1st or 2nd; the order of finite temperature phase transition of Nf=2 QCD from effective theory analysis

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with
Sinya Aoki (Kyoto University)
Hidenori Fukaya (Osaka University)

Introduction

- Chiral symmetry in QCD
  - Broken in two different ways

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$$

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Spontaneous breaking

[Nambu 1961]

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Anomaly (explicit breaking)

[Adler 1969, Bell, Jackiw 1969]

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Anomaly

(explicit breaking)

[Adler 1969, Bell, Jackiw 1969]

$$SU(N_f)_V \times U(1)_V$$

We have shown a possibility of BOTH restoration at the SAME temperature.

(Aoki, Fukaya, Taniguchi; PRD 86, 114512)

Introduction

As was discussed by Pisarski and Wilczek; PRD 29 (1984) 338

Restoration of $U(1)_A$ ⟷ Order of phase transition

- For Nf=2 QCD
- Phase transition is first order if $U(1)_A$ is restored

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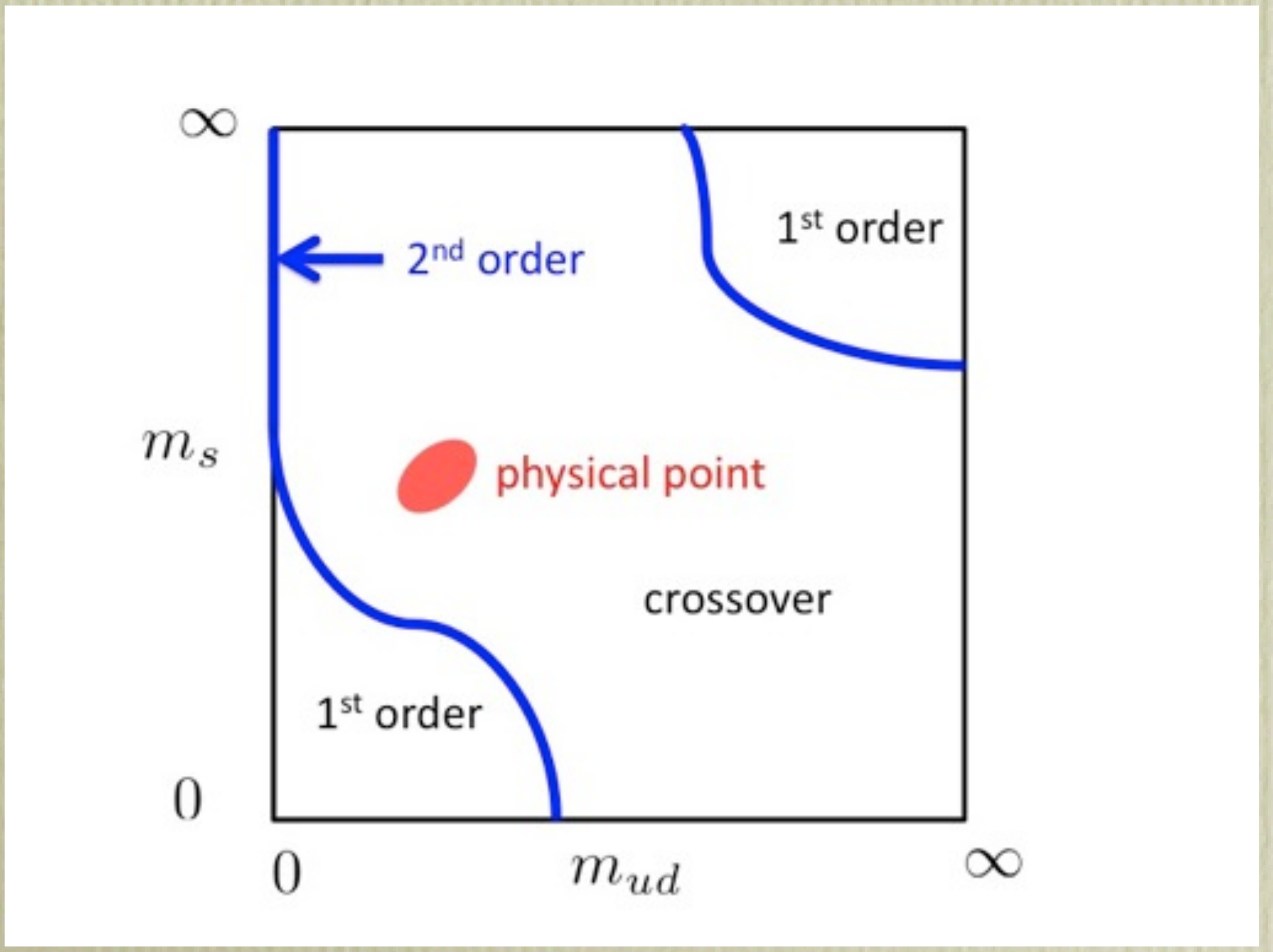

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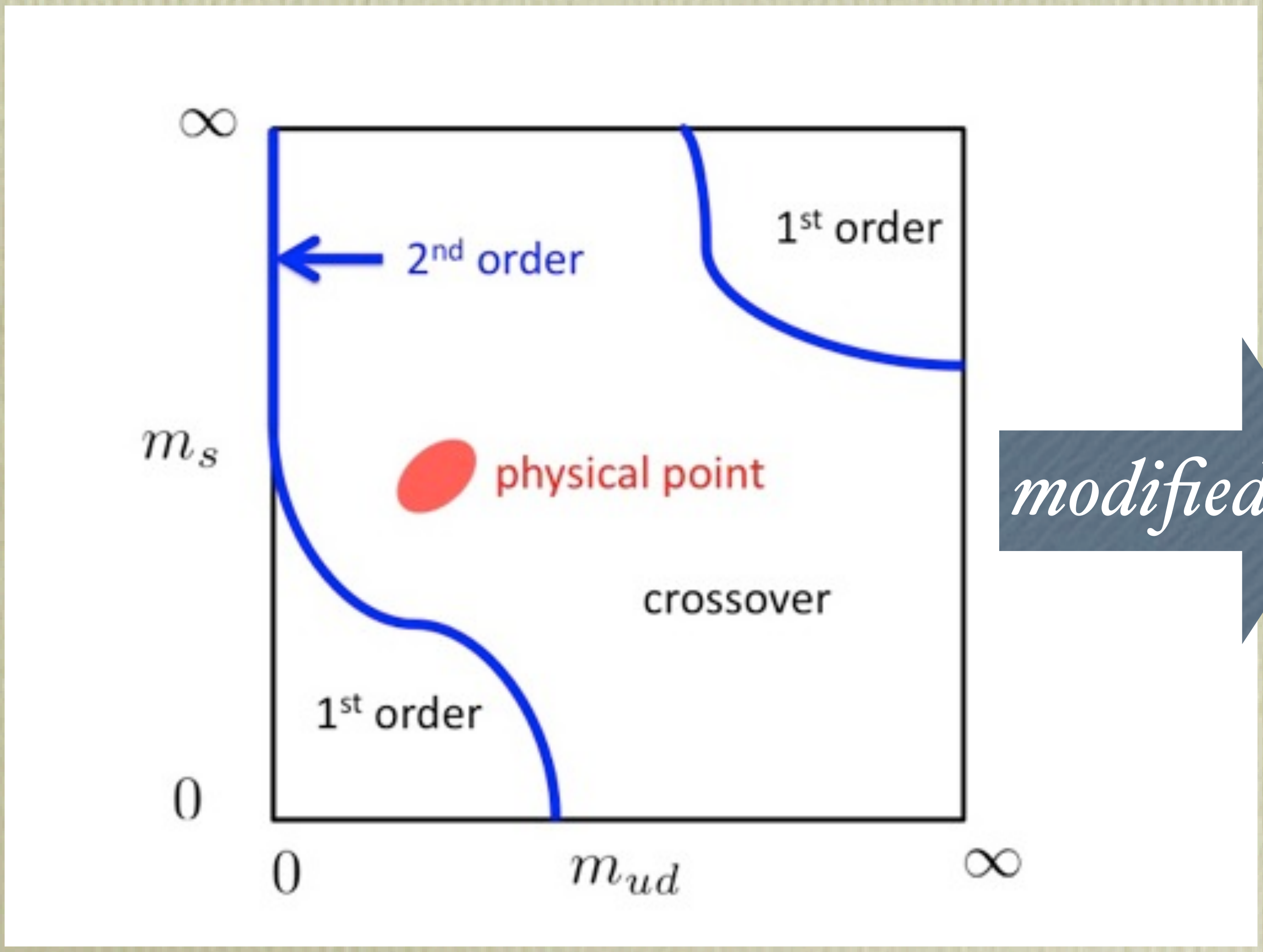

modified?

