$ tetraquark state, we need to resolve the above listed two-particle states $K + \bar{K}$ and $\eta_s + \pi$ and check, whether there is an additional 3rd state in the mass region around $980 \pm 20\,\textrm{MeV}$; to this end we need operators of two-particle type.

Two-particle creation operators (2)

- Two-particle operators:
  - Two-particle $K + \bar{K}$ type:

$$\mathcal{O}_{a_0(980)}^{K+\bar{K}\text{ two-particle}} = \left(\sum_{\mathbf{x}} \bar{s}(\mathbf{x})\gamma_5 u(\mathbf{x})\right)\left(\sum_{\mathbf{y}} \bar{d}(\mathbf{y})\gamma_5 s(\mathbf{y})\right).$$

  - Two-particle $\eta_s + \pi$ type:

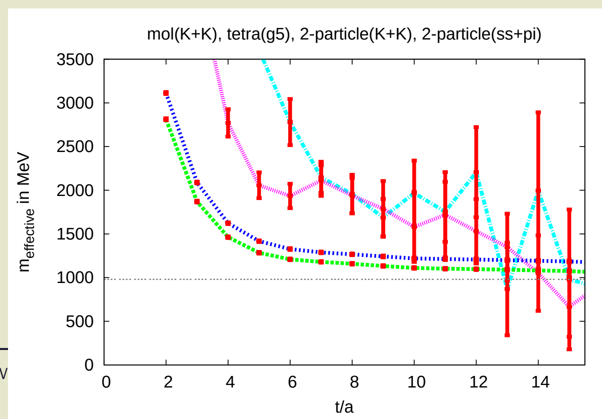
$$\mathcal{O}_{a_0(980)}^{\eta_s+\pi\text{ two-particle}} = \left(\sum_{\mathbf{x}} \bar{s}(\mathbf{x})\gamma_5 s(\mathbf{x})\right)\left(\sum_{\mathbf{y}} \bar{d}(\mathbf{y})\gamma_5 u(\mathbf{y})\right).$$

Wilson twisted mass lattice setup

- Gauge link configurations generated by the ETM Collaboration.
  [R. Baron *et al.*, JHEP **1006**, 111 (2010) [arXiv:1004.5284 [hep-lat]]]
- 2+1+1 dynamical Wilson twisted mass quark flavors, i.e. u, d, s and c sea quarks.
- Iwasaki gauge action.
- Lattice spacing $a \approx 0.086$ fm.
- Various lattice volumes.
- Various light u/d quark masses corresponding pion masses $m_\pi \approx 280 \ldots 460$ MeV.
- APE smeared links and Gaussian smeared quark fields to improve the overlap of trial states to the low lying states.
- Singly disconnected contributions neglected, i.e. no s quark propagation within the same timeslice.

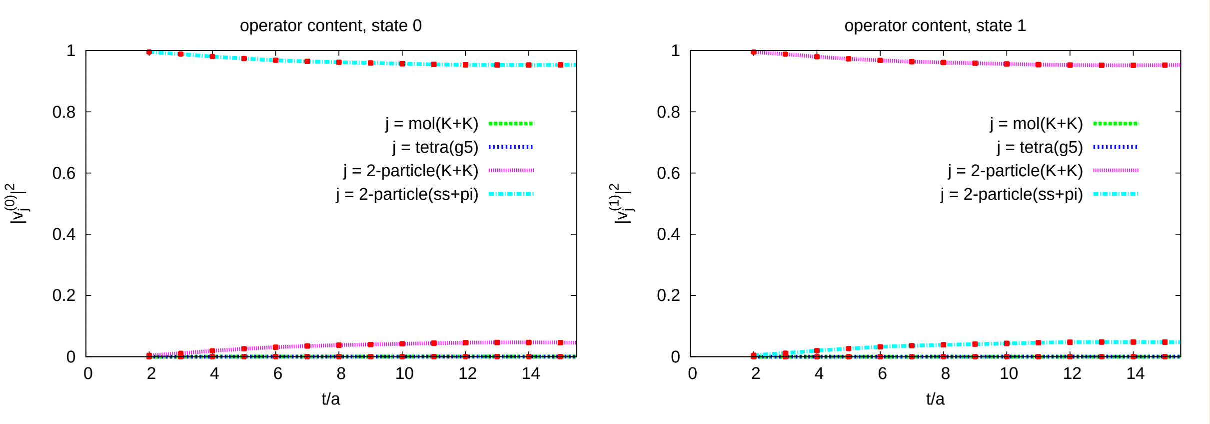
Numerical results $a_0(980)$ (1)

- Study all four operators ($K\bar{K}$ molecule, diquark, $K+\bar{K}$ two-particle, $\eta_s+\pi$ two-particle) at the same time, extract the four lowest energy eigenstates by diagonalizing a 4×4 correlation matrix.
  - Only two low-lying states around 980 ± 20 MeV, the 2nd and 3rd excitation are ≈ 750 MeV heavier.
  - The signal of the low-lying states is of much better quality than when only considering tetraquark operators
    $\rightarrow$ suggests that the observed low-lying states have much better overlap to the two-particle operators and are most likely of two-particle type.