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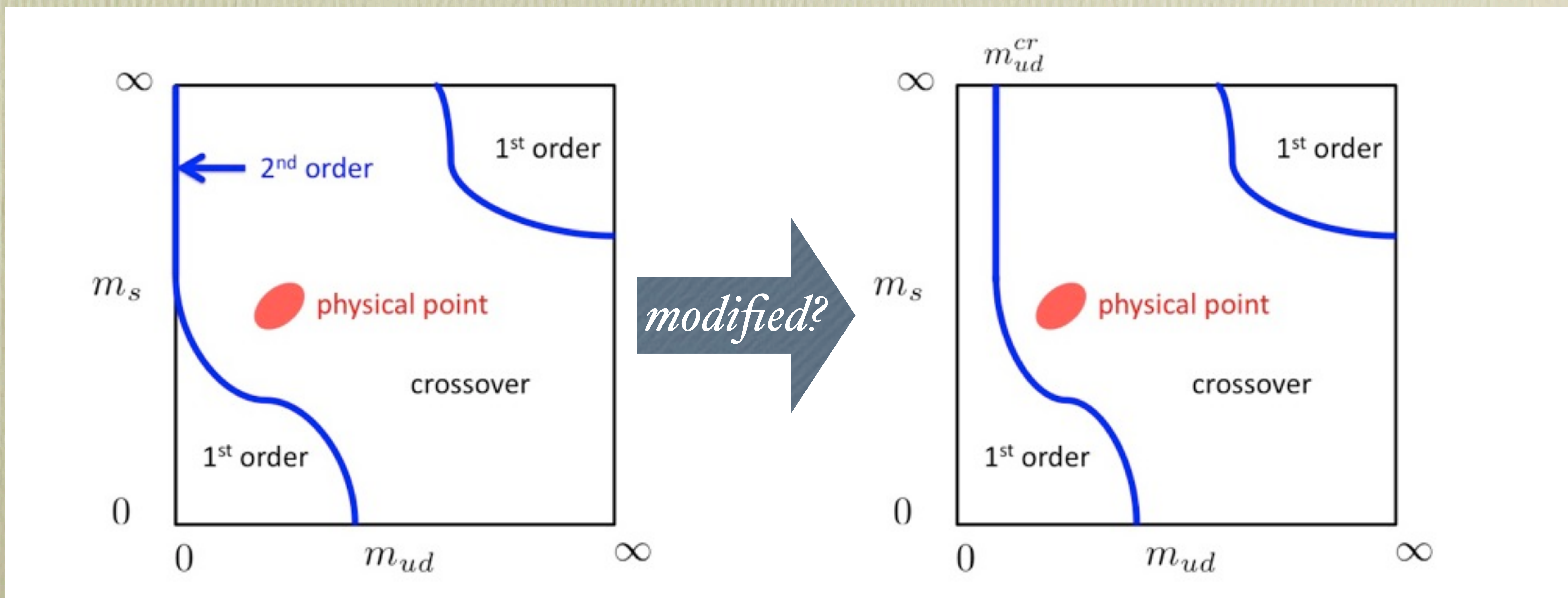


Plan of the talk

✔ 1. Introduction

2. Previous works

3. Effective theory

4. Renormalization group analysis

5. Conclusion

Previous works on $U(1)_A$ restoration

There so many works.

Previous works on U(1)A restoration

There so many works.

Cohen (1996) : YES.

U(1)A is restored above Tc

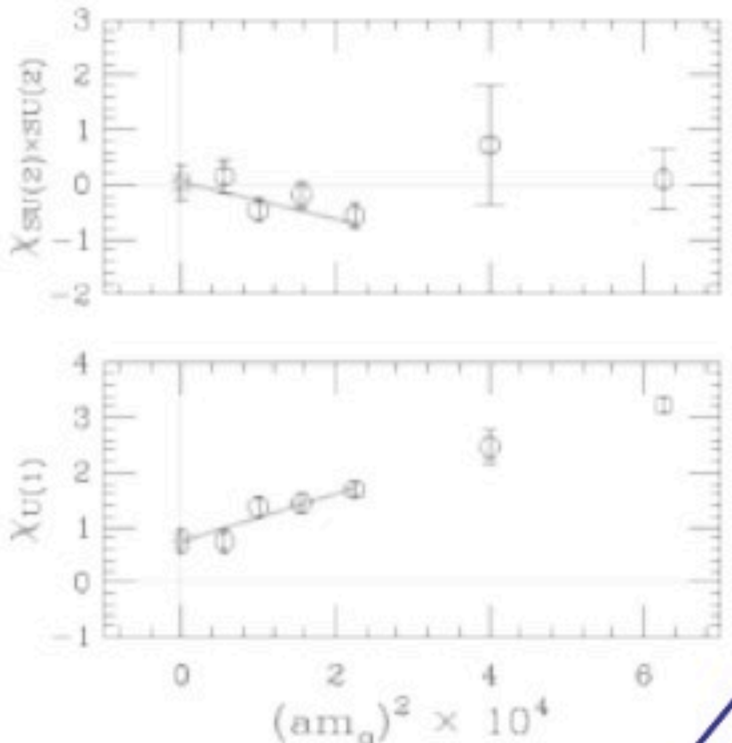
Lee & Hatsuda (1996) : NO.

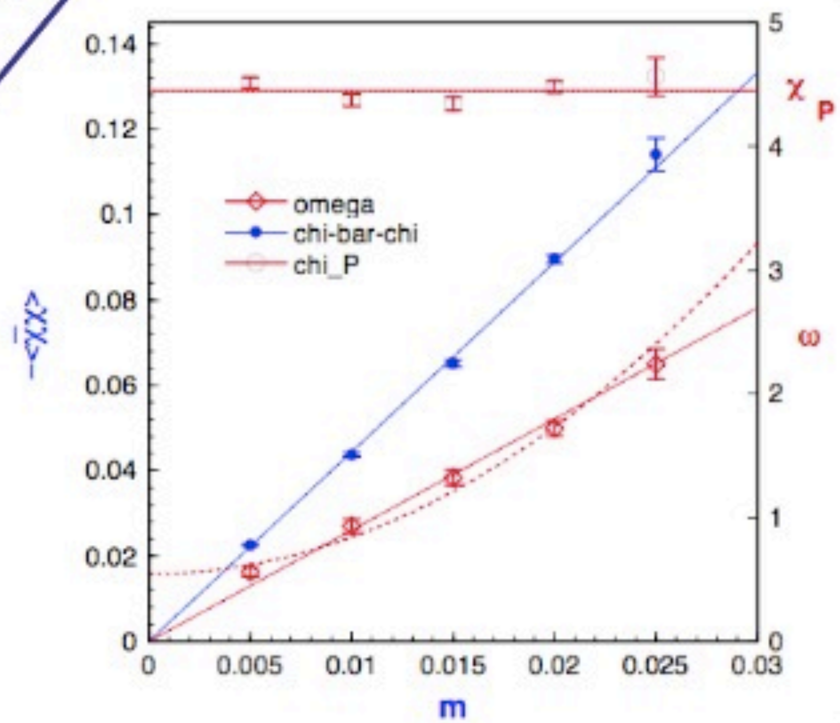
Q=±1 instanton sector does break U(1).

Previous works on U(1)A restoration

There so many works.

- Bernard *et al.* (1996): NO.
  with staggerd fermions.



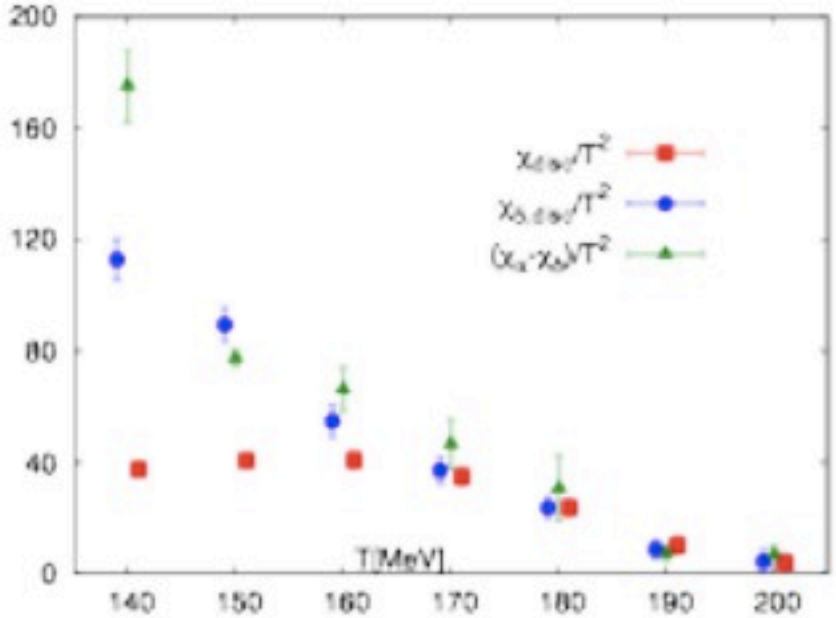


Chandrasekharan *et al.* (1998) : NO.
with staggerd fermions.

Previous works on $U(1)_A$ restoration

There so many works.

HotQCD (2011) : NO.
with domain-wall fermions.



Ohno *et al.* (2011) : NO.
with HISQ
(highly
improved
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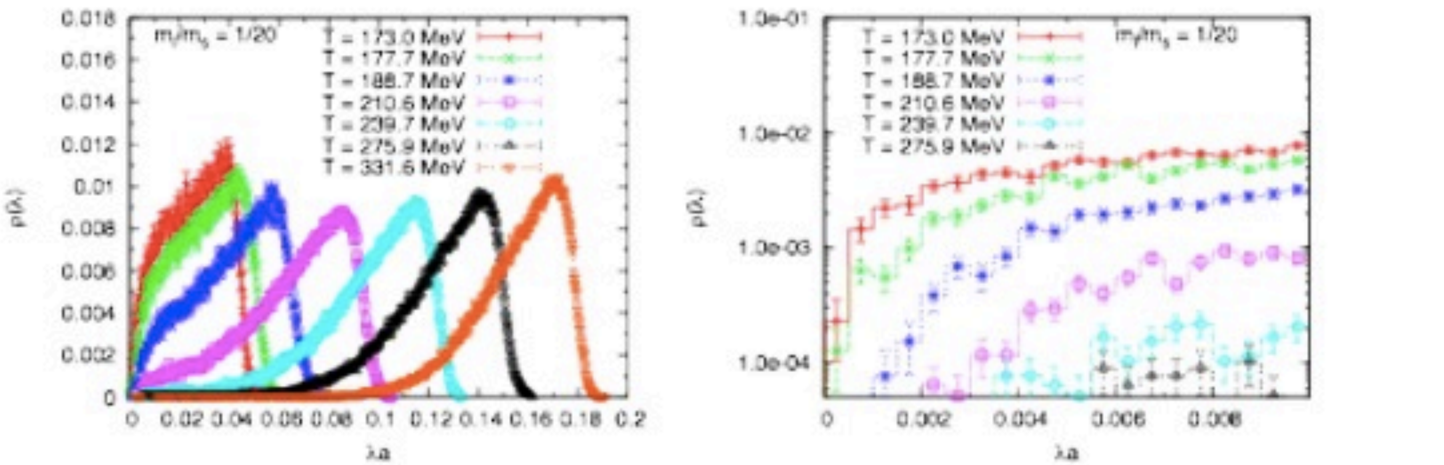

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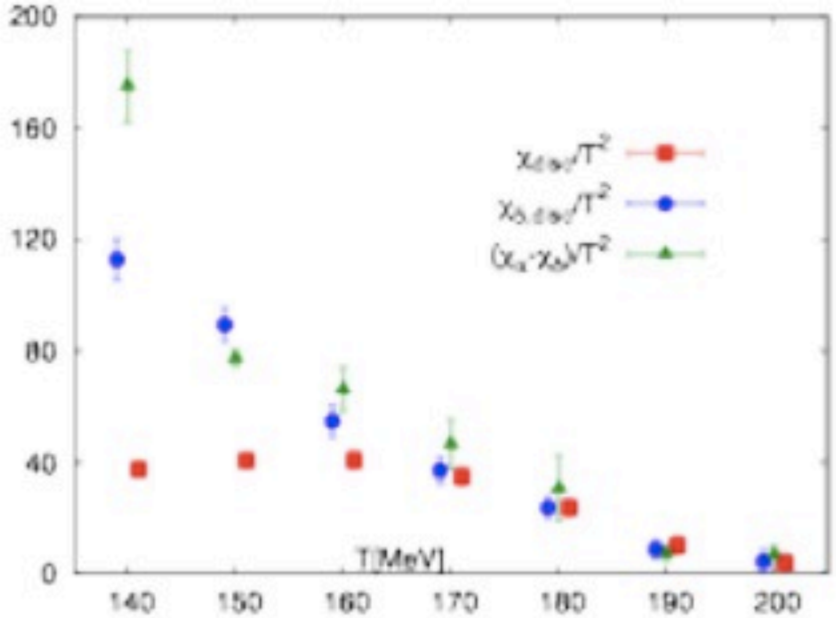
Chris Schroeder (1A)



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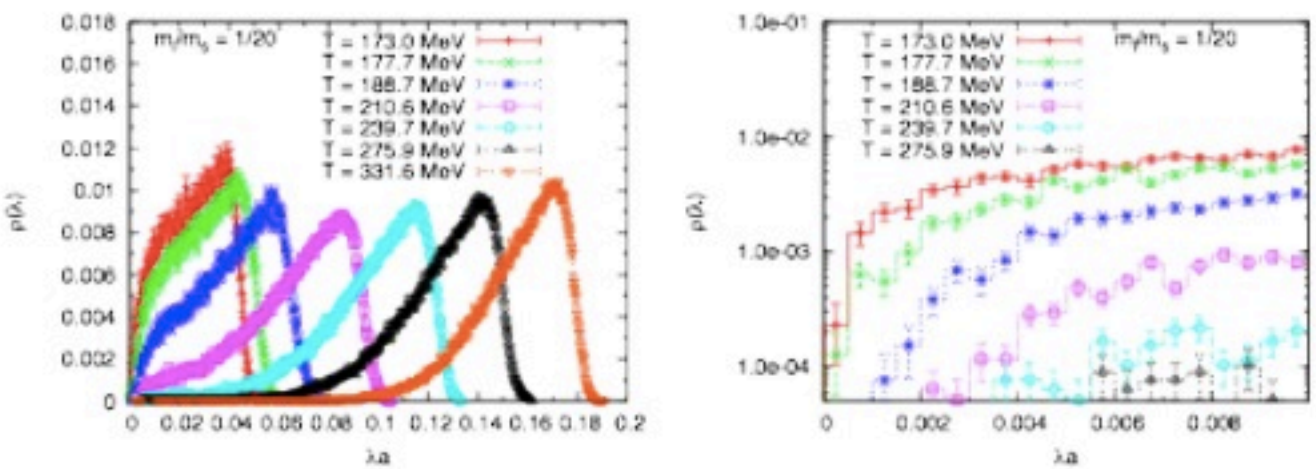

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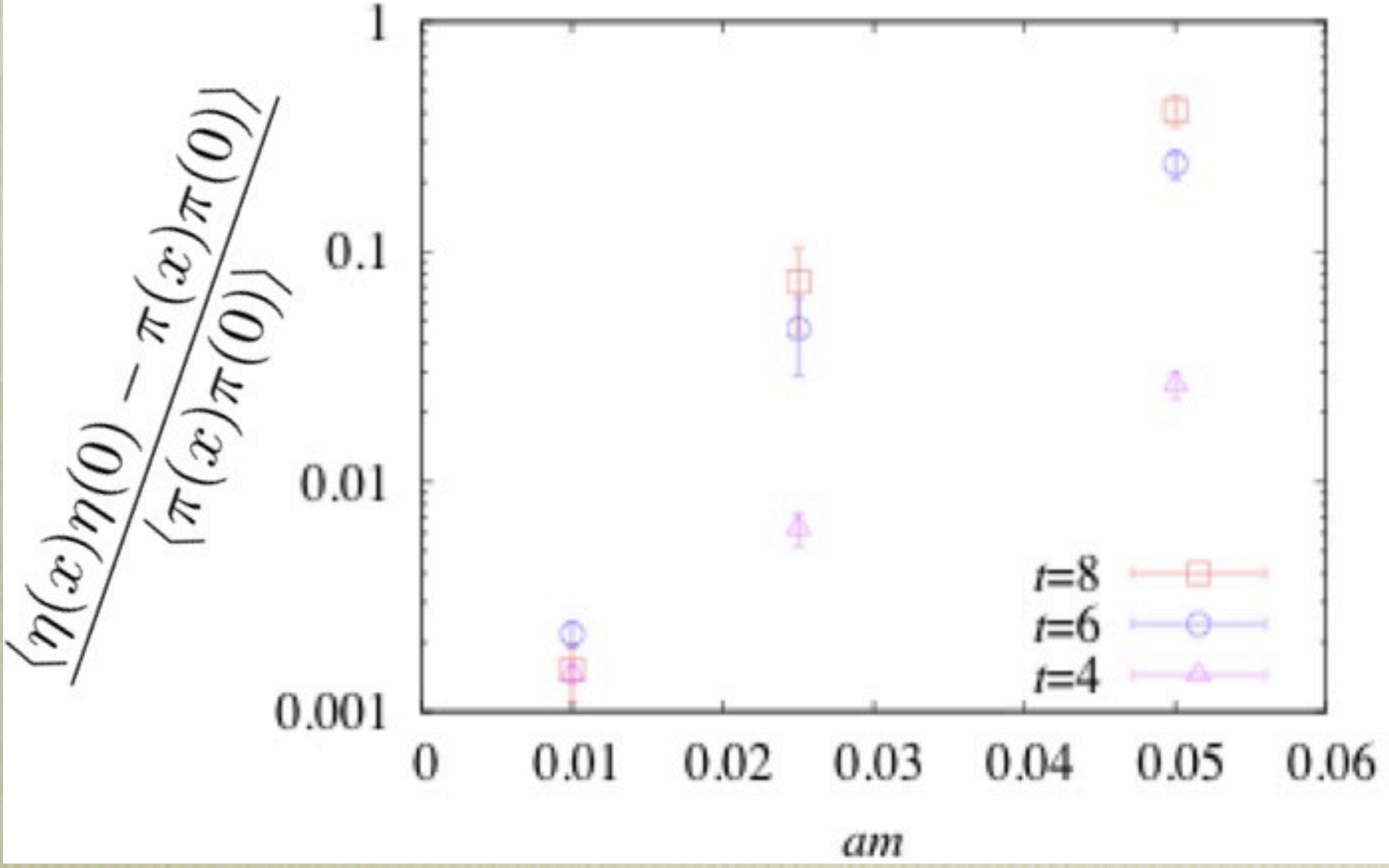