Numerical results $a_0(980)$ (2)

- When determining low-lying eigenstates from a correlation matrix, one does not only obtain their mass, but also information about their operator content, i.e. which percentage of which operator is present in an extracted state:
  - $\rightarrow$ The ground state is a $\eta_s + \pi$ state ($\gtrsim 95\%$ two-particle $\eta_s + \pi$ content).
  - $\rightarrow$ The first excitation is a $K + \bar{K}$ state ($\gtrsim 95\%$ two-particle $K + \bar{K}$ content).

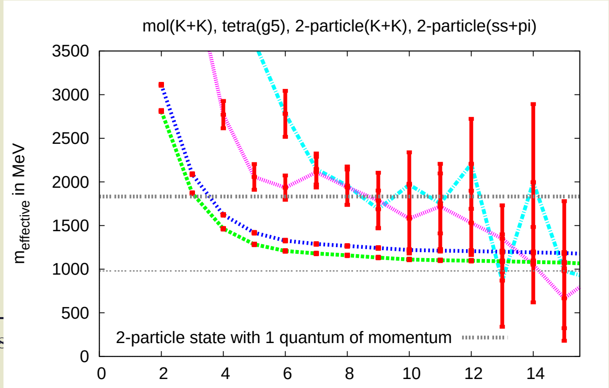


Numerical results $a_0(980)$ (3)

- What about the 2nd and 3rd excitation? ... Are these tetraquark states? ... What is their nature?

- Two-particle states with one relative quantum of momentum (one particle has momentum $+p_\text{min} = +2\pi/L$ the other $-p_\text{min}$) also have quantum numbers $I(J^{PC}) = 1(0^{++})$; their masses can easily be estimated:
  - $p_\text{min} = 2\pi/L \approx 715\,\text{MeV}$ (the results presented correspond to the small lattice with spatial extension $L = 1.73\,\text{fm}$);
  - $m(K(+p_\text{min}) + \bar{K}(-p_\text{min})) \approx 2\sqrt{m(K)^2 + p_\text{min}^2} \approx 1750\,\text{MeV}$;
  - $m(\eta(+p_\text{min}) + \pi(-p_\text{min})) \approx \sqrt{m(\eta)^2 + p_\text{min}^2} + \sqrt{m(\pi)^2 + p_\text{min}^2} \approx$
    $\approx 1780\,\text{MeV}$;

  these estimated mass values are consistent with the observed mass values of the 2nd and 3rd excitation
  $\rightarrow$ suggests to interpret these states as two-particle states.



Numerical results $a_0(980)$ (4)

- Summary:
  - In the $a_0(980)$ sector (quantum numbers $I(J^{PC}) = 1(0^{++})$) we do not observe any low-lying (mass $\lesssim 1750$ MeV) tetraquark state, even though we employed operators of tetraquark structure ($K\bar{K}$ molecule, diquark).
  - The experimentally measured mass for $a_0(980)$ is 980 ± 20 MeV.
  - Conclusion: $a_0(980)$ does not seem to be a strongly bound tetraquark state (either of molecule or of diquark type) ... maybe an ordinary quark-antiquark state or a rather unstable resonance.

Part 2:
Recent advances

Wilson + clover lattice setup

- Gauge link configurations generated by the PACS-CS Collaboration.
  [S. Aoki *et al.* [PACS-CS Collaboration], Phys. Rev. D **79**, 034503 (2009) [arXiv:0807.1661 [hep-lat]]]

- 2+1 dynamical Wilson + clover quark flavors, i.e. u, d and s sea quarks.
  $\rightarrow$ In contrast to twisted mass parity and isospin are exact symmetries, i.e. no pion and kaon mass splitting, easy separation of $P = +, -$ states, ...

- Iwasaki gauge action.

- Lattice spacing $a \approx 0.09\,\text{fm}$, lattice volume $(L/a)^3 \times T/a = 32^3 \times 64$.

- Light u/d quark masses corresponding to pion masses $m_\pi \approx 150\,\text{MeV}$ and $m_\pi \approx 300\,\text{MeV}$.
  $\rightarrow$ Computations close to physically light u/d quark masses possible.

- APE smeared links and Gaussian smeared quark fields to improve the overlap of trial states to the low lying states.

- Singly disconnected contributions included.
  $\rightarrow$ s quark propagation within the same timeslice.

Singly disconnected diagrams (1)

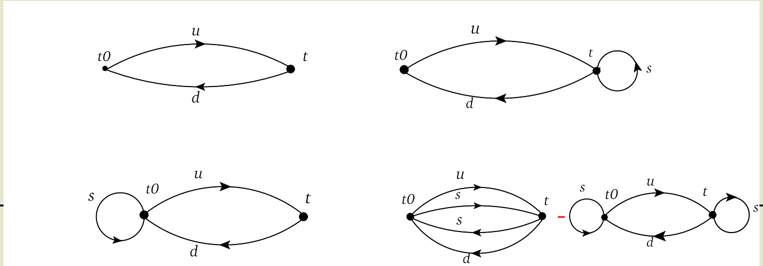
- In our previous Wilson twisted mass study of $a_0(980)$ we neglected singly disconnected contributions:

  $\rightarrow$ We could not consider a $q\bar{q}$ operator,

  $$\mathcal{O}^{q\bar{q}}_{a_0(980)} = \sum_{\mathbf{x}} \Big(\bar{d}(\mathbf{x})u(\mathbf{x})\Big),$$

  because cross correlations between this operator and any of the four-quark operators $\mathcal{O}^{K\bar{K}\text{ molecule}}_{a_0(980)}$, $\mathcal{O}^{\text{diquark}}_{a_0(980)}$, $\mathcal{O}^{K+\bar{K}\text{ two-particle}}_{a_0(980)}$ or $\mathcal{O}^{\eta_s+\pi\text{ two-particle}}_{a_0(980)}$ correspond to singly disconnected diagrams.

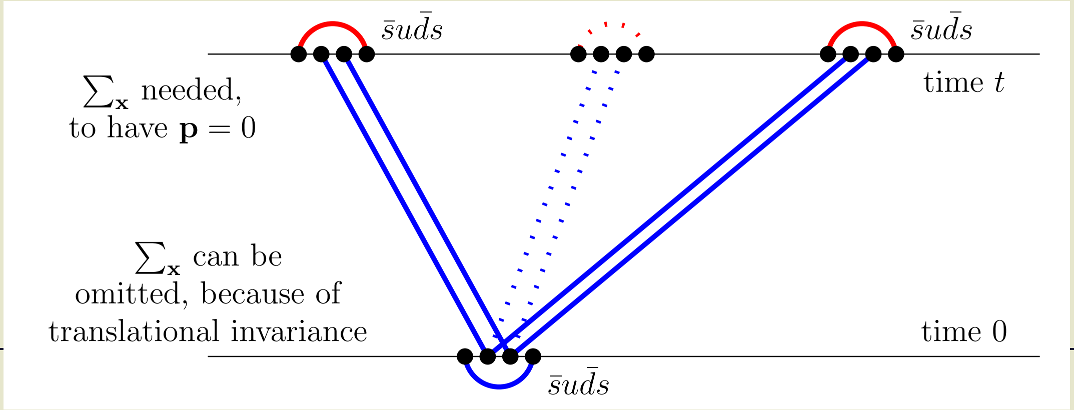
  $\rightarrow$ Also correlations between the four-quark operators include singly disconnected diagrams; therefore, we introduced a source of systematic error, which is difficult to estimate or to control.



Singly disconnected diagrams (2)

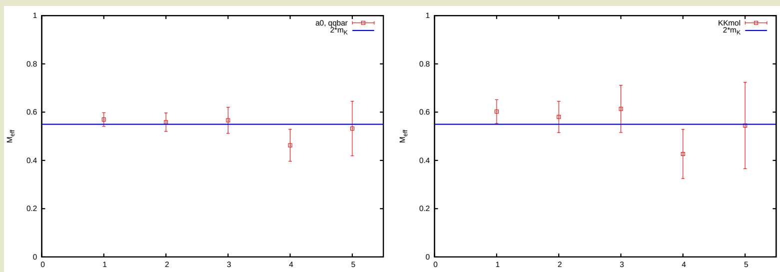
- Technical aspects of computing singly disconnected diagrams:
  - Blue: point-to-all propagators applicable.
  - Red: due to $\sum_{\mathbf{x}}$, timeslice-to-all propagators needed.
  - Timeslice-to-all propagators can be estimated stochastically.
  - Using several stochastic timeslice-to-all propagators results in a poor signal-to-noise ratio.

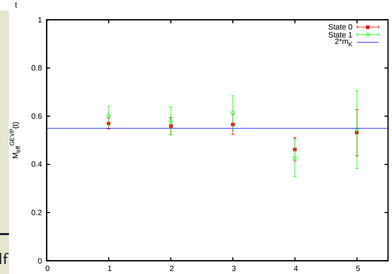
  → Combine three point-to-all (blue) and one stochastic timeslice-to-all (red) propagator.



Singly disconnected diagrams (3)

- Effective masses from a 2×2 correlation matrix ($\mathcal{O}_{a_0(980)}^{q\bar{q}}$ and $\mathcal{O}_{a_0(980)}^{K\bar{K}\text{ molecule}}$):
  - Lowest (two) energy level(s) consistent with $K + K$, $\eta + \pi$ and a possibly existing additional $a_0(980)$ state.
  - For physically interesting statements we also need to include $\mathcal{O}_{a_0(980)}^{\text{diquark}}$, $\mathcal{O}_{a_0(980)}^{K+\bar{K}\text{ two-particle}}$ and $\mathcal{O}_{a_0(980)}^{\eta_s+\pi\text{ two-particle}}$ (work in progress).





Outlook

- Enlarge correlation matrices such that
  - $q\bar{q}$ operators,
  - tetraquark operators (mesonic molecules, diquark-antidiquark pairs),
  - two-meson operators

  are included.
- Perform computations at pion mass $m_\pi \approx 150\,\text{MeV}$.
- Address various physical questions/systems (tetraquark candidates with different flavor structure, search for additional bound states, ...).