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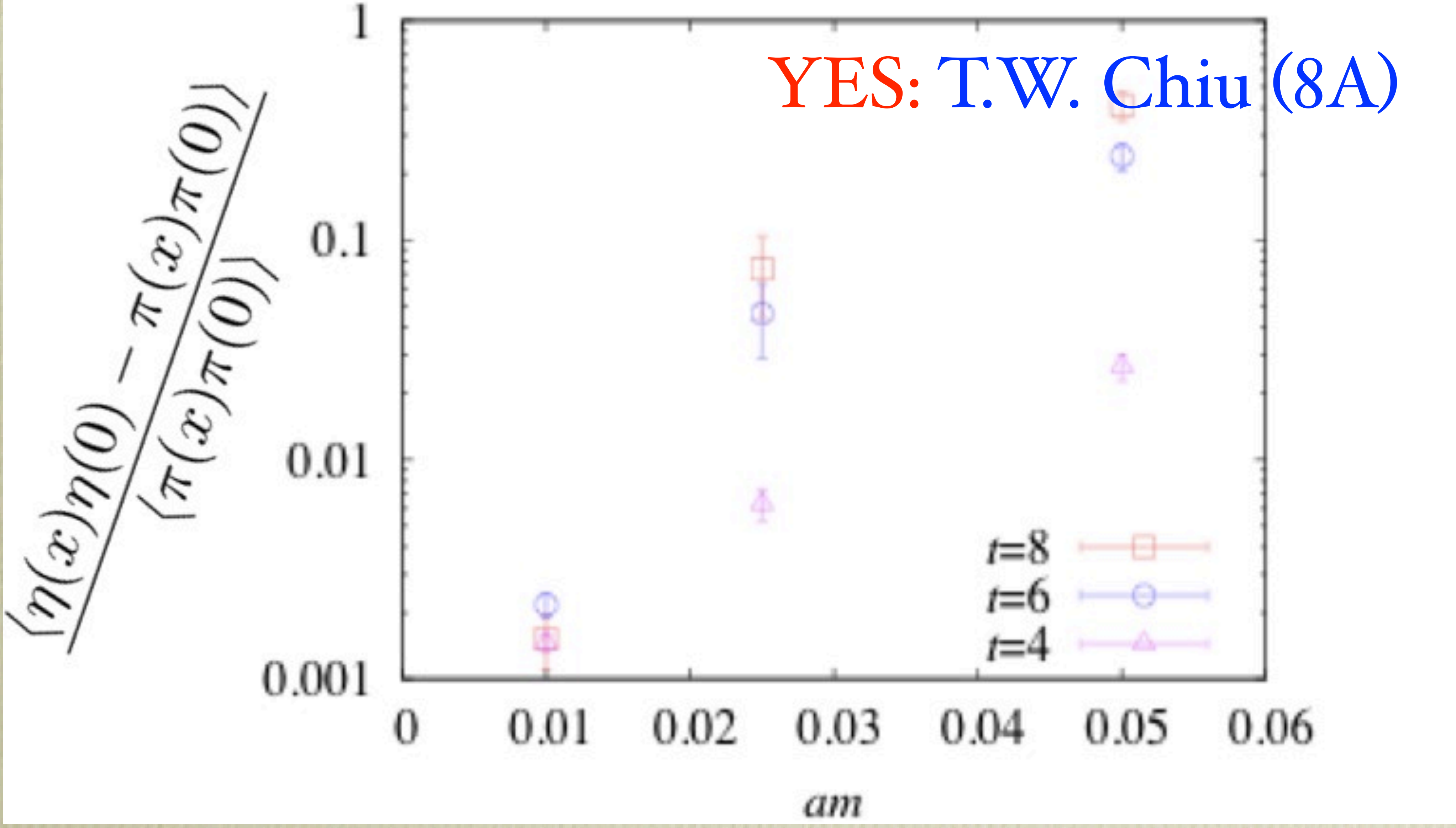

Previous works on $U(1)_A$ restoration

	U(1) restoration	instanton effect	exact chiral sym.	$V \to \infty$
Cohen	YES	×	○	○
Lee-Hatsuda	NO	○	○	×
staggered	NO	○	×	×
DFW	NO	○	△	×
overlap	YES	○	○	×
Our work	YES	○	○	○

Our previous work

- The idea:
  Eigenvalue spectrum of Dirac operator may link SU(2) SSB and U(1) anomaly

$SU(2)_L \times SU(2)_R$ breaking/restoration

Banks-Casher relation

(near) zero mode spectrum of Dirac operator

index theorem

$U(1)_A$ breaking/restoration

Our previous work

We assume: $SU(2)_L \times SU(2)_R$ is fully recovered above Tc

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Order parameter of $SU(2)_L \times SU(2)_R$

$$\frac{1}{V^k}\langle \delta^a \mathcal{O}_{n_1,n_2,n_3,n_4} \rangle = 0$$

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We assume: SU(2)$_L$×SU(2)$_R$ is fully recovered above Tc

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δ^a :
SU(2)$_A$ tr.

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non-singlet, parity odd operator

$$\mathcal{O}_{n_1,n_2,n_3,n_4} = (P^a)^{n_1}(S^a)^{n_2}(P^0)^{n_3}(S^0)^{n_4}$$

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a=1,2,3: SU(2) triplet
(sum is not taken on a)

0: singlet

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a=1,2,3: SU(2) triplet 0: singlet

(sum is not taken on a)

$$P^{(a/0)} = \int d^4x \bar{q}\gamma_5\tau^{(a/0)}q(x)$$

S: scalar

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We assume: SU(2)L×SU(2)R is fully recovered above Tc

Order parameter of SU(2)L×SU(2)R

$$\frac{1}{V^k}\langle \delta^a \mathcal{O}_{n_1,n_2,n_3,n_4}\rangle = 0$$

k: minimum number to make the VEV finite

non-singlet, parity odd operator

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constraint ↓

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- Eigenvalue density of Dirac operator

$$\rho^A(\lambda) = \lim_{V\to\infty} \frac{1}{V} \sum_n \delta\left(\lambda - \sqrt{\bar{\lambda}_n^A \lambda_n^A}\right) = \sum_{n=0}^{\infty} \rho_n^A \frac{\lambda^n}{n!}$$

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$$\rho_0^A,\ \rho_1^A,\ \rho_2^A \to 0$$

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- Number of zero mode $N_{R+L}(A)$, topological charge $Q(A)$

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constraint

we have shown

U(1)A order parameter

$$\frac{1}{V^{k'}}\langle \delta^0 \mathcal{O}'_{n_1,n_2,n_3,n_4}\rangle = 0$$

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✔ 2. Previous works

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Effective theory of Nf=2 QCD

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- $SU(2)_L\times SU(2)_R\times U(1)_A$ symmetric Lagrangian

$$\mathcal{L}_0 = \mathrm{tr}\left(\partial_\mu \Phi^\dagger \partial_\mu \Phi\right) + m_\Phi^2 \mathrm{tr}\Phi^\dagger\Phi + \frac{\lambda_1}{2}\left(\mathrm{tr}\Phi^\dagger\Phi\right)^2 + \frac{\lambda_2}{2}\mathrm{tr}\left(\Phi^\dagger\Phi\right)^2$$

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$$\mathcal{L}_{\mathrm{U}(1)_A} = c'\left(\det\Phi + \det\Phi^\dagger\right) + \frac{x}{4}\left(\mathrm{tr}\Phi^\dagger\Phi\right)\left(\det\Phi + \det\Phi^\dagger\right) + \frac{y}{4}\left((\det\Phi)^2 + \left(\det\Phi^\dagger\right)^2\right)$$

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- U(1)A breaking parameters: c', x, y

Constraint on c', x, y

Constraint on c', x, y

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example: $n_1+n_2+n_3+n_4=1$

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No constraint on c', x, y

Constraint on c', x, y

$$n_1 + n_2 + n_3 + n_4 = 2$$

Constraint on c', x, y

$$n_1 + n_2 + n_3 + n_4 = 2$$

$$\chi^{\eta-\sigma} = \frac{1}{2V^2}\langle \delta^0 \left(P^0 S^0\right)\rangle = \frac{1}{V^2}\langle P^0 P^0 - S^0 S^0 \rangle$$

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leading

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leading $\Rightarrow$

$$\chi^{\eta-\sigma} = -\left(\frac{1}{V}\langle S_0 \rangle\right)^2 + \mathcal{O}\left(\frac{1}{V}\right) = 0$$

$$\chi^{\pi-\delta} = \frac{1}{2V}\langle \delta^0 \left(P^a S^a\right)\rangle = \frac{1}{V}\langle P^a P^a - S^a S^a \rangle$$

Constraint on c', x, y

$$n_1 + n_2 + n_3 + n_4 = 2$$

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leading $\Rightarrow$ $$\chi^{\eta-\sigma} = -\left(\frac{1}{V}\langle S_0\rangle\right)^2 + \mathcal{O}\left(\frac{1}{V}\right) = 0$$

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$\chi^{\pi-\delta} = 0$ *constraint* → $c' = 0$

Constraint on c'

- One loop correction

$$\langle \pi^a \pi^a (p=0) \rangle = \frac{1}{m_\Phi^2 + \delta m_\Phi^2 + c' + \delta c'}$$

$$\langle \xi^a \xi^a (p=0) \rangle = \frac{1}{m_\Phi^2 + \delta m_\Phi^2 - (c' + \delta c')}$$

$\chi^{\pi-\xi} = 0$ → *constraint* → $c'_{\mathrm{R}} = c' + \delta c' = 0$

fine tuning of bare parameter c'

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No constraint on x, y

Effective theory of Nf=2 QCD

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$$\mathcal{L}_0 = \mathrm{tr}\left(\partial_\mu \Phi^\dagger \partial_\mu \Phi\right) + m_\Phi^2 \mathrm{tr}\Phi^\dagger\Phi + \frac{\lambda_1}{2}\left(\mathrm{tr}\Phi^\dagger\Phi\right)^2 + \frac{\lambda_2}{2}\mathrm{tr}\left(\Phi^\dagger\Phi\right)^2$$

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$$\mathcal{L}_{\mathrm{U}(1)_A} = \frac{x}{4}\left(\mathrm{tr}\Phi^\dagger\Phi\right)\left(\det\Phi + \det\Phi^\dagger\right) + \frac{y}{4}\left(\left(\det\Phi\right)^2 + \left(\det\Phi^\dagger\right)^2\right)$$

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- No mass splitting between π and η
  - all mesons contribute to the running
- The interaction has $U(1)_A$ breaking effect

Plan of the talk

✔ 1. Introduction

✔ 2. Previous works

✔ 3. Effective theory

4. Renormalization group analysis

5. Conclusion

Renormalization group analysis

Wilson and Kogut, Zinn-Justin, Pisarski and Wilczek

- If the phase transition is second order at Tc
  - ★ correlation length $\xi \to \infty$
- Long range mode dominates at Tc
- Physics is described by the theory in 3D
- Theory in 3D should have stable IR fixed point
- Instead of 3D we adopt ε -expansion analysis

Renormalization group analysis

4 couplings $g_i = \{\lambda_1, \lambda_2, x, y\}$

β function $\beta_{g_i} = \mu \frac{\partial}{\partial \mu} g_i$

Dimensional regularization d=4- ε and MS scheme

$$g_i = \mu^{-\epsilon} Z_{g_i}^{-1} g_{i0}$$

β function $\beta_{g_i} = -\epsilon g_i + g_i \frac{\partial \ln Z_{g_i}}{\partial \left(\frac{1}{\epsilon}\right)}$

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$$\beta_{\lambda_1} = -\epsilon\lambda_1 + \frac{1}{8\pi^2}\left(8\lambda_1^2 + 8\lambda_1\lambda_2 + 3\lambda_2^2 + \frac{3}{2}x^2 + \frac{5}{4}y^2\right)$$

$$\beta_{\lambda_2} = -\epsilon\lambda_2 + \frac{1}{8\pi^2}\left(4\lambda_2^2 + 6\lambda_1\lambda_2 - \frac{3}{4}x^2 - y^2\right)$$

$$\beta_x = -\epsilon x + \frac{1}{8\pi^2}x\left(12\lambda_1 + 6\lambda_2 + 3y\right)$$

$$\beta_y = -\epsilon y + \frac{1}{8\pi^2}\left(6\lambda_1 y + \frac{3}{2}x^2\right)$$

Renormalization group analysis

- Fixed points of β-function $(\lambda_1, \lambda_2, x, y)$

Fixed points	*Property*
(0,0,0,0)	UV FP
$\epsilon\pi^2$(1,0,0,0)	saddle point
$\epsilon\pi^2/3$(4,-2,0,-4)	saddle point
$\epsilon\pi^2/3$ (4,-2,0,4)	saddle point
$\epsilon\pi^2/3$(2,-1,-4,2)	saddle point
$\epsilon\pi^2/3$ (2,-1,4,2)	saddle point

No stable IR FP found!

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U(1)A order parameter vanishes $\frac{1}{V^k}\langle\delta^0\mathcal{O}_{n_1,n_2,n_3,n_4}\rangle = 0$

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Constraint on effective theory

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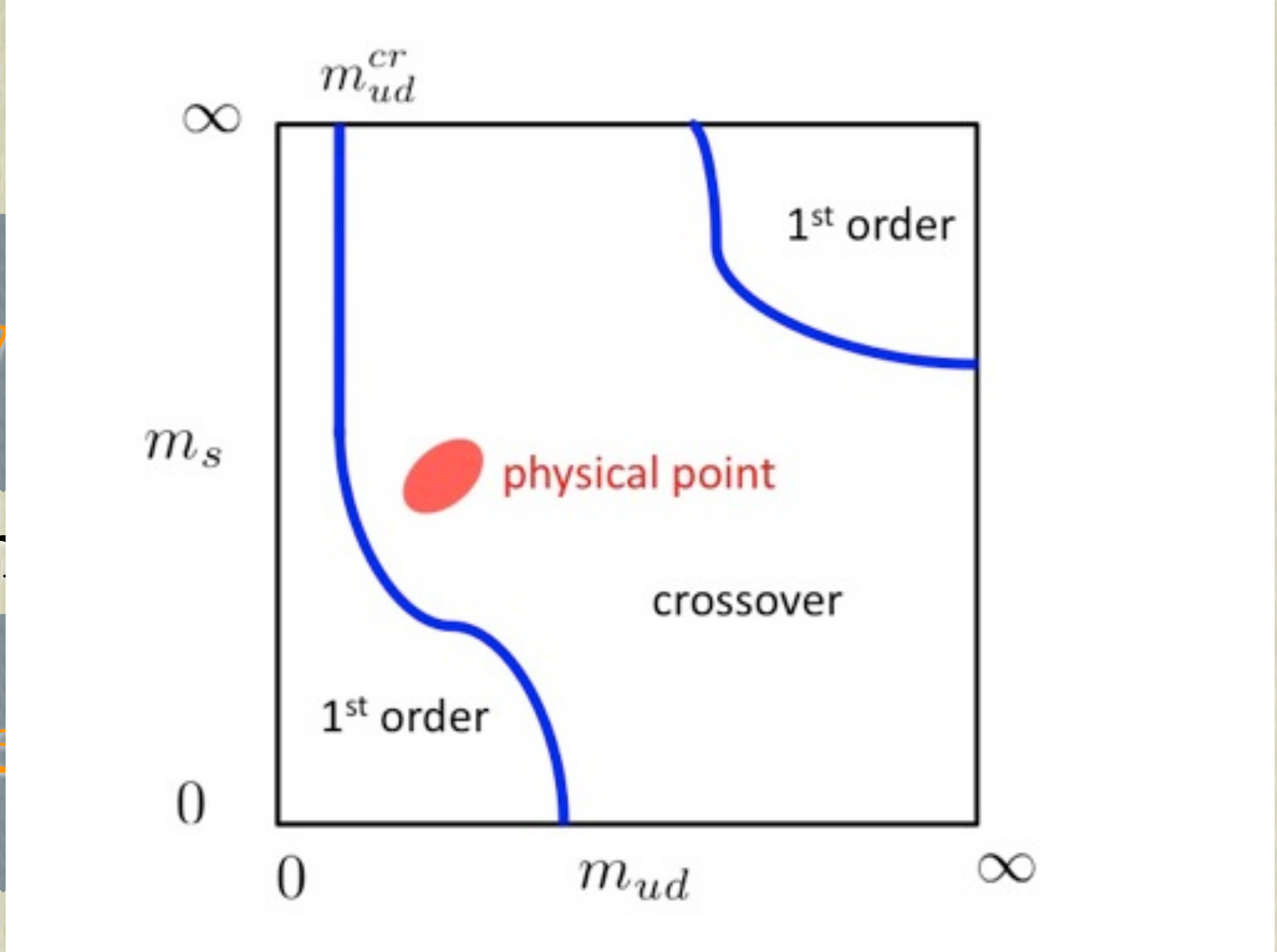

Eigenvalue density

U(1)A order parameter vanish

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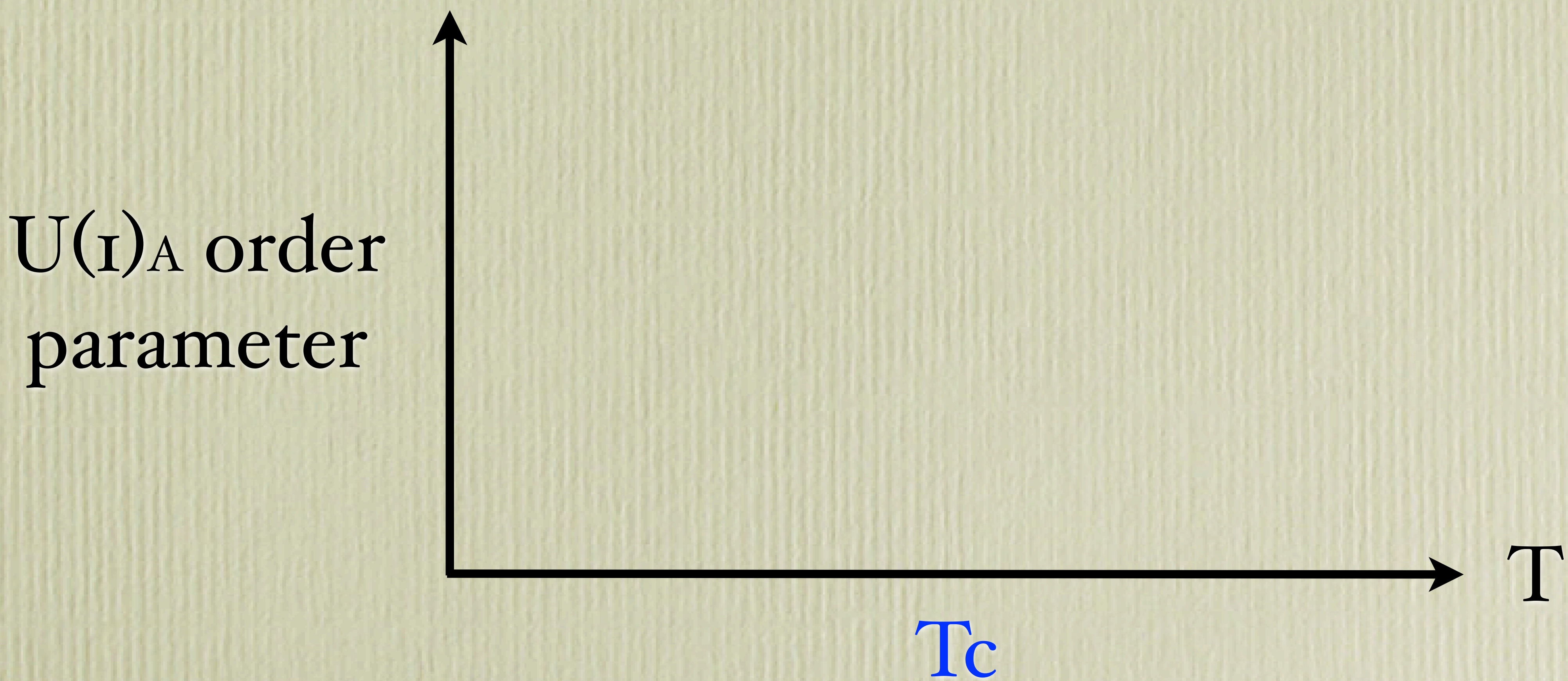
Non-perturbative scenario?

Non-perturbative β-function and $\varepsilon \to 1$

1. No stable IR FP is found → first order
2. Stable IR FP is found at $x=0$
   - Lagrangian acquire Z_4 symmetry
   - Second order but $O(4) \times Z_4$ universality class
   - $c'=0$ is trivial by the symmetry
3. Stable IR FP is found at non-zero x
   - Second order and $O(4)$ universality class
   - Fine tuning is needed for $c'=0$

2nd order? Really?

If SU(2)xSU(2) phase transition is 2nd order:



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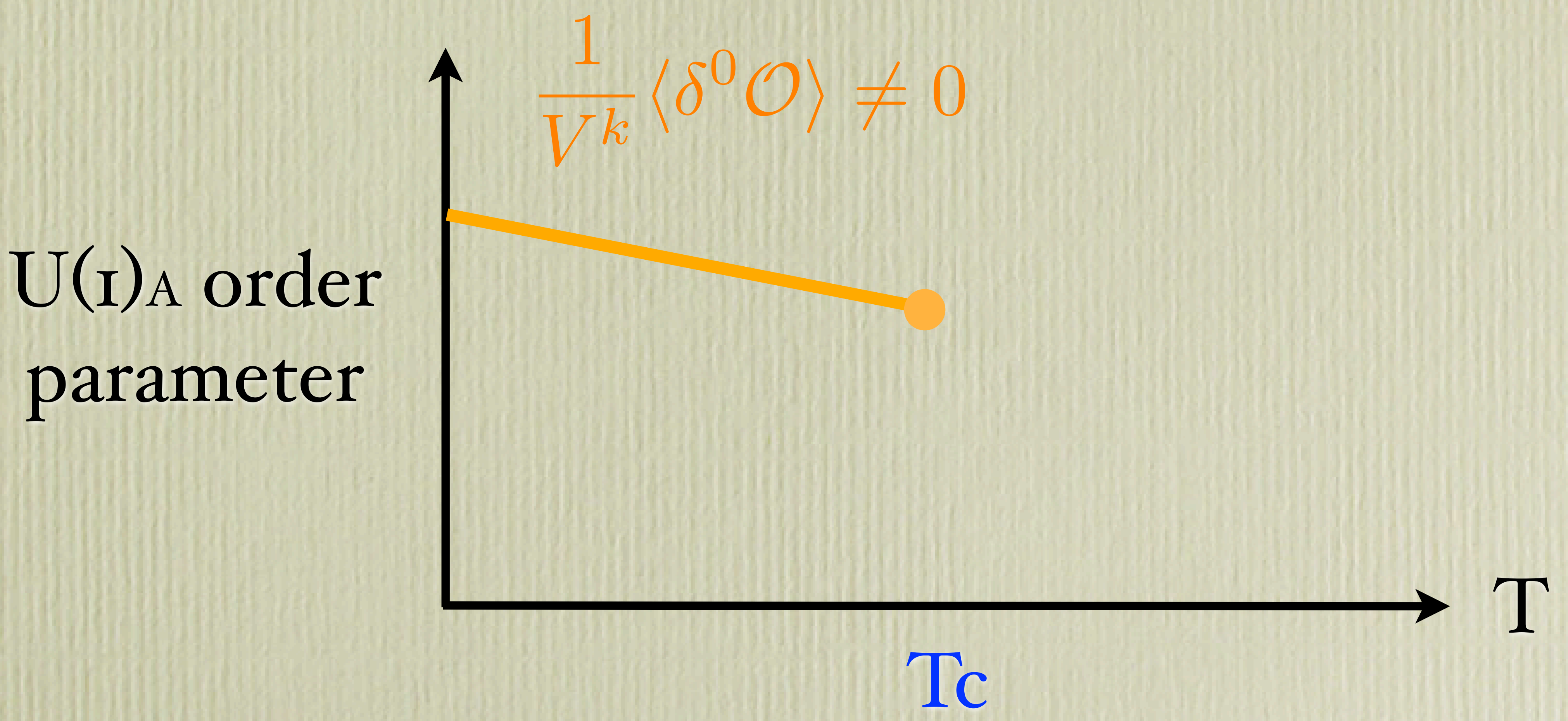

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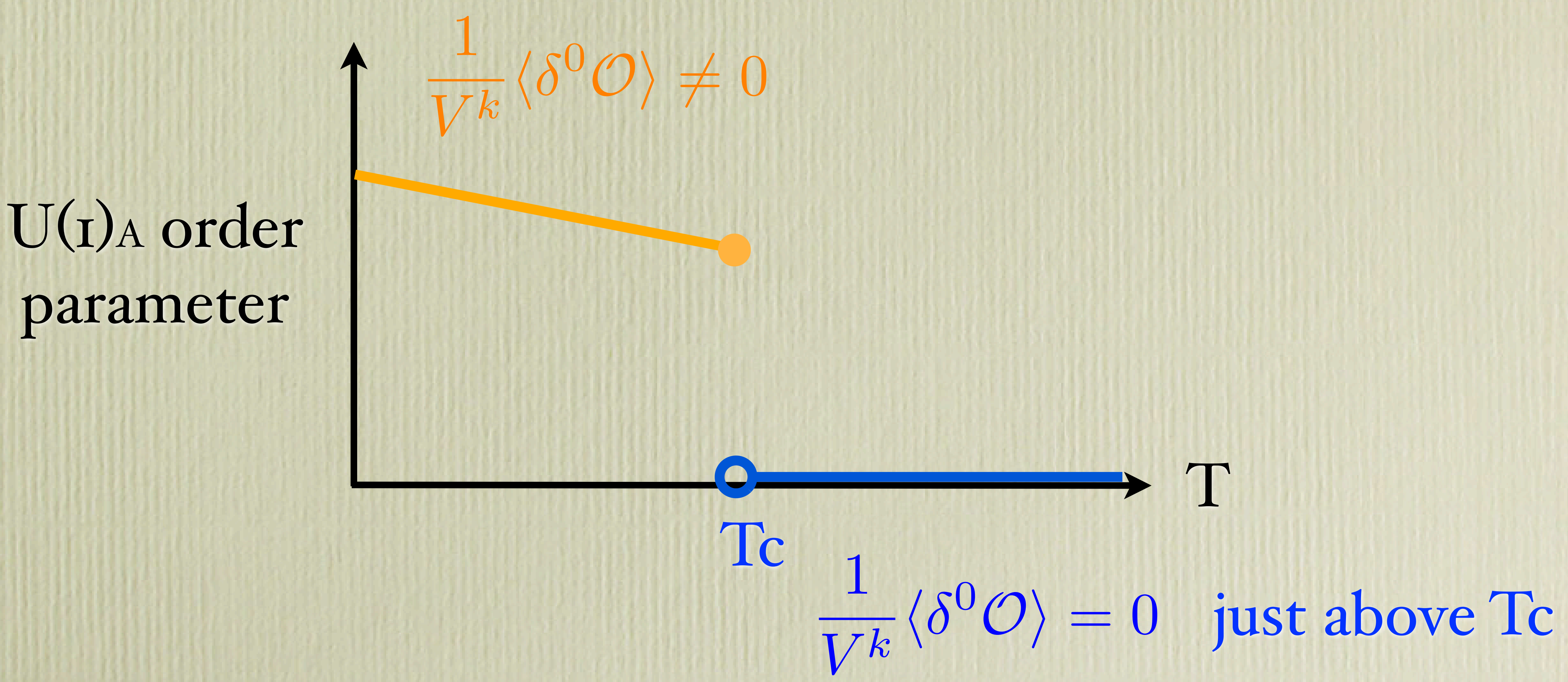

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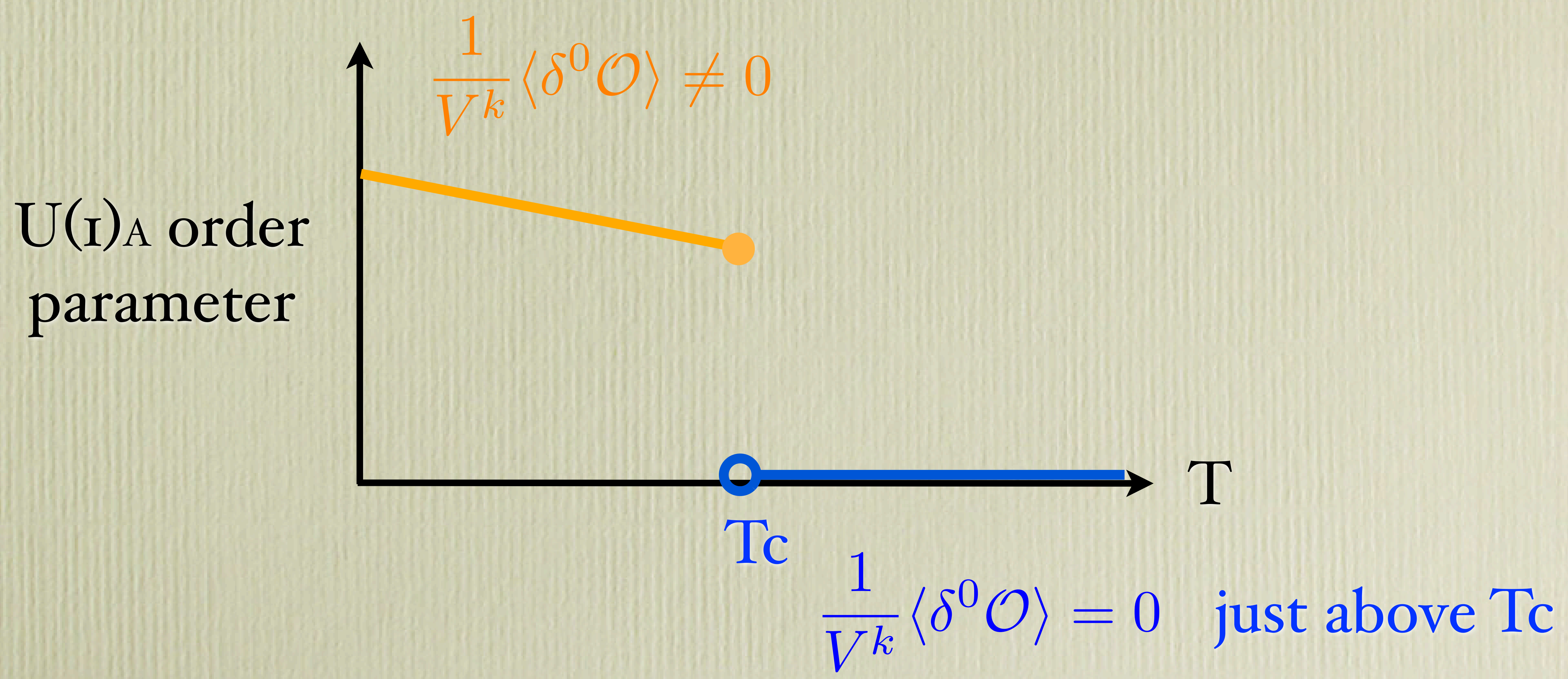

U(1)A restoration seems like first order

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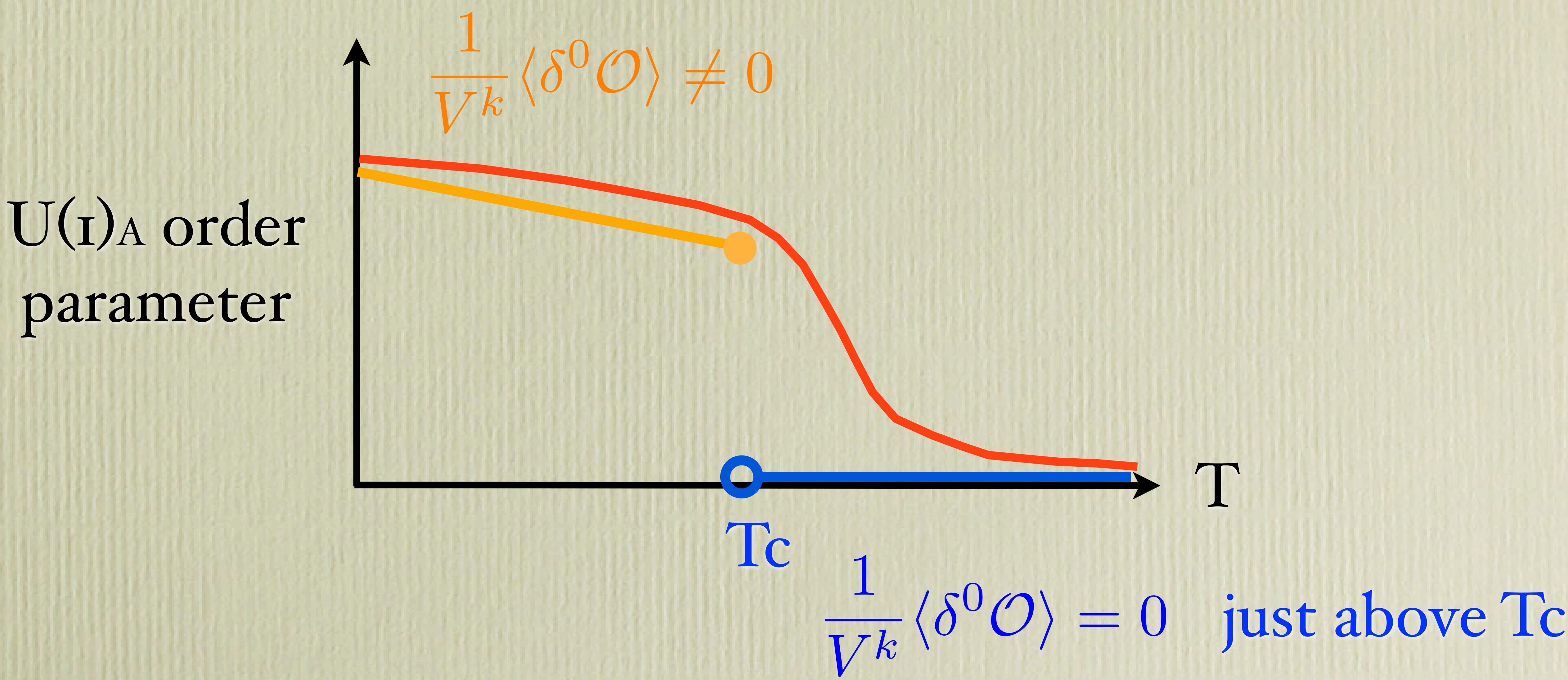

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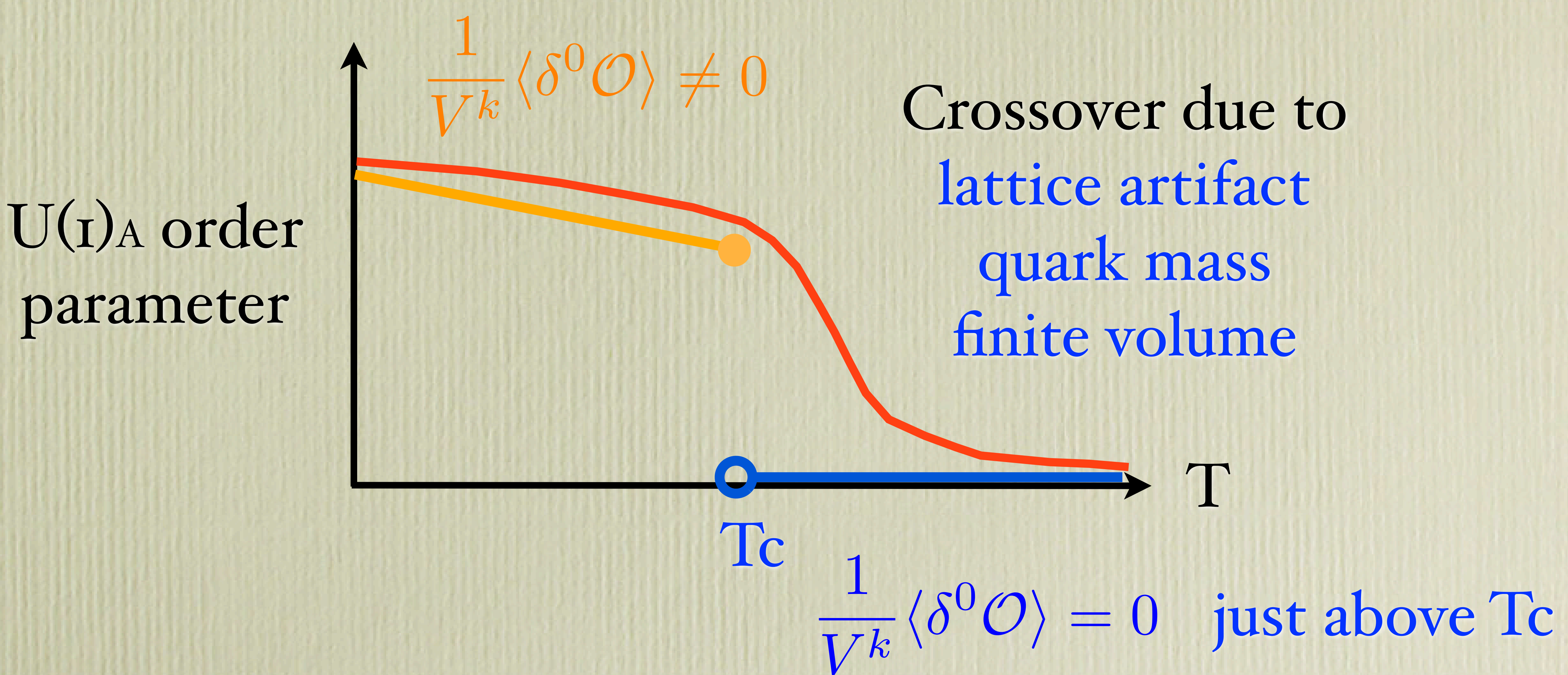

Crossover due to
lattice artifact
quark mass
finite volume

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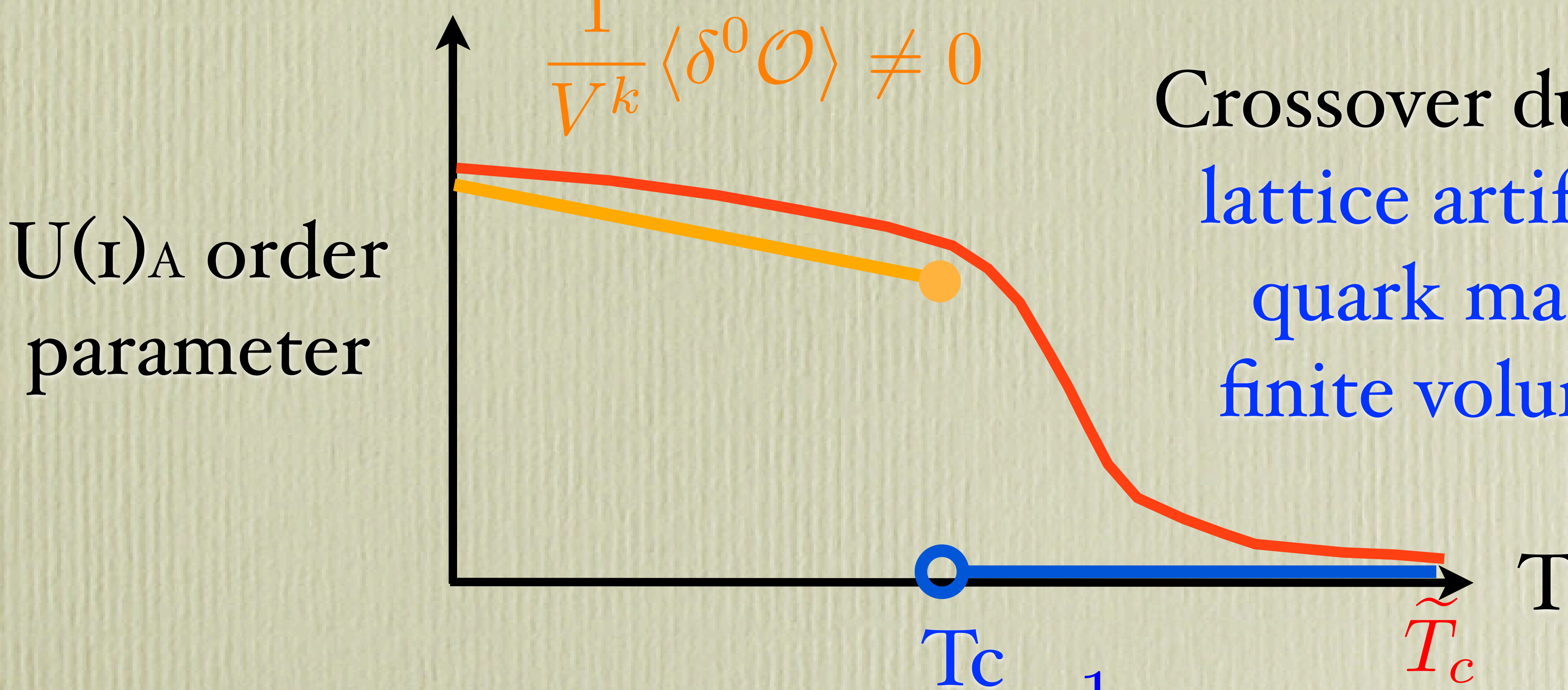


Crossover due to
lattice artifact
quark mass
finite volume

$\frac{1}{V^k}\langle\delta^0\mathcal{O}\rangle = 0$ just above Tc

U(1)A restoration seems like first order

If you can read this
I am on the wrong page.

Non-singlet and singlet chiral susceptibility

Ward-Takahashi identity

$$\frac{m}{V}\langle P^0 P^a S^a\rangle = \frac{1}{V}\langle P^0P^0 - S^aS^a\rangle = \chi^{\eta-\delta} \quad \text{(non-singlet)}$$

$$= \frac{1}{V}\langle P^aP^a - S^aS^a\rangle - 4\langle\frac{Q^2}{m^2V}\rangle$$

$\chi^{\pi-\delta}$ (singlet) — topological charge

4 point non-singlet order parameter

$$\frac{1}{2V^2}\delta^a\langle S^a(P^0)^3 + (S^a)^3P^0\rangle = \frac{1}{V^2}\langle(S^a)^4 - (P^0)^4\rangle = 0$$

$V\to\infty$ *then* m→0 ⇒ $$\left\langle\frac{Q^2}{m^2V}\right\rangle = 0$$

Our Assumptions

1. SU(2) x SU(2) fully recovered at Tc.

2. if $\mathcal{O}(A)$ is m-independent ,

$$\langle \mathcal{O}(A) \rangle_m = f(m^2) \qquad f(x) \text{ is analytic at } x = 0$$

3. if $\mathcal{O}(A)$ is m-independent and positive, and satisfies

$$\lim_{m \to 0} \frac{1}{m^{2k}} \langle \mathcal{O}(A) \rangle_m = 0$$

$$\Rightarrow \langle \mathcal{O}(A) \rangle_m = m^{2(k+1)} \underline{\int \mathcal{D}A\, \hat{P}(m^2, A) \mathcal{O}(A)}$$

finite

$$\Rightarrow \langle \mathcal{O}(A)^l \rangle_m = m^{2(k+1)} \int \mathcal{D}A\, \hat{P}(m^2, A) \mathcal{O}(A)^l = O(m^{2(k+1)})$$

4. $$\rho^A(\lambda) \equiv \lim_{V \to \infty} \frac{1}{V} \sum_n \delta\left(\lambda - \sqrt{\bar{\lambda}_n^A \lambda_n^A}\right) = \sum_{n=0}^{\infty} \rho_n^A \frac{\lambda^n}{n!} \quad \text{at } \lambda = 0 \ (\lambda < \epsilon\ )$$

(4 can be removed later.)

Lattice artifact?

We have used full SU(2)xSU(2) chiral symmetry. If we use non-chiral lattice fermion, the result should change.

$$\langle \rho^A(\lambda) \rangle_m = \langle \rho_3^A \rangle_0 \frac{\lambda^3}{3!} + \cdots$$

$$\langle \rho^A(\lambda) \rangle_m = \alpha m_{\mathrm{break}} \Lambda_{\mathrm{QCD}} \lambda + \beta m_{\mathrm{break}} \lambda^2 + \left(\langle \rho_3^A \rangle_0 + \gamma m_{\mathrm{break}} / \Lambda_{\mathrm{QCD}} \right) \frac{\lambda^3}{3!} + \cdots$$

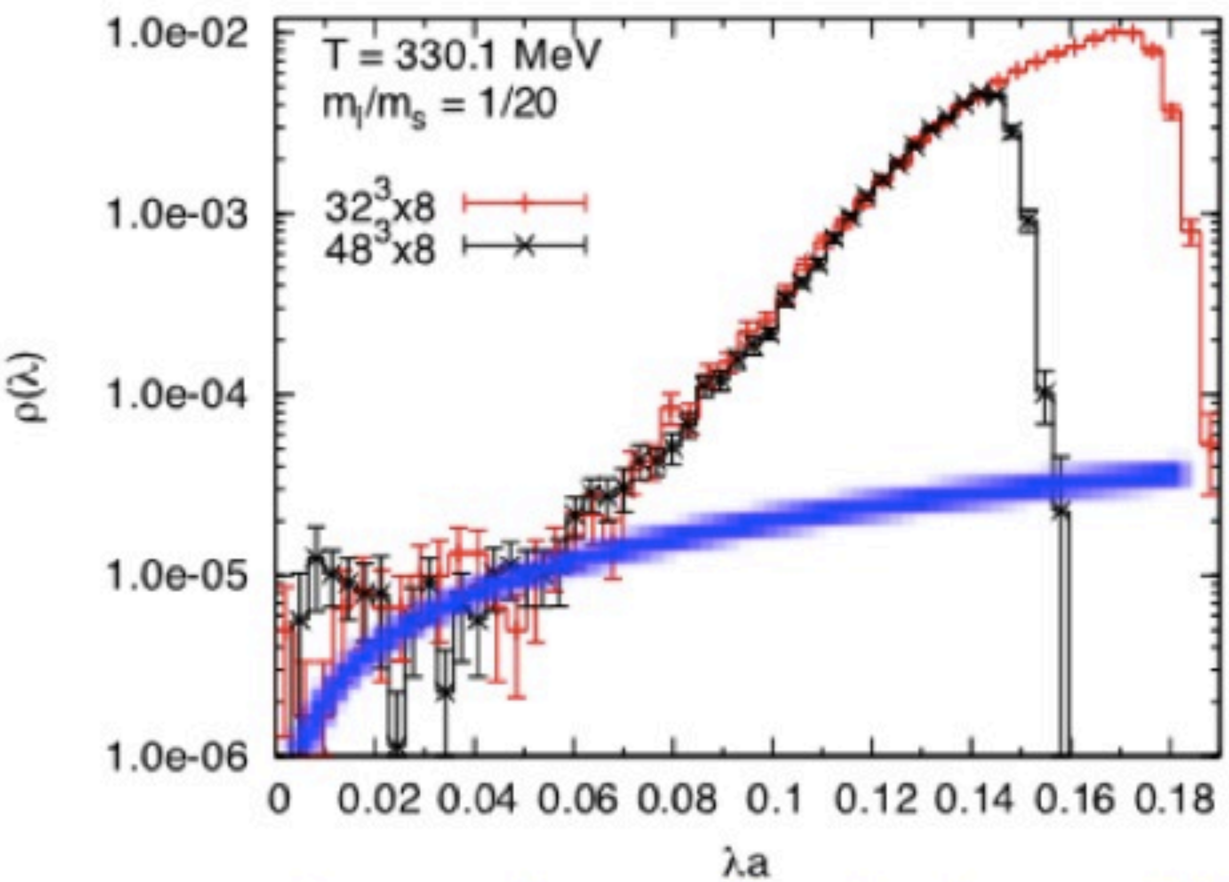
For example, staggerd fermion might have

$$m_{\mathrm{break}} \sim a^2 \Lambda_{QCD}^3$$

Lattice artifact?

Ohno et al. (2012)
$U(1)_A$ looks broken
w/ staggered fermion.

$$m_{\rm break} \sim 4{\rm MeV}$$



Our estimate for lattice artifact

Leading terms in WT identity

Ward-Takahashi identity $\frac{1}{V^k}\langle \delta^0 \mathcal{O} \rangle = \frac{4i}{V^k}\langle Q_{\text{top.}}\, \mathcal{O} \rangle$

Anomaly contribution is leading order

Need to consider the leading order term